Ranking Alternatives Expressed via Atanassov's Intuitionistic Fuzzy Sets

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Abstract

We propose a new method of ranking alternatives represented by Atanassov's intuitionistic fuzzy sets, to be called A-IFSs, for short. First, we discuss an approach based on the calculation of the distances from the ideal positive alternative which can be viewed as a counterpart of the approach in the traditional fuzzy setting. Next, we propose a new method which takes into account not only the amount of information related to an alternative (expressed by a distance from an ideal positive alternative) but also the reliability of information represented by an alternative meant as how sure the information is.

Keywords: Ranking alternatives, fuzzy sets, intuitionistic fuzzy sets.

1 Introduction

Atanassov's intuitionistic fuzzy sets (cf. Atanassov [2], [3]), can be viewed as a tool that may help better model imperfect information, especially under imperfectly defined facts and imprecise knowledge. One of the important problems in this context is the ranking of alternatives (options) obtained after a process of decision analysis, evaluation, aggregation etc. At the end a set of alternatives is expressed in such a way that each option fulfills a set of criteria to some extent μ and, on the other hand, it does not fulfill this set of criteria to some extent ν. In other words, this implies that the alternatives can conveniently be expressed via Atanassov's intuitionistic fuzzy sets [cf. Section 2]. For brevity, such alternatives will be called intuitionistic fuzzy alternatives.

The problem of ranking intuitionistic fuzzy alternatives may be solved under some additional assumptions only because there is no linear order among elements of the A-IFSs. It is a different situation to that for fuzzy sets (Zadeh [31]) for which fuzzy elements are naturally ordered because the membership degrees are real numbers from $[0, 1]$.

In the literature there are not many approaches for ranking the intuitionistic fuzzy alternatives. They were proposed by, for instance, Chen and Tan [4], Hong and Choi [5], Li et al. [6], [7], and Hua-Wen Liu and Guo-Jun Wang [8].

Here we propose another approach that is different in several respects.

First, we employ the representation of A-IFSs (i.e., intuitionistic fuzzy alternatives) taking into account all three functions (membership, non-membership, and hesitation margin). Such a representation gives intuitively appealing results (cf. e.g., Szmidt and Kacprzyk[25], [18], [27]), [28]) while constructing distance, similarity, entropy, etc. like measures that play a crucial role in virtually all information processing tasks.

Second, we propose a function for ranking intuitionistic fuzzy alternatives which depends

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on two factors: the amount of information represented by an alternative (expressed by the distance from the ideal positive alternative), and the reliability of information (i.e. how sure an alternative is) – expressed by the hesitation margin.

2 A Brief Introduction to Intuitionistic Fuzzy Sets

One of the possible generalizations of a fuzzy set in X (Zadeh [31]), given by

$$A^{'} = \{< x, \mu_{A'}(x) > | x \in X\} \quad (1)$$

where $\mu_{A'}(x) \in [0, 1]$ is the membership function of the fuzzy set $A^{'}$, is Atanassov's intuitionistic fuzzy set (Atanassov [1], [2], [3]) A given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\} \quad (2)$$

where: $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \quad (3)$$

and $\mu_A(x)$, $\nu_A(x) \in [0, 1]$ denote the degree of membership and a degree of non-membership of $x \in A$, respectively.

Obviously, each fuzzy set may be represented by the following intuitionistic fuzzy set

$$A = \{< x, \mu_{A'}(x), 1 - \mu_{A'}(x) > | x \in X\} \quad (4)$$

For each intuitionistic fuzzy set in X, we will call

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \quad (5)$$

an *intuitionistic fuzzy index* (or a *hesitation margin*) of $x \in A$ and, it expresses a lack of knowledge of whether x belongs to A or not (cf. Atanassov [3]). It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

The hesitation margin turns out to be important while considering the distances (Szmidt and Kacprzyk [12], [16], [25], entropy (Szmidt and Kacprzyk [18], [27]), similarity (Szmidt and Kacprzyk [28]) for the A-IFSs, etc. i.e., the measures that play a crucial role in virtually all information processing tasks. In this paper the hesitation margin is shown to be indispensable in ranking the intuitionistic fuzzy alternatives because it indicates how reliable (sure) the information represented by an alternative is.

