Measuring with priority based logic

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Abstract

The paper offers a first order axiomatization of priority based logic that can handle prioritized symbols. Proposed axiomatization is closely connected to the generalized prioritized fuzzy constraint satisfaction problem (GPFCSP). Developed formalism is decidable, due to the existence of a quantifier elimination procedure for it.

Keywords: priority, logic, GPFCSP.

1 Introduction

Priority is generally viewed as the importance of an object and it is often used in real time systems. Prioritized fuzzy constraint satisfaction problems (PFCSP's) were introduced in [3], an axiomatic framework (in the sense of the first order set theory) was given in [7] and applied in the agent–based automated negotiation [6]. PFCSP is actually a fuzzy constraint satisfaction problem (FCSP) in wich the notion of priority is introduced. Perhaps, the key factors in that implementation are schur-concave t-norms. They are defined in a such a way that the smallest value, usually the value with biggest priority, has the largest impact on the result given by a Schur-concave t–norm. PFCSP can only handle conjunction of constrains (see [11]). In [12, 13] PFCSP are extended in a way that they can handle disjunction and negation and generalized prioritized constraint satisfaction problems (GPFCSP).

Our approach is similar to the one given in [4] for axiomatization of reasoning about polynomial weight formulas. Let us briefly outline methodology. We start with a countably many propositional letters of the form $\langle v, p\rangle$, where $v \in \{v_n \mid n \in \mathbb{N}\}$ and $p \in \{P\} \cup \{p_n \mid n \in \mathbb{N}\}$. Here v represents the local satisfaction degree of an attribute, while p represents its priority. The set For of all fuzzy propositional formulas formulas with connectives $\neg_L, \wedge_L, \vee_L, \wedge_\Pi$ and $\vee_\Pi$ is built in the usual way[1]. In terms of PFCSP, each $\alpha \in For$ may be seen as a prioritized query. For each $\alpha \in For$ we introduce a new constant symbol $\mathtt{C}_\alpha$ (with respect to the language $\mathcal{L}_{OF} = \{+, \cdot, \leqslant, 0, 1\}$ of the ordered fields), which represents the truth degree of α. In addition, v_n, p_n and P are also treated as new constant symbols. Then we extend the theory RCF of the real closed fields with the following axioms:

- $0 \leqslant v_n \wedge v_n \leqslant 1$, $n < \omega$.
- $0 < p_0$.
- $p_n < p_m$, whenever $n < m$.
- $p_n < P$, $n < \omega$.
- $\mathtt{C}_{\langle v,p\rangle} = 1 - \frac{(1-v)p}{P}$.
- $\mathtt{C}_{\neg_L \alpha} = 1 - \mathtt{C}_\alpha$.

[1] $\neg_L, \wedge_L, \vee_L, \wedge_\Pi$ and $\vee_\Pi$ stands for Łukasiewicz negaiton, Łukasiewicz conjunction, Łukasiewicz disjunction, product conjunction and product disjunction, respectively.

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- $\mathtt{C}_{\alpha\wedge_L\beta} = \max(\mathtt{C}_\alpha + \mathtt{C}_\beta - 1, 0)$.
- $\mathtt{C}_{\alpha\vee_L\beta} = \min(\mathtt{C}_\alpha + \mathtt{C}_\beta, 1)$.

Recall that min and max are formally definable in the theory of ordered fields. For instance, the formula

$$(x = z \wedge y \leqslant x) \vee (y = z \wedge x \leqslant y)$$

defines the function max. Developed theory $RCF_{\text{L}\Pi}$ is consistent and it admits the quantifier elimination. As a consequence, $RCF_{\text{L}\Pi}$ is decidable as well.

The rest of the paper is organized as follows. In Section 2 we discuss priority fuzzy constraint satisfaction problems. Section 3 deals with the axiomatization. Concluding remarks are in Section 4.

2 Priority Fuzzy Constraint Satisfaction Problems (PFCSP)

The concept of constraint satisfaction problem (CSP) has been known for years [3]. A CSP consists of the set of constraints and a solution space, and its goal is to find a solution that satisfies all constraints. The prime example of this kind is a construction of a timetable.

In practice, most of constraints have inherited fuzziness (tall, bald, strong, young, age around 24, good stamina etc), and they are more naturally represented as fuzzy sets. This kind of approach leads to fuzzy constraint satisfaction problem (FCSP). Here, the degree of satisfaction of a constraint is the membership degree of its domain value on the fuzzy set that represents it. The global satisfaction degree can be obtained by the aggregation of values of each constraint, and a standard choice of the aggregation operator is suitable t–norm, s–norm or some fuzzy negation.

