H-IFS: Modelling & Querying over Hierarchical Universes

Panagiotis Chountas DKMG - HSCS University of Westminster Watford Road, Northwick Park London, HA1 3TP, UK chountp@wmin.ac.uk Ermir Rogova DKMG - HSCS University of Westminster Watford Road, Northwick Park London, HA1 3TP, UK rogovae@wmin.ac.uk Krassimir Atanassov CLBME Bulgarian Academy of Sciences, Bl. 105, Sofia-1113, BULGARIA krat@argo.bas.bg

Abstract

Query requirements led us to introduce the concept of closure of an Intuitionistic Fuzzy Set over a universe that has a hierarchical structure. The automatic recommendation of user queries or intentions according to ontological concepts is introduced and defined, in order to guide the query answering as part of an integrated database environment

Keywords: H-IFS, Flexible Querying Systems.

1 Introduction

This paper is about the propagation of preference or possibility degrees in a hierarchy. We propose an extension of the Intuitionistic Fuzzy Logic known as Hierarchical Intuitionistic Fuzzy sets (H-IFS) in which a query on a given class is also addressed to the subclasses of this class. Concerning query enlargement, several works such as [1] use a lattice of concepts to generalize unsolvable queries.

We can note two main categories of papers, in recent research: In studies about possibilistic ontologies [2], each term of ontology is considered as a linguistic label and has an associated fuzzy description. Fuzzy pattern matching between different ontologies is then computed using these fuzzy descriptions. This approach is related to those concerning the introduction of fuzzy attribute values in the object relational model [3].

Studies about fuzzy thesauri have discussed different natures of relations between concepts.

Fuzzy thesauri have been considered, for instance, in [4].

However, in our context, the terms of the hierarchy and the relations between terms are not vague. These observations led us to introduce the concept of closure of the H-IFS which is a developed form defined on the whole hierarchy. Intuitively, in the closure of the H-IFS, the "kind of" relation is taken into account by propagating the degree associated with an element to its sub-elements more specific elements in the hierarchy. The rest of the paper is organized as follows:

- Section two and three deliver the basic definitions and properties of the H-IFS
- Section three demonstrates the suitability of H-IFS' as a modelling concept with respect of the Vitis Vinifera domain
- Section four delivers an On-Line Analytical Processing (OLAP) environment based on H-IFS that incorporates background knowledge and guides query answers as part of an extended query mechanism.

2 IFS-Atanassov's Sets and H-IFS

Each element of an Intuitionistic fuzzy [5, 6] set has degrees of membership or truth (μ) and non-membership or falsity (v), which don't sum up to 1.0 thus leaving a degree of hesitation margin (π).

As opposed to the classical definition of a fuzzy set given by $A' = \{ < x, \mu_{A'}(x) > | x \in X \}$ where $\mu_A(x) \in [0, 1]$ is the membership function of the fuzzy set A', an Intuitionistic fuzzy set A is given by

$$A = \{ < x, \, \mu_A(x), v_A(x) > | x \in X \}$$

where: $\mu_A : X \to [0, 1]$ and $v_A : X \to [0, 1]$ such that $0 < \mu_A(x) + v_A(x) < 1$ and $\mu_A(x) v_A(x) \in [0, 1]$ denote a degree of membership and a degree of non-membership of $x \in A$, respectively.

Obviously, each fuzzy set may be represented by the following Intuitionistic fuzzy set

 $\begin{array}{l} A=\{<\!\!x, \mu_A'(x), (x), 1-\mu_A'(x)\!\!>\!\!|x \in X\} \\ \text{For each Intuitionistic fuzzy set in } X, \text{ we will} \\ \text{call} \quad \pi_A(x)=1-\mu_A(x)-v_A(x) \end{array}$

an Intuitionistic fuzzy index (or a hesitation margin) of $x \in A$ which expresses a lack of knowledge of whether x belongs to A or not. For each $x \in A$ $0 \le \pi_A(x) \le 1$.

The definition domains of the H-IFS [7, 9] that we propose below are subsets of hierarchies composed of elements partially ordered by the "kind of" relation. An element l_i is more general than an element l_j (denoted $l_i \sim l_j$), if l_i is a predecessor of l_j in the partial order induced by the "kind of" relation of the hierarchy. A hierarchical fuzzy set is then defined as follows:

Definition 2.1: An H-IFS is an Intuitionistic fuzzy set whose definition domain is a subset of the elements of a finite hierarchy partially ordered by the "kind of" \leq relation.

