Semantics Properties of Association Rules Based on Intuitionistic Fuzzy Special Sets

Zheng Pei
School of Mathematics & Computer, Xihua University, 610039, China
pqyz@263.net

Abstract

One of the core tasks of Knowledge Discovery in Databases (KDD) is the mining of association rules. In this paper, truth values of association rules are discussed. Firstly, two knowledge bases of association rules are fixed, *i.e.*, information system $\mathscr{A}$ and a fixed association rule (it's confidence is 1), then based on Intuitionistic fuzzy special sets (IFSS) Representation of Rough Set, IFSS representations of association rules (also called statements) are discussed based on the two knowledge bases. Finally, based on Hamming distance of IFSS, truth values of statements are obtained.

Keywords: KDD, IFSS, association rule, truth value.

1 Introduction

The aim of knowledge discovery in databases (KDD) is to support human analysts in the overall process of discovering valid, implicit, potentially useful and ultimately understandable information in databases. One of the core tasks of KDD is the mining of association rules (conditional implications). Association rules was stated by [1], so called association rules are statements and provide associations among attributes of information systems, generally, a real number from the interval $[0,1]$ is assigned to each association rule and provides measure of the confidence of the rule, *e.g.*, "the customers buying cereals and sugar also buy milk, the confidence the rule are 0.7." The task of mining association rules is to determine all rules whose confidences and supports are above user defined thresholds. Nowadays, various approaches have been proposed for an increased efficiency of rule discovery in very large databases [2]-[4]. In all of these mining methods, rough set is an important tool to extract association rules from information systems [5]-[10]. Formally, an information system is expressed as a quaternion denoted as $\mathscr{A}=(U,A,V,f)$, where U is a non-empty set of objects, A is a non-empty finite set of attributes, $V=\bigcup_{a\in A}V_a$ and V_a is the domain of a, $f: U\times A\longrightarrow V$ is information function. In $\mathscr{A}$, $\forall a\in A$ and $x_i,x_j\in U$, define $x_i\sim_a x_j$ if and only if $f(x_1,a)=f(x_2,a)$, then $\sim_a$ is an equivalence relation on U, and $\sim_A$ is intersection of all $\sim_a$ $(a\in A)$, denotes $U/\sim_A=\{U^r|r=1,\cdots,n\}$, where U^r is an equivalence class. For rough set, association rules are considered as follows: Let $\mathscr{A}$ be an information system and $T=D_1\wedge D_2\wedge\cdots\wedge D_k$ be a template, in which, $\forall k'\in\{1,\cdots,k\}$, $D_{k'}\equiv(a_{k'}=v_{i_{k'}a_{k'}})$, $a_{k'}\in A$ be an attribute, $v_{i_{k'}a_{k'}}\in V_{a_{k'}}$ value of the attribute. Association rules generated by T could be expressed by the form

$$\varphi\equiv\bigwedge_{D_l\in P}D_l\longrightarrow\bigwedge_{D_j\in Q}D_j, \tag{1}$$

in which, $\{P,Q\}$ is a partition of $\{D_1,D_2,\cdots,D_k\}$. The confidence of association rule

L. Magdalena, M. Ojeda-Aciego, J.L. Verdegay (eds): Proceedings of IPMU'08, pp. 1686–1692
Torremolinos (Málaga), June 22–27, 2008

φ is defined by

$$Con_{\mathscr{A}}(\varphi) = \frac{Sup_{\mathscr{A}}(T)}{Sup_{\mathscr{A}}(\bigwedge_{D_l \in P} D_l)}, \tag{2}$$

in which, $Sup_{\mathscr{A}}(*)$ means the number of objects satisfying $*$. One problem in mining association rules is the large number of rules which are usually returned. Two basic steps used in generating process are a fixed threshold c for the confidence of association rule and $c-$irreducible [10].