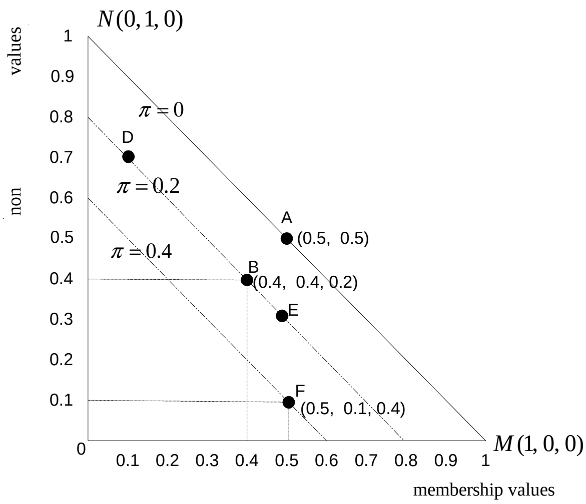

Figure 1: Geometrical representation

The application of A-IFSs instead of fuzzy sets means the introduction of another degree of freedom (non-memberships) into a set description. Such a generalization of fuzzy sets gives us an additional possibility to represent imperfect knowledge what leads to describing many real problems in a more adequate way. Applications of intuitionistic fuzzy sets to group decision making, negotiations, voting and other situations are presented in Szmidt and Kacprzyk [11], [13], [14], [17], [19], [21], [20], [22], [26], Szmidt and Kukier [29], [30]. (because of the different approaches presented in the works cited above, we are not able to discuss details here, and refer the interested reader directly to them).

2.1 Geometrical representation

One of the possible geometrical representations of an intuitionistic fuzzy sets is given in Fig. 1 (cf. Atanassov [3]). It is worth noticing that although we use a two-dimensional figure (which is more convenient to draw in our further considerations), we still adopt our approach (e.g., Szmidt and Kacprzyk [16], [25], [18], [27]), [28]) taking into account all three functions (membership, non-membership and hesitation margin values) describing an intuitionistic fuzzy set. Any element belonging to an intuitionistic fuzzy set may be represented inside an MNO triangle. In other words, the MNO triangle represents a surface where the coordinates of any element belonging to an A-IFS can be represented. Each point belonging to the MNO triangle is described by the three coordinates: (μ, ν, π). Points M and N represent crisp elements. Point $M(1,0,0)$ represents elements fully belonging to an A-IFS as $\mu = 1$, and may be seen as the representation of the ideal positive element. Point $N(0,1,0)$ represents elements fully not belonging to an A-IFS as $\nu = 1$. Point $O(0,0,1)$ represents elements about which we are not able to say if they belong or not belong to an A-IFS (intuitionistic fuzzy index $\pi = 1$). Such an interpretation is intuitively appealing and provides means for the representation of many aspects of imperfect information. Segment MN (where $\pi = 0$) represents elements belonging to classical fuzzy sets ($\mu + \nu = 1$). For example, point $A(0.5, 0.5, 0)$ (Figure 1), like any element from segment MN represents an element of a fuzzy set. A line parallel to MN describes the elements with the same values of the hesitation margin. In Figure 1 we can see point $B(0.4, 0.4, 0.2)$ representing an element with the hesitation margin equal 0.2, like $D(0.1, 0.7, 0.2)$, $E(0.5, 0.3, 0.2)$ and all elements on the line pointed out by any two from B, E, D. The closer a parallel line to MN is to O, the higher the hesitation margin.

Remark: We use the capital letters (e.g., A, B, C) for the geometrical representation of x_i's (Figure 1) on the plane. The same abbreviations (capital letters) mean in this paper the sets but we always explain the current meaning of a symbol used.