Definition 2.2: Let F be a H-IFS defined on a subset D of the elements of a hierarchy L. Its degree is denoted as $\langle \mu, \nu \rangle$. The closure of F, denoted clos(F), is a H-IFS defined on the whole set of elements of L and its degree $\langle \mu, \nu \rangle_{clos(F)}$ is defined as follows.

For each element I of L, let $S_L = \{l_1, ..., l_n\}$ be the set of the smallest super-elements of in D :

If S_L not empty then

 $<\mu,\nu>_{clos(F)}(S_L) =$

 $< \max_{1 \le i \le n} (\mu(L_i)), \min_{1 \le i \le n} (\nu(L_i))$ else $< \mu, \nu >_{clos(F)} (S_L) = <0, 0>$

In other words, the closure of a H-IFS F is built according to the following rules. For each element l_1 of L:

- If l_I belongs to F, then l_I keeps the same degree in the closure of F (case where $S_L = \{ l_I \}$).
- If l_I has a unique smallest super-element l_I in F, then the degree associated with l_I is propagated to L in the closure of F, S_L= { l_I } with l_{I >} l_I)
- If L has several smallest super-elements {*l*₁,,*l*_n} in F, with different degrees, a choice has to be made concerning the degree that will be associated with *l*₁ in the closure. The

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proposition put forward in Definition 2.2, consists of choosing the maximum degree of validity μ and minimum degree of non validity v associated with $\{l_1, ..., l_n\}$.

- All the other elements of L, i.e., those that are more general than, or not comparable with the elements of F, are considered as non-relevant. The degree <0,0> is associated with them.
- In a given equivalence class (that is, for a given closure C), a hierarchical fuzzy set is said to be **minimal** if its closure is C and if none of the elements of its domain is derivable.

Obtaining the Minimal H-IFS

Step 1: Assign Min-H-IFS $\leftarrow \emptyset$. Establish an order so that the sub-elements $\{l_1, \dots, l_n\}$ of the hierarchy L are examined after its super-elements.

Step 2: Let l_1 be the first element and $(l_1)/<\mu$, $\nu > \neq (l_1)/<0$, 0> then add l_1 to Min-H-IFS and $<\mu$, $\nu >_{clos(Min-HIFS)} (l_1)=(l_1)/<\mu$, $\nu >$.

Step 3: Let us assume that K elements of the hierarchy L satisfy the condition $\langle \mu, \nu \rangle_{clos(Min-HIFS)}$ (\mathbf{l}_i)=(\mathbf{l}_i)/ $\langle \mu, \nu \rangle$. In this case the Min-H-IFS do not change. Otherwise go to next element \mathbf{l}_{k+1} and execute Step 4.

Step 4: The $l_{k+1} / <\mu_{k+1}$, $\nu_{k+1} >$ associated with l_{k+1} . In this case l_{k+1} is added to Min-H-IFS with the corresponding $<\mu_{k+1}$, $\nu_{k+1} >$.

Step 5: Repeat steps three and four until **clos**_(Min-HIFS)=**C**.

Work reported in [10] in parallel to our framework is considering the problem of obtaining a family of fuzzy clusters with clear overlapping by allowing objects to fully belong to several classes. In this framework, the hesitation margin denoting to what extent the overlapping occurs was not considered and cannot be represented directly in the fuzzy hierarchies, classes/clusters. Furthermore the H-IFS notion as proposed in [8] can capture semantics that are not represented by [10].

