Image segmentation using A-IFSs

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Abstract

The problem of segmentation in spite of all the work over the last decades, is still an important research field and also a critical preprocessing step for image processing, mostly due to the fact that finding a global optimal threshold that works well for all kind of images is indeed a near impossible task that, probably, will never be accomplished.

During the past years, fuzzy logic theory has been successfully applied to image thresholding. Moreover, considering that for segmentation purposes, in most cases, image pixels have an inherent ambiguity in the predicate that they must fulfill in order to belong to an object, which results in the experts uncertainty in assigning the pixel to that object. In the context of fuzzy sets theory, Atanassov's Intuitionistic fuzzy sets are a relevant and interesting extension since, uncertainty is one of the underlying ideas behind this theory. In this paper we describe a thresholding technique using Atanassov's intuitionistic fuzzy sets (A-IFSs). This approach uses Atanassov's intuitionistic index values for representing the uncertainty of the expert in determining that the pixel belongs to the background or that it belongs to the object. We first introduce the general framework of this approach and then its natural extension to multilevel thresholding.

Segmentation experimental results and their performance evaluation for the calculation of one, two and three thresholds are presented.

Keywords: Fuzzy Sets Theory Applications, Atanassov's Intuitionistic Fuzzy Sets (A-IFSs), Pattern Recognition, Digital Image Processing.

1 Introduction

Many image analysis techniques take as starting point a segmentation of the image, that is, the image is decomposed into meaningful parts for further analysis, resulting in the partition of the set of pixels in the image into a finite set of regions (subsets) according to a certain methodology.

The segmentation of digital images is the process of dividing an image into disjoined regions. The most commonly used strategy for segmenting images is global thresholding that refers to the process of dividing the pixels in an image on the basis of their intensity levels of gray. This division is made by establishing a threshold value.

Extensive research has been conducted in this research field over the last years, and many types of segmentation techniques have been proposed in the literature, each one of them based on a certain methodology to classify the regions [1, 2, 3, 4, 5].

In this context, within the framework of fuzzy theory [5, 6, 7, 8, 9, 10, 11] the most popular thresholding algorithms are those that use the concept of fuzzy entropy [6, 8, 12]. The main problem of this kind of approaches lies on the experts uncertainty when assigning the pixels either to the background or to the object trough the choice of

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the membership functions. Moreover, this choice has proven to be of uttermost importance regarding the algorithm's performance.

In order to overcome this problem, we present an approach to image thresholding using Atanassov's intuitionistic fuzzy sets (A-IFSs). This approach uses the Atanassov's intuitionistic index values for representing the uncertainty of the expert in determining that the pixel belongs to the background or that it belongs to the object.

The segmentation result is evaluated trough the use of a uniformity measure. This algorithm was implemented for the determination of one, two and three threshold values segmenting the image into one background and one, two or three objects.

As we will see in section 4 the performance of the presented algorithms are identical to Otsu's algorithm performance regarding the used uniformity measure [21, 22, 23].

Other techniques using A-IFSs in image thresholding have been presented in [13].

2 General framework for A-IFSs based image threshold

Being (x, y) the coordinates of each pixel on the image Q, and being $q(x, y)$ the gray level of the pixel (x, y) so that $0 \leq q(x, y) \leq L-1$ for each $(x, y) \in Q$ where L is the image grayscale, many methods have been proposed for determining the threshold t of an image considering fuzzy set theory as an efficient tool in order to obtain a good segmentation of the image considered.

We now introduce a general framework for image thresholding using A-IFSs and restricted dissimilarity functions.

Each image pixel is associated with three numerical values:

- A value for representing its membership to the background, which we will interpret as the expert's knowledge of the membership of the pixel to the background. We obtain this value by means of the membership function associated with the set that represents the background. This function will be constructed by the expert using restricted dissimilarity functions (see [14]).