2.2 Distances between A-IFSs

In Szmidt and Kacprzyk [16], Szmidt and Baldwin [9, 10], and especially in Szmidt and Kacprzyk [25] it is shown why when calculating distances between A-IFSs we should take into account all three functions describing the A-IFSs. In [25] not only the reasons why we should take into account all three functions are given but also some possible serious problems that can occur while taking into account two functions only.

In our further considerations we will use the normalized Hamming distance between the A-IFSs A, B in $X = \{x_1, \ldots, x_n\}$ Szmidt and Baldwin [9, 10], Szmidt and Kacprzyk [16], [25]:

$$\begin{aligned} l_{IFS}(A,B) &= \\ &= \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \\ &+ \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \qquad (6) \end{aligned}$$

For (6) we have: $0 \le l_{IFS}(A,B) \le 1$. Clearly the normalized Hamming distance (6) satisfies the conditions of the metric.

3 Ranking the Alternatives

In Section 2 we have pointed out some possible applications of the A-IFSs and mentioned, among others, those related to voting. Now we will try to propose how to rank the voting alternatives expressed via intuitionistic fuzzy elements.

3.1 Ranking Alternatives via Distances from the Ideal Positive Alternative

Let an element x belonging to an A-IFS characterized via (μ, ν, π) expresses a voting situation: μ means the proportion (from $[0,1]$) of voters who vote for x, ν the proportion of those who vote against, and π of those who abstain. The simplest idea to compare different voting situations (rank the alternatives) might be to use a distance measure from the ideal voting situation $M = (x, 1, 0, 0)$ (100% voting for, 0% vote against and 0% abstain) to the alternatives considered. We will call M the ideal positive alternative. Let
$A = (x, 0.5, 0.5, 0)$ – 50% vote for, 50% against, and 0% abstain,
$B = (x, 0.4, 0.4, 0.2)$ – 40% vote for, 40% vote against and 20% abstain,

$C = (x, 0.3, 0.3, 0.4)$ – 30% vote for, 30% vote against and 40% abstain.

Certainly, the method of calculating distances between two A-IFSs A and B using the membership and non-membership values only (7) does not work properly (cf. Szmidt and Kacprzyk [16], [25], Szmidt and Baldwin [9], [10]) in this case, too:

$$l_2(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|) \quad (7)$$

The results from (7), i.e., the distances for the above voting alternatives represented by points A, B, C (cf. Figure 2) from the ideal positive alternative represented by $M(1,0,0)$ are, respectively:

$$l_2(M,A) = 0.5(|1-0.5| + |0-0.5|) = 0.5 \quad (8)$$
$$l_2(M,B) = 0.5(|1-0.4| + |0-0.4|) = 0.5 \quad (9)$$
$$l_2(M,C) = 0.5(|1-0.3| + |0-0.3|) = 0.5 \quad (10)$$

The results seems to be counterintuitive as (7) suggests that all the alternatives (represented by) A, B, C seem to be "the same". On the other hand, the normalized Hamming distance (6) taking into account besides the membership and non-membership also the hesitation margin, gives:

$$l_{IFS}(M,A) = 0.5(|1-0.5| + |0-0.5| + |0-0|) = 0.5 \quad (11)$$
$$l_{IFS}(M,B) = 0.5(|1-0.4| + |0-0.4| + |0-0.2|) = 0.6 \quad (12)$$
$$l_{IFS}(M,C) = 0.5(|1-0.3| + |0-0.3| + |0-0.4|) = 0.7 \quad (13)$$

The results (11)–(13) seem to reflect our intuition: alternative A seems to be the best in the sense that the distance $l_{IFS}(M,A)$ is the smallest (we know for sure that 50% vote for, 50% vote against). The situation is given in Fig. 2. The alternative represented by point A is just a fuzzy alternative (A lies on MN where the values of the hesitation margin are equal 0). On the other hand, alternatives B and C are "less sure" (with the hesitation margin equal 0.2, and 0.4, respectively).