For example, let E be a finite universe with the form

 $E = \{c_1, c_2, c_3, \{c_1, c_2\}, \{c_1, c_3\}, \{c_1, c_2, \{c_1, c_3\}\}\}$ Therefore, the H-IFS A over E will have the form:

$$\begin{split} &A = \{ < e_1, \mu_A(e_1), v_A(e_1) >, < e_2, \mu_A(e_2), v_A(e_2) >, \\ &< e_3, \mu_A(e_3), v_A(e_3) >, < \{e_1, e_2\}, \mu_A(\{e_1, e_2\}), v_A(\{e_1, e_2\}) >, \\ &< \{e_1, e_3\}, \mu_A(\{e_1, e_3\}), v_A(\{e_1, e_3\}) >, < \{e_1, e_2, \{e_1, e_3\}\}, \\ &\mu_A(\{e_1, e_2, \{e_1, e_3\}\}), v_A(\{e_1, e_2, \{e_1, e_3\}\}) > \end{split}$$

Let E_I be

 $E = \{e_1, e_2, e_3, \{f_1, f_2\}, \{f_1\}, \{g_1, g_2, \{g_1, g_3\}\}\}$ We can tell that elements e_1, e_2, e_3 are "elements from first level", elements f_1, f_2, g_1, g_2

"elements from second level" and elements g_1 , g_3 –elements from third level".

If there is an order between some of the elements of E, e.g., if for i = 1...3, $e_i = i$, this order (\leq or <) cannot be extend over the rest E-elements. If the order is \subset , it will be valid for fourth and sixth elements of E, but will not be possible for the rest of E-elements. Finally, the order \in will be valid, e.g. for the fifth and sixth elements, but not for the third and sixth elements.

Now, for H-IFS E that has *n* levels and for every natural number $i \le n$ we can introduce set *support*_i(E) that contains all E-elements that are from i-th level and that are not sets of elements of (i + 1)-th level.

$$support(E) = \bigcup_{i \le n} support_i(E) \cdot$$

We see that the e-elements are from *different hierarchical levels* and this is our reason to use the name of H-IFS for such sets.

3 The Vitis Vinifera Domain

The Vitis Vinifera domain is a case of multidimensional modelling. According to Multidimensional paradigm [11].

Further analysis of the Vitis Vinifera domain will require operations to aggregate based on levels of aggregation alternatively known as dimension hierarchies. So, improving decision making process involves well defined and rich hierarchies. Then the main task is on addressing the following question-issue, "How to define dimension hierarchies"?

There are several possible approaches in developing a hierarchy:

A top-down development process starts with the definition of the most general concepts in the domain and subsequent specialization of the concepts. A bottom-up development process starts with the definition of the most specific elements, the leaves of the hierarchy, with subsequent grouping of these classes into more general concepts. A combination development process is a combination of the top-down and bottom-up approaches: We define the more significant concepts first and then generalise or specialise them appropriately. We might start

with a few top-level concepts such as Wine, and a few specific concepts, such as Syrah. We can then relate them to a middle-level concept, such as Rhone.

This is not a simple task for the following reasons:

- Hierarchies could not be specified as many terms and data required by users are not included in the operational sources, i.e. consider a wine-sales database.
- Some kind of guidance is needed to enrich hierarchies by adding levels of aggregation, when referring to complex modelling domains like Vitis Vinifera.
- Use knowledge provided by the domain Vitis Vinifera to improve quality of dimension hierarchies. This will allow the inclusion of new hierarchy aggregation levels, which in return will allow DW users to achieve their analysis information needs and better support the decision-making process.

A proposed solution

Automatically complete hierarchies using relationships among concepts provided by an H-IFS for the following reasons:

- Dimension hierarchies represent semantic relations between values. i.e. Red Bordeaux are Red wines.
- H-IFS can express generalisation of two important properties: "is-a-kind-of" and aggregation or "is-a-part-of". For example, Cabernet Sauvignon, Cabernet Franc and Merlot are kind of Bordeaux Grapes and they are part of Red Bordeaux wines.
- These semantic relations allow us to organize concepts into hierarchical structures. We are interested in "kind-of" and "is part of" relations between concepts since they are the most useful relationships in a dimension hierarchy, and could be used to extent dimension hierarchies.
- In this context, H-IFS, which are more generally used to represent concepts whose borders are not strictly delimited, can be used to define flexible selection criteria, by associating a preference with every candidate value. The hierarchical structure can be used to enlarge the users' queries in case of empty answers, while respecting the preference order defined by the users in their selection criteria.

In the Vitis Vinifera domain, the following are the possible competency questions:

- Which wine characteristics should I consider when choosing a wine?
- Is Bordeaux a Red or White wine?
- Is Muscat a Red or White wine?
- Are Pinot Gris and Pinot Noir clones of Pinot Grapes?
- What makes Red Bordeaux a special wine?