- A value for representing its membership to the object, which we will interpret as the expert's knowledge of the membership of the pixel to the object. This value is obtained by means of the membership function associated with the set that represents the object. This function will also be constructed using restricted dissimilarity functions.

- A value for representing the unknowledge/ignorance of the expert in determining the membership functions described in the first two items. This value will be represented by Atanassov's intuitionistic index.

Under these conditions, if Atanassov's intuitionistic index associated with a pixel has a value of zero, it means that the expert is positively sure that the pixel belongs either to the background or to the object. However if the expert does not know if the pixel belongs to the background or to the object he must represent its membership to both with the value 0.5, and under these conditions we can say that the expert has used the greatest unknowledge/ignorance/intuition allowed in the construction of the functions of membership to the background and to the object respectively. Hence, the Atanassov's intuitionistic index value increases with respect to the unknowledge/ignorance of the expert as to whether the pixel belongs to the background or the object.

In general terms the algorithm we propose for calculating the best threshold value of an image Q is made up of the following steps:

(A) Construct L fuzzy sets $\tilde{Q}_{Bt}$ associated with the image Q. These sets represent the background of the image Q. Each one is associated with a level of intensity t, ($t = 0, 1, \cdots, L-1$), of the grayscale L used.

(B) Construct L fuzzy sets $\tilde{Q}_{Ot}$ associated with the image Q. These sets represent the object of the image Q considered. Each one is associated with a level of intensity t, ($t = 0, 1, \cdots, L-1$), of the grayscale L used.

(C) Represent the unknowledge/ignorance of the expert in the construction of the sets corresponding to items (A) and (B) by means of Atanassov's intuitionistic fuzzy index.

(D) Construct the L intuitionistic fuzzy sets of Atanassov Q_{Bt} associated with the background of the image.

(E) Calculate the entropy ε_T of each one of the L intuitionistic fuzzy sets of Atanassov Q_{Bt}.

(F) Take as best threshold the value of t associated with the intuitionistic fuzzy set of Atanassov Q_{Bt} of lowest entropy ε_T.

The gray level t associated with the Atanassov intuitionistic fuzzy set with the lowest entropy is selected for the best threshold since, in this methodology, entropy on A-IFSs is interpreted as a measure of the degree of a A-IFS that a set has with respect to the fuzziness of the said set (see [15]) and, under these conditions, the entropy will be null when the set is a fuzzy set and will be maximum when the set is totally intuitionistic.

We now present one possible implementation of this methodology [16, 20].

2.1 (*Steps A and B*)

Construct L fuzzy sets $\tilde{Q}_{Bt}$ associated with the background and L fuzzy sets $\tilde{Q}_{Ot}$ associated with the object. Each one of these fuzzy sets is associated with a gray level t of the grayscale L used. The membership functions of these sets are defined by means of restricted dissimilarity functions and the expressions are:

$$\mu_{\tilde{Q}_{Bt}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{\mathrm{B}}(t)}{L-1}\right)\right)$$

$$\mu_{\tilde{Q}_{Ot}}(q) = F\left(d\left(\frac{q}{L-1}, \frac{m_{\mathrm{O}}(t)}{L-1}\right)\right)$$

where

$$m_{\mathrm{B}}(t) = \frac{\sum_{q=0}^{t} qh(q)}{\sum_{q=0}^{t} h(q)}\ , m_{\mathrm{O}}(t) = \frac{\sum_{q=t+1}^{L-1} qh(q)}{\sum_{q=t+1}^{L-1} h(q)}$$

being $h(q)$ the number of pixels of the image with the gray level q, $F(x) = 1 - 0.5x$ and, the restricted dissimilarity function $d(x, y) = |x - y|$ for all $x \in [0, 1]$ (see [14, 16, 20]).

Note that $F(x)$ and $d(x, y)$ are only ones of the set of possibilities that could be used (see [14, 16, 20]).