Definition 2.1 *(see [7]) A fuzzy constraint satisfaction problem (FCSP) is a triple* $\langle X, D, C\rangle$ *such that:*

- $X = \{x_1, \ldots, x_n\}$ *is a set of variables.*
- $D = \{d_1, \ldots, d_n\}$ *is a set of domains. Each domain* d_i *is a finite set of possible values for the corresponding variable* x_i.
- C *is a finite nonempty set of elements called fuzzy constraints, where each* $c \in C$ *has a form*

$$c = \{c_1, \ldots, c_n\} \cup \{f_1, \ldots, f_n\}.$$

Here each c_i *is a subset of* $\{1, \ldots, n\}$ *and each* f_i *maps* $\prod_{j\in c_i} d_j$ *into* $[0, 1]$. □

The membership degree of each constraint indicates the local degree to which the constraint is satisfied. In order to obtain the global satisfaction degree, local degrees are aggregated using certain t–norm. Adding priorities to the FCSP and allowing constraints to be aggregated by any logical formula produces the GPFCSP.

Definition 2.2 *Let* (X, D, C^f) *be defined as in Definition 2.1. Next, let* $\rho : R^f \to [0, \infty)$ *and a compound label* v_X *of all variables in X, and ,* $g : [0, \infty) \times [0, 1] \to [0, 1]$.

Generalized PFCSP is defined as a tuple $(X, D, C^f, \rho, g, \wedge, \vee, \neg)$.

An elementary formula in generalized PFCSP is a pair $(x, \rho(C_i))$ *where* $C_i \in C^f$, $x \in Dom(C_i)$ *represents the satisfaction degree of a constraint* C_i *and* $p_i = \rho(C_i)$ *represents its priority.*

A formula in GPFCSP is defined in the following way:

(i) An elementary formula is a formula.

(ii) If f_1 *and* f_2 *are formulas then also* $\wedge(f_1, f_2)$, $\vee(f_1, f_2)$ *and* $\neg(f_1)$ *are formulas.*

For each valuation v_X *a satisfaction degree* $\alpha_F(v_X)$ *of a formula F is calculated depending on the interpretation of connectives.*

A system is a GPFCSP if

1. *Let* $F = \wedge_{i\in\{1,\ldots,n\}} f_i$ *be a formula in GPFCSP where* f_i, $i \in \{1, \ldots, n\}$ *are elementary formulas and let* C^f *be a set of*

constrains that appear in the formula. If for the fuzzy constraint R^f_{max} we have

$$\rho_{max} = \rho(R^f_{max}) = \max\{\rho(R^f) \mid R^f \in C^f\},$$

then for each formula F we have:

$$\mu_{R^f_{max}}(v_X) = 0 \Rightarrow \alpha_F(v_X) = 0.$$

2. *If $\forall R^f \in C^f$, $\rho(R^f) = \rho_0 \in [0,1]$, then for each formula F we have:*

$$\alpha_F(v_X) = F_{\mathcal{L}}(v_X)$$

where $F_{\mathcal{L}}$ is the interpretation of the logical formula F in fuzzy logic $\mathcal{L}(\wedge, \vee, \neg)$).

3. *For R^f_i, $R^f_j \in C^f$, assume $\rho(R^f_i) \geq \rho(R^f_j)$, $\delta > 0$ and assume that there are two different compound labels v_X and v'_X such that:*
   - *if $\forall R^f \neq R^f_i$ and $\forall R^f \neq R^f_j$, then $\mu_{R^f}(v_X) = \mu_{R^f}(v'_X)$,*
   - *if $R^f = R^f_i$, then $\mu_{R^f}(v_X) = \mu_{R^f}(v'_X) + \delta$,*
   - *if $R^f = R^f_j$, then $\mu_{R^f}(v'_X) = \mu_{R^f}(v_X) + \delta$.*

   Then the following properties hold for $F = \wedge_{k=1,\ldots,n}(x_k, \rho(R_k))$, $x_k \in Dom(R_k)$ or $F = \vee_{i=1,\ldots,n}(x_k, \rho(R_k))$, $x_k \in Dom(R_k)$:

$$\alpha_F(v_X) \geq \alpha_F(v'_X).$$

4. *Assume that two different compound labels v_X and v'_X such that $\forall R^f \in C^f$ satisfy*

$$\mu_{R^f}(v_X) \geq \mu_{R^f}(v'_X).$$

   If formula F that has no negation connective, then holds

$$\alpha_F(v_X) \geq \alpha_F(v'_X).$$

5. *Let there be a compound label such that $\forall R^f \in C^f$, $\mu_{R^f}(v_X) = 1$.*

   If F is a formula $F = \wedge_{i=1,\ldots,n} f_i$ or $F = \vee_{i=1,\ldots,n} f_i$ where f_i, $i \in \{1, \ldots, n\}$ are elementary formulas then

$$\alpha_F(v_X) = 1.$$

On of the problems is to find a triple $\wedge, \vee, \neg$ that satisfy the previous properties. The following theorem proven in [13] gives the answer to this question.