On the other hand, when inference based on association rules in formal logic system, *e.g.*, multi-valued logic systems, is considered, two aspects are needed: a) one is sematic; b) the other is syntax. Sematic of formal logic system discusses truth of propositions which is obtained by truth of simple statements, *e.g.*, for association rule (1), truth of φ could be obtained by truth of $\bigwedge_{D_l \in P} D_l$ and $\bigwedge_{D_j \in Q} D_j$. How to obtain truth of $\bigwedge_{D_l \in P} D_l$, or more detail, truth of $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$ which is understood as simple statement, are problem under background of an dynamic information system. It could be noticed that the confidence of association rule φ is not suitable to truth of φ, in fact, the confidence isn't even in contact with implication " $\rightarrow$ " of φ.

Due to discussion of association rules in an information system, we affirm that truth of $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$ could be decided in the information system. From rough set point of view, there exist two sources to represent and evaluate simple statement $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$, one is the information system itself which provides major premise for all statements; the other is association rule φ which provides minor premise for simple statement $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$. This is similar to hypothetical syllogism, *i.e.*, when we consider truth of simple statement $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$, we could compare representations of $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$ based major premise and minor premise, respectively, intuitively, the more in keeping with representations, the very true $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$ due to knowledge by inheritance. Based on above analysis, for a fixed information system $\mathscr{A} = (U, A, V, f)$, truth of simple statement $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$ could be obtained by finishing the follows steps:

- According to $U/\sim_A$, rough set of $\{u \in U | f(u, a_{k'}) = v_{i_{k'}a_{k'}}\}$ could be obtained, denoted as $(\underline{X_{D_{k'}}}, \overline{X_{D_{k'}}})$, based on IFSS representation of rough set, IFSS representation of $\{u \in U | f(u, a_{k'}) = v_{i_{k'}a_{k'}}\}$ could be obtained, denoted as $\langle U, \underline{X_{D_{k'}}}, U - \overline{X_{D_{k'}}}\rangle$;
- According to $U/\sim_\varphi$, where φ is decided by (1) and $\sim_\varphi$ is decided by attributes set $\{a_{k'} | (a_{k'} = v_{i_{k'}a_{k'}}) \in \varphi\}$, rough set and IFSS representation of $\{u \in U | f(u, a_{k'}) = v_{i_{k'}a_{k'}}\}$ could be obtained, denoted as $(\underline{X_{D_{k'}}}, \overline{X_{D_{k'}}})$ and $\langle U, \underline{X^{\varphi}_{D_{k'}}}, U - \overline{X^{\varphi}_{D_{k'}}}\rangle$, respectively;
- Based on distance of IFS, *e.g.*, Hamming distance, distance between $\langle U, \underline{X_{D_{k'}}}, U - \overline{X_{D_{k'}}}\rangle$ and $\langle U, \underline{X^{\varphi}_{D_{k'}}}, U - \overline{X^{\varphi}_{D_{k'}}}\rangle$ could be obtained;
- Based on the above distance, truth of simple statement $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$ could be suitable defined.

The organization of this paper is as follows: In the next Section, Intuitionistic fuzzy special sets Representation of Rough Set are showed. In the Section III, IFSS representation of simple statement $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$ is discussed, Hamming distance of IFSS is provided. In the Section IV, truth of simple statement $D_{k'} \equiv (a_{k'} = v_{i_{k'}a_{k'}})$ is discussed. Example is in the Section V.

2 IFSS Representation of Rough Set

Intuitionistic fuzzy special subset (IFSS) is $A = \{X, A_1, A_2\}$, in which, $X \neq \emptyset$, $A_1 \subseteq X$, $A_2 \subseteq X$ and $A_1 \cap A_2 = \emptyset$ [11]. In [12] and [13], IFSS Representation of Rough Set are discussed based on information systems. Based on $U/\sim_A$, $\forall X \subseteq U$, define

$$\underline{X} = \bigcup\{U^k \in U/\sim_A | U^k \subseteq X\}, \tag{3}$$

$$\overline{X} = \bigcup\{U^k \in U/\sim_A | U^k \cap X \neq \emptyset\}, \tag{4}$$

when $\underline{X} \neq \overline{X}$, $(\underline{X}, \overline{X})$ is $\sim_A$ rough set. In (3), due to $\underline{X} \subseteq \overline{X}$, $\underline{X} \cap (U - \overline{X}) = \emptyset$ is obviously. Hence, $\langle U, \underline{X}, U - \overline{X} \rangle$ is IFSS based on $\sim_A$, this means that in the framework of $U/\sim_A$, $\forall X \subseteq U$, there is the follows one to one mapping

$$(\underline{X}, \overline{X}) \longleftrightarrow \langle U, \underline{X}, U - \overline{X} \rangle. \quad (5)$$