In providing an answer to these questionsqueries one has to recognise that off the shelf products cannot answer the above questions simply because hierarchies provide only levels of summarisation but not any knowledge about the domain. On the other hand, H-IFS provide an ontological view of the modelled domain as well as efficient ways of summarising operational data as part of data warehouse.

The H-IFS structure for Vitis Vinifera Domain has been constructed as follows: applying elementary generalisation of the initial set of an H-IFS structure into an H-IFS of extended structure to create a new hierarchy level. The process is repeated until the required level of aggregation is achieved.

Thus the first elementary generalisation for the H-IFS structure for Vitis Vinifera domain starts at level n-3. This elementary generalisation allows us to relate grape varieties as "kind of" regional wine types. This corresponds to the level n-2 of the H-IFS.

The second elementary generalisation starts at level n-2. This consecutive elementary generalisation allows us to relate regional wine types as kind of the general type wines (Black, Brown, Violet, Red). This corresponds to the level n-1 of the H-IFS. If we wish to further generalise the n-1 H-IFS, then the next elementary generalisation, level n, will produce the whole production for all types of wines.

In the next section we present an OLAP querying mechanism that utilises knowledge in the form of H-IFS for the Vitis Vinifera domain.

4 Querying the Vitis Vinifera Domain

Let us consider a sample multidimensional model, (Fig.4.1) in the form of a star schema that describes sales of the Vitis Vinifera type wines.

Considering the Wine Sales star schema and the product dimension, the attribute H-Name corresponds to Fig 4.3, H-IFS structure for Vitis Vinifera domain. So far no fuzziness with respect to data displayed in Fig. 4.1.



Figure 4.1: Sample of a Star schema

At the same time let us recall the main focus of the multi-dimensional approach which is the subject area that is most important for analysis in our case sales of bottled wines. To put it differently the focus is on the strategic business questions and not on operational issues. To this extent let us consider the following questions: Which wines red or white are increasing in popularity? What is the trend for Red Bordeaux in comparison White Bordeaux?

Traditional OLAP tools like Oracle Express, etc. are currently not capable of answering this query for the following reasons:

- If we observe the H-Name attribute in Fig 4.1, it can be seen that there are no direct matches for red or white wine. So a traditional OLAP query will return no answers for question I. Moreover Muscat type wines can either be classified as red or white. Same applies to question III.
- Similarly, question II cannot be answered by traditional OLAP tools because there is not direct match for Red Bordeaux and White Bordeaux.

The following diagram (Fig 4.2) represents the query dilemmas for a traditional OLAP Tools.



Figure 4.2: Allocation strategies

•Bottles B1 Red Bordeaux, B4 Merlot can be classified as Red Bordeaux or Red wines with absolute certainty.

•Bottles B6 Pinot Gris, B5p Sauvignon and B7 are White Wine Types, for sure. Only bottle B7 is White Bordeaux. Bottle B5p, Sauvignon can be either classified as White Bordeaux or as Chateau d'Yquem, B8. Chateau d'Yquem is a White wine but not a White Bordeaux

•Bottles B2p, known as Muscat can be either Red or White wines.

The above queries show the importance of H-IFS for two reasons, firstly they allow us to extend the scope of the query and secondly they permit us to consider mixed concepts i.e. Muscat when we answer the queries above.

At this point it is important to estimate the total confidence in B5p being White Bordeaux or as Chateau d'Yquem. Similarly we need to estimate the total confidence in B2p, being white or red wine. The measure can be formulated as following:

C _{Ip, Region1 =}	Sum Regio	n1 lp∉	Sum Region
C _{B5p, Chateau d'Yqr}	Sum Region1 + Sum B8 4 	Region2 00 = 4 + 600 10	_
C _{BSp, White Bordea}	B7 B7 + B8	600 600 + 400	<u>- 6</u> 10
$C_{B2p, Red Wine} =$	= B1 + B4 B1 + B4 + B5 + B6 + B7 + B8	1160 1160 + 14	<u>= 116</u> 00 256
C _{B2p, White Wine} =-	B5 + B6 + B7 + B8 B1 + B4 + B5 + B6 + B7 + B8	1400 1160 + 1400	_ <u>140</u>

As an alternative for the Sum, the Count measure can be utilised.