Theorem 2.1 *The following system $(X, D, C^f, \rho, g, \wedge, \vee, \neg, \diamond)$ where $\wedge = T_L$, $\vee = S_L$, $\neg = N_S$ and finally $\diamond(x_i, c_i) = S_P(x_i, 1 - p_i)$ is a GPFCSP. The global satisfaction degree of a valuation v_X for a formula F is obtained in the following way:*

$$\alpha_F(v_X) = \mathcal{F}\{\diamond(v_{x_i}, \frac{\rho(R^f)}{\rho_{\max}})|R^f \in C^f\},$$

where C^f is the set of constraints of formula F , $\rho_{\max} = \max\{\rho(R^f), R^f \in C^f\}$ and $\mathcal{F}$ is the interpretation of formula F in GPFCSP.

The proof is an immediate consequence of the fact that N_s is the standard negation, T_L is a t-norm and S_L is a t-conorm dual to T_L. Theorem 2.1 gives us a concrete GPFCSP.

3 The theory $RCF_{L\Pi}$

In order to give a syntactical approach to GPFCSP, we will interpret the Łukasiewicz logic and the Product logic into the first order theory of the reals. Besides complete axiomatization, this method also gives decidability of the underlying theory.

Why *RCF* (the first order theory of the reals)? There are several important reasons:

- *RCF* is the best possible first-order approximation of the ordered field $\mathbb{R}$ of the reals. On the other hand, $\mathbb{R}$ provides natural semantics for fuzzy logics.

- The most important t-norms (i.e. the product norm, the Łukasiewicz norm and the Gödel norm), as well as the corresponding t-conorms, residuated implications and negations are formally definable in RCF. This fact alone enables the interpretation of the underlying fuzzy logics into RCF.

- RCF is decidable, with EXPSPACE containment for the general decision procedure, and PSPACE containment for the decision procedure of the existential part.

There are Hilbert-style axiomatizations of the Łukasiewicz logic, Product logic and Gödel logics and the reader may find them in the Hajek's book [5]. Our aim is to use the interpretation method in order to obtain a complete axiomatization of the $L\Pi$ logic (a combination of the Łukasiewicz logic and the Product logic, see [8]) in the following sense: if *phi* is an arbitrary $L\Pi$-formula, then its maximal satisfaction degree is s iff $C_\phi = s$ is a theorem of $T_{L\Pi}$. In this way we can formally speak about the truth degree of a fuzzy formula within the framework of the first order theory of the reals.

What is connection with the prioritized fuzzy constraint satisfaction problem? Basic attributes may be seen as a propositional letters (primitive propositions), so any query may be seen as a propositional formula. In the non-prioritized case this would be enough for the axiomatization. However, priorities introduce additional technical complications. To avoid even more cumbersome notation then the present one, we have decide to adopt the following convention: as an input, we have the following data:

- The local satisfaction of the attribute, computed with respect to the hard and soft constraints (soft constraints introduces fuzzy quantities).

- The priority of the attribute.

Thus, our propositional letters are pairs of the form $\langle v, p\rangle$, where the first coordinate refers to the local satisfaction degree of an attribute, while the second coordinate refers to its priority. A query will be an $L\Pi$-formula over the introduced propositional letters.

Now we will start with the technical details. Concerning model theoretical notions, our notation and terminology is standard and follows [9]. Let $\mathcal{L}_{OF} = \{+, -, \cdot, ^{-1}, \leqslant, 0, 1\}$ be the language of the ordered fields, let RCF be the first order $\mathcal{L}_{OF}$–theory of the real closed fields, and let $\mathcal{V} = \{v_n \mid n < \omega\}$, $\mathcal{P} = \{P\} \cup \{p_n \mid n < \omega\}$ and $\mathcal{C} = \{\langle v, p\rangle \mid v \in \mathcal{V}$ and $p \in \mathcal{P}\}$. The letters u, v and w will be variables for the elements of $\mathcal{V}$, while p, q and r will be variables for the elements of $\mathcal{P}$. We define the set For of fuzzy propositional formulas recursively as follows:

- $For_0 = \mathcal{C}$.