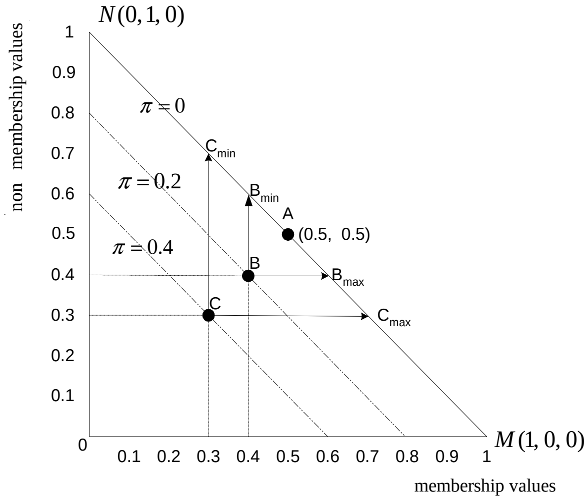

Figure 2: Geometrical representation of IFSs

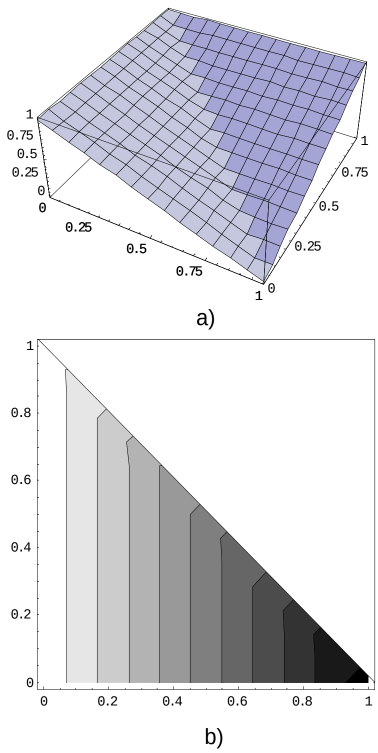

Figure 3: a) Distances (6) of any IFS element from ideal alternative M; b) contour plot

However, a weak point of ranking the alternatives by calculating the distances from the ideal positive alternative represented by M is that for a given value of the membership function, (6) gives just the same value (for example, if the membership value μ is equal 0.8, for any intuitionistic fuzzy element, i.e. such that its non-membership ν and hesitation margin π fulfill $\nu + \pi = 0.2$, is equal 0.2). It is shown in Figures 3. To better see this, the distances (6) for any alternative from M (Figure 3a) are presented for μ and ν for the whole range $[0, 1]$ (instead for $\mu + \nu \leq 1$ only). For the same reason (to better see the effect), in Fig. 3b) the contour plot of the distances (6) is given only for the range of μ and ν for which $\mu + \nu \leq 1$).

The conclusion is that the distances from the ideal positive alternative alone do not make it possible to rank the alternatives in the intended way.

3.2 A New Method of Ranking Alternatives

Let us analyze the sense of a voting alternative (expressed via an intuitionistic fuzzy element) using the operators of (cf. Atanassov [3]): *necessity* ($\Box$), *possibility* ($\Diamond$), $D_\alpha(A)$ and $F_{\alpha,\beta}(A)$ given as:

- The *necessity* operator ($\Box$)

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in X\} \quad (14)$$

- The *possibility* operator ($\Diamond$)

$$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in X\} \quad (15)$$

- Operator $D_\alpha(A)$ (where $\alpha \in [0, 1]$)

$$D_\alpha(A) = \{\langle x,\ \mu_A(x) + \alpha\pi_A(x),\ \nu_A(x) + (1-\alpha)\pi_A(x)\rangle\, | x \in X\} \quad (16)$$

- Operator $F_{\alpha,\beta}(A)$ (where $\alpha, \beta \in [0, 1]$; $\alpha + \beta \leq 1$)

$$F_{\alpha,\beta}(A) = \{\langle x,\ \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x)\rangle\, | x \in X\} \quad (17)$$

For example, for alternative $B(0.4, 0.4, 0.2)$ we obtain $\Box B = B_{min}$, where $B_{min} = (0.4, 0.6)$, and $\Diamond B = B_{max}$, where $B_{max} = (0.6, 0.4)$ (Figure 2). Operator $F_{\alpha,\beta}(A)$ makes it possible for alternative B to become any alternative represented in triangle $BB_{max}B_{min}$. A similar reasoning leads to the conclusion that alternative C (Figure 2) might become any alternative represented in triangle $CC_{max}C_{min}$, and alternative $O(0, 0, 1)$ (because of the hesitation margin equal 1) may become any alternative (the whole area of the triangle MNO).