Definition 1 *[13] Let $\mathscr{A}$ be an information system, $\forall X \subseteq U$, $\langle U, \underline{X}, U - \overline{X} \rangle$ is called IFSS representation of $(\underline{X}, \overline{X})$.*

As a special case, $(U^i, U^i) \longleftrightarrow \langle U, U^i_\theta, (U^i_\theta)^c \rangle = \langle U, U^i_\theta, \bigcup_{j \neq i} U^j_\theta \rangle$.

Definition 2 *[13] Let $\mathscr{A}$ be an information system, $U/\sim_A = \{U^k | k = 1, \cdots, n\}$. If $\langle U, U^{k_1}, U^{k_2} \rangle$ such that $U^{k_1}, U^{k_2} \in U/\sim_A$ and $k_1 \neq k_2$, then $\langle U, U^{k_1}, U^{k_2} \rangle$ is called basic IFSS based on $\sim_A$. Denote $B_{\sim_A} = \{\langle U, U^{k_1}, U^{k_2} \rangle | U^{k_1}, U^{k_2} \in U/\sim_A, k_1 \neq k_2\}$.*

Let a class of recursive set $\Phi(B_{\sim_A})$ be such that 1) $\forall \langle U, U^{k_1}, U^{k_2} \rangle \in B_{\sim_A}, \langle U, U^{k_1}, U^{k_2} \rangle \in \Phi(B_{\sim_A})$; 2) If $\langle U, A_1, B_1 \rangle, \langle U, A_2, B_2 \rangle \in \Phi(B_{\sim_A})$, then $\langle U, A_1, B_1 \rangle \cap \langle U, A_2, B_2 \rangle = \langle U, A_1 \cap A_2, B_1 \cup B_2 \rangle \in \Phi(B_{\sim_A})$; 3) If $\langle U, A_1, B_1 \rangle \in \Phi(B_{\sim_A})$, then $\overline{\langle U, A_1, B_1 \rangle} = \langle U, B_1, A_1 \rangle \in \Phi(B_{\sim_A})$.

Property 1 *[13] a) $\forall \langle U, A_1, B_1 \rangle \in \Phi(B_{\sim_A})$, $A_1 \cap B_1 = \emptyset$, i.e., $\langle U, A_1, B_1 \rangle$ is IFSS.*

b) $\Phi(B_{\sim_A})$ is an intuitionistic fuzzy special σ-algebra generated by $B_{\sim_A}$.

c) In $\Phi(B_{\sim_A})$, 1) $\langle U, \emptyset, \emptyset \rangle \in \Phi(B_{\sim_A})$; 2) $\forall U^i \in U/\sim_A$, $\langle U, U^i, (U^i)^c \rangle \in \Phi(B_{\sim_A})$; 3) $\forall X \subseteq U$, $\langle U, \emptyset, \underline{X} \rangle, \langle U, \emptyset, \overline{X} \rangle \in \Phi(B_{\sim_A})$; 4) $\forall X \subseteq U$, $\langle U, \underline{X}, U - \overline{X} \rangle \in \Phi(B_{\sim_A})$.

The final item (4.) means that rough set could be represented by IFSS in $\Phi(B_{\sim_A})$. From IFSS point of view, its' advantages in $\Phi(B_{\sim_A})$ are that (1) the background knowledge of $U/\sim_A$ is used, this could be seen from basic IFSS based on $\sim_A$; (2) IFSS is a special case of intuitionistic fuzzy subset (IFS), there are many papers discuss about uncertainty measures and implication of IFS [12]-[20], all of these could be used by IFSS.