2.2 (*Step C*)

As it has been said before, the unknowledge/ignorance of the expert in the construction of the fuzzy sets (in *Step A*) is represented by means of Atanassov's intuitionistic fuzzy index (π), meaning that, it is considered that $\mu_{\tilde{Q}_{Bt}}$ ($\mu_{\tilde{Q}_{Ot}}$) indicates the expert's degree of knowledge of the pixel belonging to the background (object).

If the expert is certain of the pixel belonging to the background or the object, then the value of π must be zero. The value of π increases as the unknowledge/ignorance of the expert grows. However, the unknowledge/ignorance must have the least possible influence on the choice of the membership degree, so, in this implementation, in the worst case, the unknowledge will have a maximum influence of 50 percent.

Under these conditions, the following expression is used to calculate π:

$$\pi(q) = \wedge(1 - \mu_{\tilde{Q}_{Bt}}(q), 1 - \mu_{\tilde{Q}_{Ot}}(q)) \qquad (1)$$

Again, this expression is only one of the possible ones (see [16]).

2.3 (*Step D*)

Construct an A-IFS, using π, with each one of the fuzzy sets $\tilde{Q}_{Bt}$ and $\tilde{Q}_{Ot}$.

$Q_{Bt} =$
$\{(q, \mu_{Q_{Bt}}(q), \nu_{Q_{Bt}}(q)) | q = 0, 1, \cdots, L-1\}$,
$\mu_{Q_{Bt}}(q) = \mu_{\tilde{Q}_{Bt}}(q)$
$\nu_{Q_{Bt}}(q) = 1 - \mu_{Q_{Bt}}(q) - \pi(q)$
and,
$Q_{Ot} =$
$\{(q, \mu_{Q_{Ot}}(q), \nu_{Q_{Ot}}(q)) | q = 0, 1, \cdots, L-1\}$,
$\mu_{Q_{Ot}}(q) = \mu_{\tilde{Q}_{Ot}}(q)$
$\nu_{Q_{Ot}}(q) = 1 - \mu_{Q_{Ot}}(q) - \pi(q)$

2.4 (*Step E*)

At this step we are going to calculate the entropy ε_T of each one of the L intuitionistic fuzzy sets of Atanassov Q_{Bt} and Q_{Bt}. We will use the intuitionistic entropy defined by Burillo and Bustince (see [15, 16]), by means of the following expression:

$$\varepsilon_T(Q_{Bt}) = \frac{1}{N \times M} \sum_{q=0}^{L-1} h(q)\pi(q) \qquad (2)$$

where $N \times M$ are the image dimensions in pixels.

2.5 (*Step F*)

Finally, the gray level associated with the Atanassov's intuitionistic fuzzy set Q_{Bt} of lowest entropy ε_T is chosen as the best threshold.

We now justify our choice for the minimum value of ε_T.

Under our constructions, for each $t \in \{0, 1, \cdots, L-1\}$,

$$0 \leq \varepsilon_T(Q_{Bt}) \leq 0.5$$

holds.

From our constructions we deduce the following two items:

1. If $\mu_{\tilde{Q}_{Bt}}(q) \to 1$, then $d\left(\frac{q}{L-1}, \frac{m_B(t)}{L-1}\right) \to 0$, therefore $q \approx m_B(t)$. In this case the pixels with intensity q are such that this intensity is very close to the average intensity of the pixels that represent the background. This fact enables us to assure that the pixel in question belongs to the background.

2. If $\mu_{\tilde{Q}_{Ot}}(q) \to 1$, then $d\left(\frac{q}{L-1}, \frac{m_O(t)}{L-1}\right) \to 0$, therefore $q \approx m_O(t)$. In this case the pixels with intensity q are such that this intensity is very close to the average intensity of the pixels that represent the object. This fact enables us to assure that the pixel in question belongs to the object.