- $For_{n+1} = For_N \cup \{\neg_L \alpha \mid \alpha \in For_n\} \cup \{(\alpha * \beta) \mid \alpha, \beta \in For_n\}$, where $* \in \{\wedge_L, \wedge_\Pi, \vee_L, \vee_\Pi\}$.

- $For = \bigcup_{n \in \mathbb{N}} For_n$.

As it is usual, we will omit the uttermost brackets in any fuzzy formula. The elements of For will be denoted by α, β and γ, indexed or primed if necessary.

Definition 3.1 *Let* $\mathcal{L}^* = \mathcal{L}_{OF} \cup \mathcal{V} \cup \mathcal{P} \cup \{\mathtt{C}_\alpha \mid \alpha \in For\}$. *Here the elements of* $\mathcal{L}^* \backslash \mathcal{L}_{OF}$ *are treated as new constant symbols. We define the theory* $RCF_{L\Pi}$ *as an* $\mathcal{L}^*$*–theory with the following axioms:*

1. *All axioms of* RCF.
2. $0 \leqslant v_n \wedge v_n \leqslant 1$, $n < \omega$.
3. $0 < p_0$.
4. $p_n < p_m$, *whenever* $n < m$.
5. $p_n < P$, $n < \omega$.
6. $\mathtt{C}_{\langle v,p\rangle} = 1 - (1 - v) \cdot p \cdot P^{-1}$.
7. $\mathtt{C}_{\neg_L \alpha} = 1 - \mathtt{C}_\alpha$.
8. $\mathtt{C}_{\alpha \wedge_L \beta} = \max(\mathtt{C}_\alpha + \mathtt{C}_\beta - 1, 0)$.

9. $\mathtt{C}_{\alpha\vee_L\beta} = \min(\mathtt{C}_\alpha + \mathtt{C}_\beta, 1)$.

Let us briefly comment the above axiomatization. Pairs of the form $\langle v, p\rangle$ are typical for priority language. First coordinate represents the local satisfaction degree of an attribute, while the second one represents its priority. $\mathtt{C}_\alpha$ stands for prioritized satisfaction degree of the query $\alpha \in For$. Axiom (2) states that each local satisfaction degree is between 0 and 1. Axioms (3), (4) and (5) states that priorities form a positive sequence[2] whose order type is $\omega + 1$. Axiom (6) introduces priority in the calculation of the global satisfaction degree. The rest of the axioms follows the usual truth functions for Łukasiewicz negation, Łukasiewicz conjunction and Łukasiewicz disjunction. It is important to say that in this context, $+, -, \cdot, ^{-1}$, max and min are purely syntactical symbols.

Theorem 3.1 *$RCF_{L\Pi}$ is consistent.*

Proof 3.1 We use the compactness argument. That is, in order to prove consistency of $RCF_{L\Pi}$, it is sufficient to prove consistency of its arbitrary finite subset. Suppose that Γ is an arbitrary finite subset of $RCF_{L\Pi}$. Let $\alpha_1, \dots, \alpha_n$ be all fuzzy formulas appearing (as indices) in Γ. We construct the model $\mathcal{M}$ for Γ as follows:

- The universe M of $\mathcal{M}$ is the universe of some fixed real closed field $\mathbb{M}$. The language $\mathcal{L}_{OF}$ is interpreted as in $\mathbb{M}$. We may assume (without any loss of generality) that $\mathbb{Q} \subseteq \mathbb{M}$.
- Each v_m appearing in $\alpha_1, \dots, \alpha_n$ is interpreted as $\frac{1}{n+1}$.
- Each p_m is interpreted as $m + 1$. If k is the maximum of all such interpretations, then P is interpreted as $k + 1$.
- $\mathtt{C}^{\mathcal{M}}_{\langle v,p\rangle} = 1 -^{\mathbb{M}} (1 -^{\mathbb{M}} v^{\mathcal{M}}) \cdot^{\mathbb{M}} p^{\mathcal{M}} \cdot P^{\mathcal{M}-1^{\mathbb{M}}}$.
- $\mathtt{C}^{\mathcal{M}}_{\neg_L\alpha} = 1 -^{\mathbb{M}} \mathtt{C}^{\mathcal{M}}_\alpha$.
- $\mathtt{C}^{\mathcal{M}}_{\alpha\wedge_L\beta} = \max^{\mathbb{M}}(\mathtt{C}^{\mathcal{M}}_\alpha +^{\mathbb{M}} \mathtt{C}^{\mathcal{M}}_\beta -^{\mathbb{M}} 1, 0)$.
- $\mathtt{C}^{\mathcal{M}}_{\alpha\vee_L\beta} = \min^{\mathbb{M}}(\mathtt{C}^{\mathcal{M}}_\alpha +^{\mathbb{M}} \mathtt{C}^{\mathcal{M}}_\beta, 1)$.