3 Truth Values of Statements

According to information systems $\mathscr{A} = (U, A, V, f)$ and association rule $\varphi = \bigwedge_{D_l \in P} D_l \longrightarrow \bigwedge_{D_j \in Q} D_j$ with $Con_{\mathscr{A}}(\varphi) = 1$, equivalence relations $\sim_A$ and $\sim_\varphi$ on U, which are decided by attribute sets $\{a_{k'} | (a_{k'} = v_{i_{k'} a_{k'}}) \in A\}$ and $\{a_{k'} | (a_{k'} = v_{i_{k'} a_{k'}}) \in \varphi\}$, respectively, could be obtained, let

$$U/\sim_A = \{U^1_A, \cdots, U^m_A\}, \quad (6)$$
$$U/\sim_\varphi = \{U^1_\varphi, \cdots, U^n_\varphi\}. \quad (7)$$

Due to $\{a_{k'} | (a_{k'} = v_{i_{k'} a_{k'}}) \in \varphi\} \subset \{a_{k'} | (a_{k'} = v_{i_{k'} a_{k'}}) \in A\}$, from equivalence classes point of view, $U/\sim_A$ is thinner than $U/\sim_\varphi$. Let all considered statements is denoted by $S(\mathscr{A})$.

Definition 3 *Let information system $\mathscr{A}$. $S(\mathscr{A})$ could be defined recursively as follows*

1. *$\forall a_k \in A$, $(a_k = v_{i_k a_k}) \in S(\mathscr{A})$;*
2. *If $P, Q \in S(\mathscr{A})$, then $P \wedge Q, P \vee Q, P \longrightarrow Q, \neg P \in S(\mathscr{A})$.*

Definition 4 *$\forall P \in S(\mathscr{A})$, extension of P, denoted by X_P, is defined recursively as follows: (1) $\forall D_k = (a_k = v_{i_k a_k}) \in S(\mathscr{A})$, $X_{D_k} = \{u \in U | f(u, a_{k''}) = v_{i_{k''} a_{k''}}\}$; (2) $X_{P \wedge Q} = X_P \cap X_Q$; (3) $X_{P \vee Q} = X_P \cup X_Q$; (4) $X_{P \longrightarrow Q} = \overline{X_P} \cup X_Q$; (5) $X_{\neg P} = \overline{X_P}$. In which, $\overline{X_P} = U - X_P$.*

In this paper, there is no different between statement ψ and it's extension X_ψ. Obviously, extensions of statements are subset of U, i.e., $X_\psi = \{u \in U | u \text{ satisfies } \psi\}$. According to (5), (6) and (7), $\langle U, \underline{X_\psi}, U - \overline{X_\psi} \rangle$ and $\langle U, \underline{X^\varphi_\psi}, U - \overline{X^\varphi_\psi} \rangle$ could be obtained, and define Hamming distance as follows

$$\begin{aligned} d^H_{X_\psi X^\varphi_\psi} &= \frac{1}{2|U|} \sum_{s=1}^{|U|} (|\underline{X_\psi}(x_s) - \underline{X^\varphi_\psi}(x_s)| + \\ &\quad |(U - \overline{X_\psi})(x_s) - (U - \overline{X^\varphi_\psi})(x_s)| \\ &\quad + |\pi_{X_\psi}(x_s) - \pi_{X^\varphi_\psi}(x_s)|), \end{aligned} \quad (8)$$

in which, $\pi_{X_\psi}(x_s) = 1 - \underline{X_\psi}(x_s) - (U - \overline{X_\psi})(x_s)$, $\pi_{X^\varphi_\psi}(x_s) = 1 - \underline{X^\varphi_\psi}(x_s) - (U -$