Having the above considerations in mind we could say that the smaller the area of the triangle $Y_iY_{i,min}Y_{i,max}$ (Figure 4) the better alternative Y_i from a set Y of the alternatives considered. Alternatives having their representations on segment MN are the best in the sense that:

- the hesitation margin is equal 0 here, which means that the alternatives are fully reliable in the sense of the information represented, and
- the alternatives are ordered – the closer an alternative to ideal positive alternative $M(1, 0, 0)$, the better it is (it is an obvious fact as fuzzy alternatives are univocally ordered).

The above reasoning suggests that a promising way of ranking the intuitionistic fuzzy alternatives Y_i with the same values of π_i is converting them into the fuzzy alternatives (which may be easily ranked). For alternatives Y_i with different values of π_i the simplest way to rank the alternatives is seems to use the information carried by triangles $Y_iY_{i,min}Y_{i,max}$.

Y_i^* indicates the amount of information connected with Y_i (the amount of information is indicated by "the position" of triangle $Y_iY_{i,min}Y_{i,max}$ inside triangle MNO – expressed by the projection on segment MN). The value of the hesitation margin π_{Y_i} indicates how sure (reliable) is the information represented by Y_i^*.

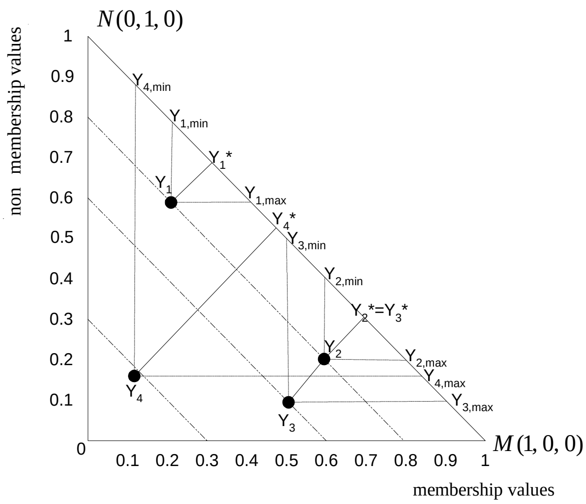

Figure 4: Ranking alternatives Y_i

Y_i^* are the orthogonal projections of Y_i on MN. Szmidt and Kacprzyk [15] considered such an orthogonal projection of the intuitionistic fuzzy elements belonging to an intuitionistic fuzzy set A. This orthogonal projection may be obtained via operator $D_\alpha(A)$ (16) with parameter α equal 0.5.

It is worth noticing that all the elements from segment OA (Figure 2) are transformed by $D_{0.5}(A)$ (16) into $A(0.5, 0.5)$ which reflects a lack of differences between the membership and non-membership, no matter what the value of the hesitation margin is.

In this context, a reasonable measure R that can be used for ranking the alternatives (represented by) Y_i seems to be

$$R(Y_i) = 0.5(1 + \pi_{Y_i}) l_{IFS}(M, Y_i^*) \qquad (18)$$

where $l_{IFS}(M, Y_i^*)$ is the distance (6) from ideal positive alternative $M(1, 0, 0)$, Y_i^* is the orthogonal projection of Y_i on MN. Constant 0.5 was introduced in (18) to ensure that $0 < R(Y_i) \leq 1$. The values of function R for any intuitionistic fuzzy element are presented in Figure 5a, and the counterpart contour plot – in Figure 5b. Unfortunately, the obtained results (18) do not rank the alternatives in the intended way. (The maximum value of (18) is not obtained for the alternative $(0, 0, 1)$ but for $(0, 1/2, 1/2)$.)