The rationale is the following: Bottle B5p is disputed from two "regions": White Bordeaux and Chateau d'Yquem. So the confidence that B5p is a kind of Chateau d'Yquem is the sum of sales for Chateau d'Yquem over the sum of all sales for White Bordeaux and Chateau d'Yquem. The item of dispute B5p is excluded from the sum. The same applies for Bottle B2p.

The estimations below are based on the sample data from Fig 4.1.

When it comes to White Bordeaux and Chateau d'Yquem we have the following: we know for certain that we have 600 Bottles of White Bordeaux wine and possibly some more, with a confidence of 6/10, out of 200 Sauvignon. Therefore:

	600		120		
$\mu_{\text{White Bordeaux, White Wine}}$	= = 1856	0.32	π _{White Bordeaux, White Wine} = 1856	=	0.07

As far as Chateau d'Yquem concerns we have 400 Bottles for certain and possibly some more with a confidence of 4/10 out of 200 Sauvignon.

$$\mu_{\text{Chateau d'Yquem, White Wine}} = \frac{400}{1856} = 0.21 \xrightarrow{\pi_{\text{Chateau d'Yquem, White wine}} = 0.04} 1856$$

Based on the above calculations we can build a weighted H-IFS sub-domain (Fig 4.3) suitable for modelling and querying needs of complex/mixed concepts and sample data of the star schema displayed in Fig 4.1.



Figure 4.3: The Vitis Vinifera H-IFS

Based on the H-IFS domain presented above and data from the star schema of Fig 4.1, we will present an OLAP querying mechanism capable of dealing with mixed concepts, knowledge and summarising data according to a specified level. The hierarchy used to represent the data, as well as to express queries in the retrieval system, is organized into a hierarchy of terms that corresponds to the taxonomy derived from the Vitis Vinifera Domain.

Following the summarisation paths definition the sales star schema presented in Fig 4.1 is reconstructed as follows:

	Sum.	Paths	
Level_4	Level_3	Level_2	Level_1
Wine	Red	Red Bordeaux	Merlot
Wine	Red <.34,.55>	Med. Muscat<.34,.55>	Muscat
Wine	White<.41,.45>	Med. Muscat<.41,.45>	Muscat
Wine	White	White Bordeaux	Semillon
Wine	White	Alsace	Pinot Gris
Wine	White	Friuli	Pinot Gris
Wine	White	Chateau d'Yquem <.21,.75>	Sauvignon<.21,.75>
Wine	White	White Bordeaux <.32,.61>	Sauvignon<.32,.61>

Figure 4.4: Summarisation paths

With respect to the reconstructed table in Fig 4.5, it is worth to mentioning two different kinds of rows:

- Rows with precise data values carrying a weight of 1,0.
- Rows with plausible data values carrying a degree of membership less than one.



Figure 4.5: Reconstructed star schema

For example if we consider Red Bordeaux, there is no dispute about it. However in the case of Medit. Muscat we can have it as either Red or White Wine. Again, Sauvignon can be classified either as White Bordeaux or Chateau d'Yqeum. To this extent let us consider the following question/queries in conventional SQL-OLAP,

like: "What are the sales of White Wines"?

In conventional OLAP querying systems this query will be formulated as follows:

Select Bottle_Id, Date, Sum(Quantity) Where Bottle-id='White' Group by Bottle Id, Date

However this query cannot be answered by conventional OLAP querying systems because with respect to data in Fig 4.1 there are no bottles exactly labelled as White.

Based on the extended formalisms used in IFPG, [12] to be able to represent, respectively, illknown data and flexible queries as Intuitionistic fuzzy sets defined on a hierarchical domain, the above query can be re-written as follows:

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Select Bottle Id, Tuple Weight.MSHIP, Date, Sum(Quantity) Where Bottle-id='Friuli' OR Bottle*id='Medit.Muscat'* Group by Bottle Id, Date HAVING Weight.MSHIP=0.41 Order by Date Union Select Bottle Id, Tuple Weight.MSHIP, Date, Sum(Quantity) Where Bottle-id='Sauvignon' OR Bottleid = White Bordeaux'Group by Bottle Id, Date HAVING Weight.MSHIP=0.32 Order by Date Union Select Bottle Id, Tuple Weight.MSHIP, Date, Sum(Quantity) Where Bottle-id='Sauvignon' OR Bottleid='Chateau d'Yquem' Group by Bottle Id, Date HAVING Weight.MSHIP=0.21 Order by Date

At this point let us observe the intermediate results of the query before applying the Sum function followed by Order, Fig 4.6.