$\overline{X^{\varphi}_{\psi}})(x_s)$, $\underline{X^*_*}$ and $(U - \overline{X^*_*})$ are characteristic functions, *i.e.*,

$$\underline{X^*_*}(x) = \begin{cases} 1, & \text{if} \quad x \in \underline{X^*_*}, \\ 0, & \text{if} \quad x \notin \underline{X^*_*}. \end{cases} \tag{9}$$

$$(U - \overline{X^*_*})(x) = \begin{cases} 1, & \text{if} \quad x \in U - \overline{X^*_*}, \\ 0, & \text{if} \quad x \notin U - \overline{X^*_*}. \end{cases} \tag{10}$$

Definition 5 *A valuation of ψ is as follows*

$$v(\psi) = 1 - d^H_{X_\psi X^{\varphi}_{\psi}}, \tag{11}$$

in which, $d^H_{X_\psi X^{\varphi}_{\psi}}$ is decided by (8), the valuation of ψ is called truth value of ψ based on information systems $\mathscr{A}$ and association rule φ with $Con_{\mathscr{A}}(\varphi) = 1$.

3.1 Truth Values of Exact Statements

Let $D_{k''} \equiv (a_{k''} = v_{i_{k''} a_{k''}})$ be a simple statement, it's extension is subset of U, *i.e.*, $X_{D_{k''}} = \{u \in U | f(u, a_{k''}) = v_{i_{k''} a_{k''}}\}$. If $D_{k''}$ is such that $a_{k''} \in \{a_{k'} | (a_{k'} = v_{i_{k'} a_{k'}}) \in \varphi\}$, according to knowledge of rough sets, $D_{k''}$ is an exact concept corresponding to association rule φ, the follows are obviously,

$$\begin{aligned} & \langle U, \underline{X_{D_{k''}}}, U - \overline{X_{D_{k''}}} \rangle \\ = & \ \langle U, X_{D_{k''}}, U - X_{D_{k''}} \rangle, \end{aligned} \tag{12}$$

$$\begin{aligned} & \langle U, \underline{X^{\varphi}_{D_{k''}}}, U - \overline{X^{\varphi}_{D_{k''}}} \rangle \\ = & \ \langle U, X_{D_{k''}}, U - X_{D_{k''}} \rangle, \end{aligned} \tag{13}$$

hence, If any simple statement $D_{k''}$ such that $a_{k''} \in \{a_{k'} | (a_{k'} = v_{i_{k'} a_{k'}}) \in \varphi\}$, then $\nu(D_{k''}) = 1$. From rough set point of view, every $D_{k''}$ is an exact concept, this coincides with $v(D_{k''}) = 1$. From logic systems point of view, if association rule φ is understood as axiom, then any knowledge which could be decided (or defined) by φ is true, simple statements are such that the condition.

Let $\psi = (a_{k_1} = v_{i_{k_1} a_{k_1}}) * \cdots * (a_{k_r} = v_{i_{k_r} a_{k_r}})$, $* \in \{\wedge, \vee\}$, it's extension is subset of U, *i.e.*, $X_\psi = \{u \in U | (f(u, a_{k_1}) = v_{i_{k_1} a_{k_1}}) * \cdots * (f(u, a_{k_r}) = v_{i_{k_r} a_{k_r}})\}$. According to knowledge of rough sets, ψ is an exact concept under information system $\mathscr{A}$, if ψ is such that $\forall D_{k_{r'}} \in \psi \Longrightarrow a_{k_{r'}} \in \{a_{k'} | (a_{k'} = v_{i_{k'} a_{k'}}) \in \varphi\}$, then $v(\psi) = 1$.

Let $\psi = P \longrightarrow Q$, it's extension is $X_\psi = (U - X_P) \cup X_Q$, in which, X_P and X_Q are extensions of P and Q, respectively. According to (11), $v(\psi) = 1$ if and only if $d^H_{X_\psi X^{\varphi}_{\psi}} = 0$ if and only if X_ψ is an exact concept corresponding to information system $\mathscr{A}$ and association rule φ.

Let $\psi = \neg P$, it's extension is $X_\psi = U - X_P$, hence, $v(\psi) = 1$ if and only if $U - X_P$ is an exact concept if and only if X_P is an exact concept.