Therefore, the most representative set of the background $\tilde{Q}_{Bt}$ is that whose membership degrees are closest to one. Identical reasoning can be made for the most representative set of the object $\tilde{Q}_{Ot}$. In any case these sets are obtained by taking, from among all the sets constructed (one for each value of t), the set with the lowest intuitionistic fuzzy entropy $\varepsilon_T(Q_{Bt})$. This is due to the fact that expression (2) is close to zero when for each q the following holds:

$$\mu_{\tilde{Q}_{Bt}}(q) \to 1 \qquad \text{or} \qquad \mu_{\tilde{Q}_{Ot}}(q) \to 1$$

which is the best possible situation as has been made clear in the two items above.

Please note that it can never happen that $\mu_{\tilde{Q}_{Bt}}(q) = 1 = \mu_{\tilde{Q}_{Ot}}(q)$; the pixel either belongs to the background or to the object, never to both at the same time.

3 Multilevel Image Segmentation using A-IFSs

Since, usually an image contains more than one object and, therefore, need to be classified in more than two regions, in this section we extend our approach to the determination of more than one threshold value in order to be able to separate all the objects in the image. The proposed methodology is extended to multilevel thresholding using the same framework presented in the above section and where the number of thresholds is predefined by the expert.

Under the same conditions described in section 2 let's consider an image Q that needs to be classified into n meaningful regions and, therefor requires $n-1$ threshold levels $t_1, \cdots t_{n-1}$ such that $0 \leq t_1 \leq \cdots \leq t_{n-1} \leq L-1$.

We now discard the concept of background and object and will refer to the considered regions as object 1 up to object n. Thus, for each image Q we will construct L fuzzy sets $\tilde{Q}_{O1t}, \cdots, \tilde{Q}_{O(n-1)t}$ associated with each object. In the same line of reasoning of section 2, the membership function of each element to these sets must express the relationship between the intensity q of the pixel and its membership to its corresponding object.

For each possible combinations of $t_1, \cdots, t_{n-1} \in \{0, 1, \cdots, L-1\}$, the mean of the intensities of gray of the pixels that belong to each object are given by the following expressions:

$$m_{O1}(t) = \frac{\sum_{q=0}^{t_1} qh(q)}{\sum_{q=0}^{t_1} h(q)},$$

$$m_{O2}(t) = \frac{\sum_{q=t_1+1}^{t_2} qh(q)}{\sum_{q=t_1+1}^{t_2} h(q)},$$
$$\cdots,$$
$$m_{O(n-1)}(t) = \frac{\sum_{q=t_n+1}^{L-1} qh(q)}{\sum_{q=t_n+1}^{L-1} h(q)}.$$

We then construct the membership functions of each possible combinations of intensities $t_1, \cdots t_{n-1}$ in the above mentioned conditions, using the function $F(x)$ and the restricted dissimilarity function $d(x, y)$ used in section 2.

Like in section 2, the constructed membership functions are always greater than or equal to 0.5 and, the lesser the distance between a pixel's intensity q and the mean of intensities of the object considered, the greater the value of its membership to that object.

Under the same interpretation of π used in section 2, in this multilevel approach we used the following expression for $\pi(q)$:

$$\pi(q) = \wedge(1 - \mu_{\tilde{Q}_{O1t}}(q), \cdots, 1 - \mu_{\tilde{Q}_{O(n-1)t}}(q)) \tag{3}$$

The expression (3) fulfils the conditions mentioned in section 2 since, $\pi(q) = 0$ if and only if the expert is positively sure that the pixel belongs to one of the objects and, $\pi(q) = 0.5$ if and only if the expert has the greatest unknowledge/ignorance in determining to which object the pixel belongs to.

Hence,considering expression (3),

$$0 \leq \pi(q) \leq 0.5$$

Under the same conditions presented in section 2 we will associate an A-IFS with each one of the fuzzy sets $\tilde{Q}_{O1t}, \cdots, \tilde{Q}_{O(n-1)t}$ and, finally, the entropy ε_T of each one of the L intuitionistic fuzzy sets of Atanassov $Q_{O1t}, \cdots, Q_{O(n-1)t}$ is calculated by means of the following expression:

$$\varepsilon_T(Q_{O1t}) = \frac{1}{N \times M} \sum_{q=0}^{L-1} h(q)\pi(q) \tag{4}$$

where π is obtained with equation (3).