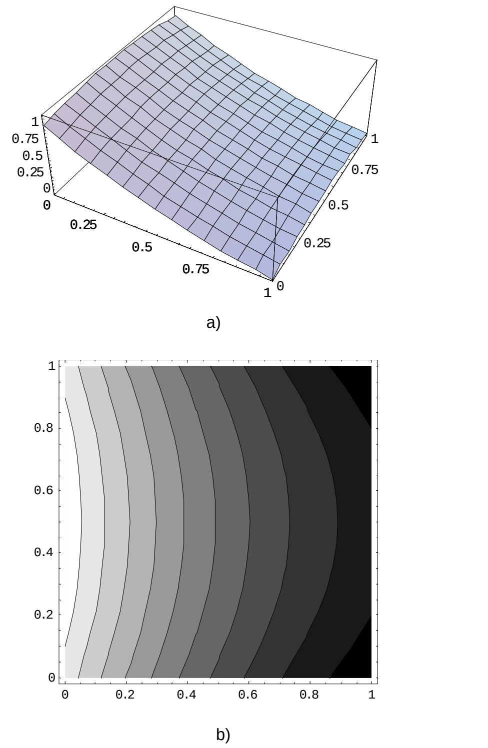

Figure 5: a) $R(Y_i)$ as a function of a distance Y_i^* from M and a hesitation margin; b) contour plot

A better measure R that can be used for ranking the alternatives (represented by) Y_i seems to be

$$R(Y_i) = 0.5(1 + \pi_{Y_i}) l_{IFS}(M, Y_i) \qquad (19)$$

where $l_{IFS}(M, Y_i)$ is the distance (6) Y_i from ideal positive alternative $M(1, 0, 0)$.

Equation (19) tells us about the "quality" of an alternative – the lower the value of $R(Y_i)$, (19), the better the alternative in the sense of the amount of the positive information included, and reliability of the information.

The best is alternative $M(1, 0, 0)$ for which $R(M) = 0$. For alternative $N(0, 1, 0)$ we obtain $R(N) = 0.5$ (alternative N is fully reliable as the hesitation margin is equal 0, but the distance $l_{IFS}(M, N) = 1$). Alternative A (Figures 1, 2) gives $R(A) = 0.25$. In general, on MN, the values of R decrease from 0.5 (for alternative N) to 0 (for the best alternative M). The maximal value of R, i.e. 1, we obtain for $O(0, 0, 1)$ for which both the distance from M and hesitation margin are

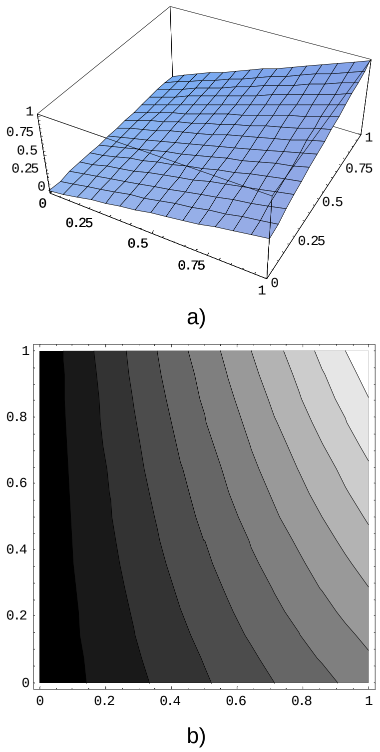

Figure 6: a) $R(Y_i)$ as a function of a distance Y_i from M and a hesitation margin; b) contour plot

equal 1 (alternative O "indicates" the whole triangle MNO). All other alternatives Y_i "indicate" smaller triangles $Y_i Y_{i,min} Y_{i,max}$ (Figure 4), so their counterpart values of R are smaller (better in the sense of the amount of the reliable information).

4 Conclusions

We have proposed a new method of ranking intuitionistic fuzzy alternatives. The method takes into account the amount of the information connected with an alternative (measured by a distance to the positive ideal alternative), and how reliable the information is (which is measured by the alternative's hesitation margin).

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