3.2 Truth Values of vague Statements

So called vague statement ψ could be understood by $v(\psi) < 1$. According to (11), $v(\psi) < 1$ if and only if $d^H_{X_\psi X^{\varphi}_{\psi}} > 0$. For any simple statement $D_k \equiv (a_k = v_{i_k a_k})$, due to it is an exact concept under information system $\mathscr{A}$, *i.e.*, $\langle U, \underline{X_{D_k}}, U - \overline{X_{D_k}} \rangle = \langle U, X_{D_k}, U - X_{D_k} \rangle$, $D_k \equiv (a_k = v_{i_k a_k})$ is vague simple statement if and only if $\langle U, \underline{X^{\varphi}_{D_k}}, U - \overline{X^{\varphi}_{D_k}} \rangle \neq \langle U, X_{D_k}, U - X_{D_k} \rangle$, *i.e.*, X_{D_k} is rough set under association rule φ. It is obvious that if $D_k \equiv (a_k = v_{i_k a_k})$ is vague simple statement, than $a_k \notin \{a_{k'} | (a_{k'} = v_{i_{k'} a_{k'}}) \in \varphi\}$. Compound statement ψ is vague statement if there exists vague simple statement in ψ, truth of vague statement ψ is $v(\psi) \in [0, 1)$, in which, if IFSS represents of ψ under information system $\mathscr{A}$ and association rule φ are $\langle U, U, \emptyset \rangle$ and $\langle U, \emptyset, U \rangle$, respectively, than $v(\psi) = 0$.

4 Example

Consider the follows information system with 18 objects and 8 attributes (see Table 1)) which is discussed in. Let association rule φ be as follows

$$\begin{aligned} \varphi \ = \ & (a_1 = 0) \wedge (a_3 = 2) \wedge \\ & (a_6 = 0) \longrightarrow (a_4 = 1 \wedge a_8 = 1). \end{aligned}$$

According to Table 1 and φ, the follows could be obtained,

$$U / \sim_A \ = \ \{\{x_1\}, \{x_2, x_4\}, \{x_3, x_8, x_{13}\}, \{x_5\},$$

$$\begin{aligned}&\{x_6\},\{x_7\},\{x_9,x_{17}\},\{x_{10},x_{16}\},\\&\{x_{11}\},\{x_{12}\},\{x_{14}\},\{x_{15}\},\{x_{18}\}\}.\\ U/\sim_\varphi \; = \; &\{\{x_1\},\{x_2,x_3,x_4,x_8,x_9,x_{10},x_{13},\\&x_{15},x_{16},x_{17}\},\{x_5\},\{x_6\},\{x_7\},\\&\{x_{11}\},\{x_{12}\},\{x_{14}\},\{x_{18}\}\}.\end{aligned}$$

Consider simple statement $D_1 \equiv (a_1 = 0)$, it's extension is as follows

$$X_{D_1} = \{x_1,x_2,x_3,x_4,x_6,x_8,x_9,x_{10},x_{11},x_{12},x_{13},x_{14},x_{15},x_{16},x_{17}\},$$

IFSS representations of X_{D_1} based on $U/\sim_A$ and $U/\sim_\varphi$ are the same form, *i.e.*, $\langle U, X_{D_1}, \{x_5,x_7,x_{18}\}\rangle$. According to (8),

$$d^H_{X_{D_1}X^\varphi_{D_1}} = 0 \text{ and } v(D_1) = 1.$$

Consider statement $\psi = (a_1 = 0) \longrightarrow (a_6 = 1)$, it's extension is as follows by Table 1

$$\begin{aligned} X_\psi &= (U - X_{(a_1=0)}) \cup X_{(a_6=1)}\\ &= \{x_5,x_7,x_{18}\} \cup \{x_5,x_6,x_7\}\\ &= \{x_5,x_6,x_7,x_{18}\},\end{aligned}$$

it is an exact concept, and $v(\psi) = 1$.