Again, for the same reasons presented in section 2, the gray levels combination set $t_1, \cdots, t_{n-1}$ associated with the Atanassov's intuitionistic fuzzy set Q_{Ot} of lowest entropy ε_T is chosen as the best thresholds.

Therefore, from among all the sets constructed (one for each possible combinations of $t_1, \cdots, t_{n-1} \in \{0, 1, \cdots, L-1\}$, such that $t_1 < \cdots < t_{n-1}$, the set with the lowest intuitionistic fuzzy entropy $\varepsilon_T(Q_{Ot})$. This is due to the fact that expression (4) is close to zero when for each q the membership function of one of the sets is closest to 1.

Note that is impossible that $\mu_{Q_{O1t}}(q) = \cdots = \mu_{Q_{O3t}}(q) = 1$. The pixel can only belong to one of the regions, never to more than one at the same time.

4 Results and Evaluation

In order to test the performance of the proposed approach, fifty six randomly selected images from the image database:`http://www.cs.cmu.edu/~cil/vision.html` were used as test images. To illustrate the obtained results, in Fig. 1 we present a ten images subset of the original fifty six image set. In all the other presented Figures, the order of the images within the Figures is always the same as the order of the images in Fig. 1.

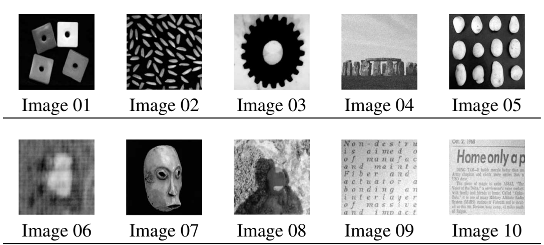

Figure 1: Original images.

To evaluate the performance of the proposed algorithms we used the following generalized uniformity measure [2, 21, 22]:

$$UM = 1 - \frac{1}{N \times M} NU \tag{5}$$

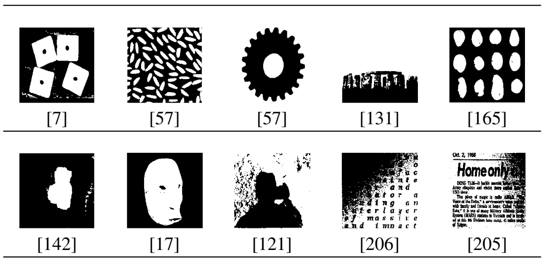

Figure 2: Segmented images (1 threshold).

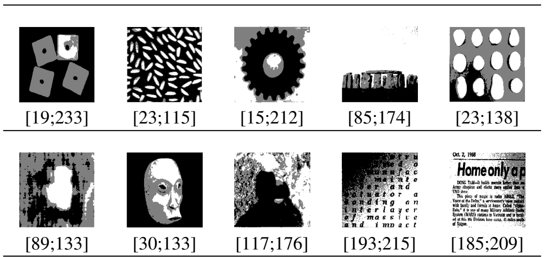

Figure 3: Segmented images (2 thresholds).

with,

$$NU = \sum_{j=0}^{T} \sum_{(x,y)\in R_j} \frac{|A_j|}{|A|} \cdot \frac{(q(x,y) - \nabla_j)^2}{(q_{max} - q_{min})^2}$$

Where, T is the number of thresholds, R_j is the j^{th} segmented region, $|A_j|$ is the pixel area of the j^{th} segmented region, $|A|$ is image pixel area, $q(x, y)$ is the gray level of the pixel (x, y), ∇_j is the mean of gray levels of the pixel in the segmented region j, $N \times M$ is the image total number of pixels and, q_{max} and q_{min} are, respectively, the maximum and minimum gray levels of the image pixels.