Sales	Bottle-Id	Quantity	Date	Weight	Tuple Weight
	Medit.Muscat	16	09-Dec-99	0.41,0.45	0.41,0.45
	Friuli	200	09-Dec-99	0.41, 0.45	1,0
	Medit.Muscat	440	12-Dec-07	0.41,0.45	0.41,0.45
Sales	Bottle-Id	Quantity	Date	Weight	Tuple Weight
Sales	Bottle-Id Sauvignon	Quantity	Date 09-Dec-99	Weight 0.32,0.61	Tuple Weight 0.32,0.61
Sales	Bottie-Id Sauvignon White Bordeux	Quantity 120 600	Date 09-Dec-99 12-Dec-07	<i>Weight</i> 0.32,0.61 0.32, 0.61	<i>Tuple Weight</i> 0.32,0.61 1,0
Sales	Bottie-Id Sauvignon White Bordeux	Quantitγ 120 600	Date 09-Dec-99 12-Dec-07	<i>Weight</i> 0.32,0.61 0.32, 0.61	<i>Tuple Weight</i> 0.32,0.61 1,0
Sales Sales	Bottle-Id Sauvignon White Bordeux Bottle-Id	Quantity 120 600 Quantity	Date 09-Dec-99 12-Dec-07 Date	Weight 0.32,0.61 0.32, 0.61 Weight	<i>Tuple Weight</i> 0.32,0.61 1,0 <i>Tuple Weight</i>
Sales Sales	Bottle-Id Sauvignon White Bordeux Bottle-Id Sauvignon	Quantity 120 600 Quantity 80	Date 09-Dec-99 12-Dec-07 Date 09-Dec-99	Weight 0.32,0.61 0.32, 0.61 Weight 0.21,0.75	Tuple Weight 0.32,0.61 1,0 Tuple Weight 0.21,0.75

Figure 4.6: Query answer/results

The answer block consisting of three sub-blocks provides the sales of all white wines ordered by date.

The first sub-block presents the quantity of Friuli wines with certainty <1,0> and two more with membership and non membership of <0.41,0.45> representing possible quantities of white Muscat wine.

The second sub-block provides the quantity of White Bordeaux with certainty <1,0> and those Sauvignons that can possibly be considered as White Bordeaux with a membership and non membership of <0.32,0.61>. So have we answered query II? Just to remind us, query II is about the sales of White Bordeaux. Conclusively query II is already answered. This is expected since the breaking of the initial selection condition wine="White" is restricted based on

the path formulation as represented through the summarization metadata table in Fig 4.4.

The third sub-block provides the quantity of Chateau d'Yquem with certainty <1,0> and those Sauvignons that can be considered as Chateau d'Yquem with a membership and non membership of <0.21,0.76>. So have we answered query III? Just to remind us, query III is about the sales of Chateau d'Yquem. Query III is already answered. Thus we do not need to define different views for executing and answering queries II and III.

Based on [13,14] one could further enhance the proposed query system in order to answer queries that refer to multi-version data warehouses where similarity between evolving hierarchies needs to be considered.

5 Conclusions

We brought forward the definition and application of H-IFS on a case study on "Vitis Vinifera" domain which has shown the following:

- The H-IFS can provide the basis for an ontological description of mixed concepts carrying a level of vagueness or imprecision.
- H-IFS can be used to extend hierarchies and enrich query definition, execution and answering when answers are not directly available.

The resolution of the sample queries presented above is important in showing how knowledge presented in the form of H-IFS can be used to enhance the definition and results of conventional OLAP type queries. It has also shown that a path based approach resolution produced a multiple query answer as part of an extended query.

Furthermore we notice that our approach can be used for the representation of Intuitionistic Fuzzy Linguistic terms.

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