Consider statement $D_2 = (a_5 = 81)$, it's extension is $X_{D_2} = \{x_2,x_3,x_4,x_5,x_6,x_8,x_{13},x_{14}\}$. Under information system $\mathscr{A}$, $D_2 = (a_5 = 81)$ is an exact concept, *i.e.*, $\langle U, \underline{X_{D_2}}, U - \overline{X_{D_2}}\rangle = \langle U, X_{D_2}, U - X_{D_2}\rangle$. However, under association rule φ, $D_2 = (a_5 = 81)$ is a vague concept, and IFSS representation of D_2 is

$$\langle U, \underline{X^\varphi_\psi}, U - \overline{X^\varphi_\psi}\rangle = \langle U, \{x_5,x_6,x_{14}\}, \{x_1,x_7,x_{11},x_{12},x_{18}\}\rangle,$$

according to (8),

$$d^H_{X_{D_2}X^\varphi_{D_2}} = \frac{5}{9},$$

hence, $v(D_2) = 1 - d^H_{X_{D_2}X^\varphi_{D_2}} = \frac{4}{9}$. Similarly, consider statement $D_3 = (a_2 = 2)$, we obtain $X_{D_3} = \{x_3,x_6,x_7,x_8,x_{12},x_{13},x_{18}\}$ and

$$d^H_{X_{D_3}X^\varphi_{D_3}} = \frac{5}{9}, \quad v(D_3) = \frac{4}{9}.$$

Consider $\psi_1 = D_1 \longrightarrow D_2$, $\psi_2 = D_2 \longrightarrow D_3$ and $\psi_3 = D_3 \longrightarrow D_2$, their extension are

$$\begin{aligned} X_{\psi_1} &= \{x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_{13},x_{14},x_{18}\},\\ X_{\psi_2} &= \{x_1,x_3,x_6,x_7,x_8,x_9,x_{10},x_{11},x_{12},x_{13},x_{15},x_{16},x_{17},x_{18}\},\\ X_{\psi_3} &= \{x_1,x_2,x_3,x_4,x_5,x_6,x_8,x_9,x_{10},x_{11},x_{12},x_{13},x_{15},x_{16},x_{17}\},\end{aligned}$$

hence, $v(\psi_1) = \frac{4}{9}$, $v(\psi_2) = \frac{4}{9}$ and $v(\psi_3) = \frac{4}{9}$.

Table 1: Information system $\mathscr{A}$

$\mathscr{A}$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
x_1	0	1	1	1	20	2	2	2
x_2	0	1	2	1	21	0	1	1
x_3	0	2	2	1	21	0	1	1
x_4	0	1	2	1	21	0	1	1
x_5	1	1	2	2	21	1	1	1
x_6	0	2	1	2	21	1	1	1
x_7	1	2	1	2	23	1	1	1
x_8	0	2	2	1	21	0	1	1
x_9	0	1	2	1	24	0	1	1
x_{10}	0	3	2	1	23	0	1	1
x_{11}	0	1	3	1	20	0	1	2
x_{12}	0	2	2	2	22	0	1	2
x_{13}	0	2	2	1	21	0	1	1
x_{14}	0	3	2	2	21	2	1	2
x_{15}	0	4	2	1	22	0	1	1
x_{16}	0	3	2	1	23	0	1	1
x_{17}	0	1	2	1	24	0	1	1
x_{18}	1	2	2	1	22	0	1	2

5 Conclusion

In the above example, many statements are checked, truth of almost every vague statement is $\frac{4}{9}$. The reason is that $U/\sim_\varphi$ is too special, in many cases, vague statement ψ under association rule φ is such that $\underline{X_\psi} \cup U - \overline{X_\psi} = \{x_1,x_5,x_6,x_7,x_{11},x_{12},x_{14},x_{18}\}$. We think that if $U/\sim_\varphi$ satisfies some properties, maybe $v(P \longrightarrow Q)$ coincides with known non-classical logic system, *e.g.*, Lukasiewicz logic system, this will be discussed in another paper.

Acknowledgements

This work was supported by the youth foundation of sichuan province (grant no. 06ZQ026-037) and the education department foundation of sichuan province (grant no. 2005A121, 2006A084).