The performance measurement value $UM \in [0, 1]$ indicates the goodness of the segmentation in such way that it assumes the value 1 when the segmentation is optimal and its value decreases as the segmentation quality decreases.

In Fig. 2 to Fig. 4 we show the segmented images, along with the calculated thresholds values, obtained with the proposed algorithm for one, two and three thresholds, corresponding to the original images presented in Fig. 1. The numerical values of the performance measure value obtained with the uniformity measure presented in equation (4) are shown in Table 1. In Table 2 we present the average of the uniformity measure results for all the fifty six images of the original set.

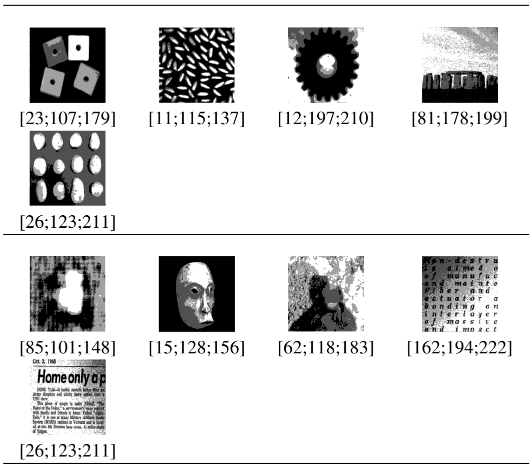

Figure 4: Segmented images (3 thresholds).

Table 1: Uniformity measure results

Image	1 th	2 ths	3ths
01	0.989705	0.996371	0.999760
02	0.996318	0.998910	0.999173
03	0.994384	0.997330	0.998630
04	0.996597	0.998972	0.999502
05	0.992553	0.997155	0.999016
06	0.994607	0.998848	0.999360
07	0.995378	0.999262	0.999514
08	0.992853	0.997129	0.998843
09	0.994562	0.997622	0.999400
10	0.995341	0.998410	0.999341

Looking at the segmented images, one can say that the algorithm perform well regarding the quality in visualization. Also, looking at Table 1 and Table 2, experimental results in terms of the tested images show that, in general, perform well regarding the uniformity measure. Since, according to Ng [23], Otsu's thresholding algorithm [18] can be considered identical to the uniformity measure proposed by Levine and Nazif [21], consequently, when we apply this measure to the proposed algorithm we are implicitly comparing it with Otsu's algorithm. Hence, one can state that, in comparison with Otsu's method, the proposed method performs equally well. Moreover, since the proposed methodology is able to incorporate uncertainty in the process of segmenting images and, the weight that this uncertainty in the determination of the threshold values can be tuned trough the definition of π, one can say that this methodology is more adaptable according to a given image segmentation problem.

Overall, we can state that, the obtained results

Table 2: Average uniformity measure results

1 th	2 ths	3ths
0,996053411	0,99843775	0,999238804

show the effectiveness of the proposed algorithm both in bi-level as in multi-level thresholding cases.

5 Conclusions and future work

The presented results show that the proposed methodology produces suitable thresholding algorithms for both bi-level and multi-level thresholding applications. Regarding the used uniformity measure, the proposed methodology gets near optimality (UM=1) as the number of thresholds increases, demonstrating that the goodness of the segmentation is directly proportional to the number of thresholds. This fact allow us to conclude that, as the number of thresholds increases, the algorithms are able to accomplish a correct separation of the image regions.

Moreover, due to the uniformity measure used to evaluate the performance of the algorithms, one can say that, in comparison with Otsu's method, the proposed method performs equally well.

Overall, experimental results demonstrate that the proposed algorithms is a suitable method for multilevel thresholding and therefore can be considered a promising and viable method for multi-thresholding applications.

Further work is intended, focusing the study of other forms to quantify the uncertainty (π) that is later used in the construction of the Atanassov Intuitionistic Fuzzy Sets in order to obtain the best segmentation results possible.

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