Clearly, $\langle \mathbb{M}, \mathtt{C}^{\mathcal{M}}_{\alpha_1}, \dots, \mathtt{C}^{\mathcal{M}}_{\alpha_n}\rangle$ is a model of Γ, so we have our claim.

Theorem 3.2 *For each sentence φ of $\mathcal{L}^*$ there is a sentence φ^* of $\mathcal{L}_{OF}$ such that $RCF_{L\Pi} \vdash \varphi$ iff $RCF \vdash \varphi^*$. In other words, $RCF_{L\Pi}$ is interpretable in RCF.*

Proof 3.2 Notice that we only need to equivalently eliminate constant symbols $\mathtt{C}_\alpha$. Obviously, each $\mathtt{C}_\alpha$ has the form

$$F(\mathtt{C}_{\langle v',p'\rangle}, \mathtt{C}_{\langle v'',p''\rangle}, \dots, \mathtt{C}_{\langle v^{(k)},p^{(k)}\rangle}), \qquad (1)$$

where F is certain composition of $+, -, \cdot, ^{-1}$, max and min. Since F is definable in RCF, it remains to give the elimination of $\mathtt{C}_{\langle v,p\rangle}$'s. It is easy to show that

$$RCF_{L\Pi} \vdash \varphi(F(\mathtt{C}_{\langle v',p'\rangle}, \mathtt{C}_{\langle v'',p''\rangle}, \dots, \mathtt{C}_{\langle v^{(k)},p^{(k)}\rangle}))$$

iff

$$RCF \vdash \exists \bar{x}, \bar{y}, \bar{z}, t(\varphi(F(\bar{z}))\wedge$$

$$\bigwedge_{i=1}^{k} z_i = 1 - \frac{(1-x_i)y_i}{t} \wedge \psi(\bar{x}) \wedge \theta(\bar{y}, t)),$$

where, $\psi(\bar{x})$ is the formula

$$0 \leqslant x_1 \leqslant 1 \wedge \dots \wedge 0 \leqslant x_k \leqslant 1$$

and $\theta(\bar{y}, t)$ is the formula

$$0 < y_1 < t \wedge \dots \wedge 0 < y_k < t.$$

Thus, we have established the elimination of new constants, so we have our claim.

Corollary 3.1 *$RCF_{L\Pi}$ is decidable.*

Proof 3.3 By the previous theorem, for each sentence φ of $\mathcal{L}$, there is a sentence φ^* of $\mathcal{L}_{OF}$ such that $RCF_{L\Pi} \vdash \varphi$ iff $RCF \vdash \varphi^*$. It is well known (see [9]) that the latter predicate is decidable. Thus, $RCF_{L\Pi}$ is decidable.

Concerning complexity containment, both RCF and $RCF_{L\Pi}$ are in EXPSPACE. However, prioritized queries can be modeled with Σ_0-sentences. Each such a sentence can be interpreted in the existential theory of the reals. In this way, using [2], we obtain a PSPACE containment for the decision procedure for Σ_0-sentences.

[2]each member of the sequence is > 0

4 Conclusion

The present paper offers a first order axiomatization of GPFCSP. Proposed methodology is similar to the one introduced in [4] for the axiomatization of reasoning about polynomial weight formulas. The main difference is in the fact that the corresponding semantics in [4] is defined through special kind of Kripke models.

From the GPFCSP point of view, the most interesting part of introduced formalism is connected with Σ_0–sentences of $\mathcal{L}^*$. Decidability of $RCF_{L\Pi}$ allows construction of a theorem prover. Though the general decision procedure is in EXPSPACE, for Σ_0–sentences of $\mathcal{L}^*$ we can do much better. Namely, the elimination procedure outlined in the proof of Theorem 3.2 produces purely existential sentence of $\mathcal{L}_{OF}$. It is well known (see[2]) that there is a decision procedure for the existential theory of reals that is in PSPACE.

In general, one can approach to the problem of the axiomatization by means of modal-like "probabilistic" operators. The main difficulty of such approach is noncompactness of any finitary axiomatization. As it is well known, this difficulty can be overcome only by some infinitary approach. Higher order languages provide strong completeness, but lack decidability. To obtain both (strong completeness and decidability), one can use infinitary inference rules, see [10].

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