References

[1] R. Agrawal, T. Imielinski, A. Swami. Mining association rules between sets of items in large databases. In *Proceedings of SIGMOD Conference*, pages 207-216, Washington, D.C., 1993.

[2] R. Agrawal, R. Srikant. Fast algorithms for mining association rules. In *Proceedings of the 20th International Conference on Very Large Data Bases (VLDB)*, pages 478-499, Morgan Kaufmann, 1994.

[3] N. Pasquier, Y. Bastide, R. Taouil, L. Lakhal. Efficient mining of association rules using closed itemset lattices. *Journal of Information Systems* 24 (1) 25-46, 1999.

[4] N. Pasquier, Y. Bastide, R. Taouil, L. Lakhal. Discovering frequent closed itemsets for association rules. In *Proceedings of the Seventh International Conference on Database Theory (ICDT)*, pages 398-416, LNCS, Springer, Heidelberg, January 1999.

[5] G. Stumme, R. Taouil, Y. Bastide, N. Pasquier, L. Lakhal. Intelligent structuring and reducing of association rules with formal concept analysis. In*Proceedings of KI 2001*, LNAI 2174, pages 335-350, Springer, Heidelberg, 2001.

[6] G. Stumme. Off to newshores: conceptual knowledge discovery and processing. *Int. J. Human-Computer Studies* 59 (2003) 287-325.

[7] G. Stumme, R. Taouil, Y. Bastide, N. Pasquier, L. Lakhal. Computing iceberg concept lattices with TITANIC. *Data & Knowledge Engineering* 42 (2002) 189-222.

[8] D. Slezak. Association reducts: a framework for mining multi-attribute dependencies. In*15th International Symposium on Methodologies for Intelligent Systems (ISMIS 2005)*, volume 3488, pages 354-363, Springer, Heidelberg, May 2005.

[9] H. S. Nguyen, S. H. Nguyen. Rough sets and association rule generation. *Fundamenta Informaticae* 40 (4) (1999) 383-405.

[10] Z. Pawlak, A. Skowron. Rough sets and Boolean reasoning. *Information Sciences* 177 (1) (2007) 41-73.

[11] E. Coskun. Systems on intuitionistic fuzzy special sets and intuitionistic fuzzy special measures. *Information Sciences* 128 105-118, 2000.

[12] Z. Pei, K. Y. Qin. Intuitionistic fuzzy special set expression of rough set and its application in reduction of attributes. *Pattern Recognition and Artificial Intelligence* 17 (3) (2004) 262-268.

[13] Z. Pei, L. Zhang, H. H. Chen. Representation of rough sets based on intuitionistic fuzzy special sets. In*12th International Fuzzy Systems Association World Congress, IFSA2007*, LNAI 4529, pages 114-123, Springer, Heidelberg, June 2007.

[14] V. Gregori, S. Romaguera, P. Veeramani. A note on intuitionistic fuzzy metric spaces. *Chaos, Solitons and Fractals* 28 (2006) 902-905.

[15] E. Szmidt, J. Kacprzyk. Distances between intuitionistic fuzzy sets. *Fuzzy Sets and Systems* 114 (2000) 505-518.

[16] Deng-Feng Li. Some measures of dissimilarity in intuitionistic fuzzy structures. *Journal of Computer and System Sciences* 68 (2004) 115-122.

[17] Deng-Feng Li. Multiattribute decision making models and methods using intuitionistic fuzzy sets. *Journal of Computer and System Sciences* 70 (2005) 73-85.

[18] W. Wang, X. L. Xin. Distance measure between intuitionistic fuzzy sets. *Pattern Recognition Letters* 26 (2005) 2063-2069.

[19] Z. Z. Liang, P. F. Shi. Similarity measures on intuitionistic fuzzy sets. *Pattern Recognition Letters* 24 (2003) 2687-2693.

[20] C. Cornelis, G. Deschrijver, E. E. Kerre. Implication in intuitionistic fuzzy and interval-valued fuzzy set theory: construction, classification, application. *International Journal of Approximate Reasoning* 35 (2004) 55-95.