MULTI CRITERIA DECISION MAKING (MCDM): MANAGEMENT OF AGGREGATION COMPLEXITY THROUGH FUZZY INTERACTIONS BETWEEN GOALS OR CRITERIA

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Abstract

Since fuzzy set theory has been suggested as a suitable conceptual framework of decision making many different approaches have been developed. However, the subject of the complexity of the aggregation process and/or of the aggregation operations is rather not discussed. Complex real world applications on the other hand do not excuse such a lack. In this paper it is discussed in which way a decision making approach based on interactions between goals helps to manage aggregation complexity. The management of aggregation complexity is explained and compared with the way how related approaches deal with complexity. Two challenging research topics are suggested. These topics may help to close the gap between these approaches and the approach presented in this paper.

Keywords: Decision making, interactions between goals, aggregation complexity.

1 Introduction

In this paper it is analyzed that many decision making approaches are limited with respect to the management of complexity. The consequence of this limitation is that these models are rather not applicable for real world problems. In contrast to this, the presented decision making model based on interactions between goals is less limited because the complexity of both the input information required and the aggregation process is not higher than polynomial. Since the model has successfully been applied to many real world problems [11] it is worked out why it helps to manage complexity of the aggregation process. The number of goals in such real world problems may vary between 50 and 100 [11]. The number of decision alternatives may be even higher by more than several thousands. In the subsequent sections first the aspect of complexity in context with well known decision making approaches is discussed. Then the approach of interactions between decision goals is described and its complexity is analyzed and compared. Finally, two challenging topics for future work are suggested and conclusions are given. The notions decision goal and criteria are used synonymously.

2 Complexity Discussion of Related Approaches

Since fuzzy set theory has been suggested as a suitable conceptual framework of decision making [2], different categories of approaches in the field of fuzzy decision making can be observed. The first category reflects the fuzzification of established approaches like linear programming [22] or dynamic programming [3]. The basic aspect of these approaches is that decision making is understood as optimization and is reduced to modeling the goals as linear functions and the decision as a linear combination of the linear goal functions. The aggregation is basically modeled as the calculation of a weighted sum of linear functions. Although the algorithms implementing this approaches are linear in the average case, they are limited to decision situations which can be described in a linear manner. In case that the goal functions are not

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linear the methods move towards the so called mathematical programming and the complexity of the algorithms implementing it becomes exponential with respect to the number of goals and decision alternatives. If the search for solutions is not analytical but random, the field of models like evolutionary computation is entered. The weighted sum then is called fitness function. The complexity of the search algorithms tends to be np-hard, too.

The second category is based on the assumption that the process of decision making can be modeled by axiomatically specified aggregation operators [5] or aggregation operators based on weighted sums [6]. When the number of decision alternatives increases the approaches become less applicable because they are based on weighted sums where the weights are normalized to 1 [1],[13]. If the number of decision alternatives increases it becomes more and more difficult to distinguish the small values of the weights. The consequence is that these approaches are useful only if the number of decision alternatives and goals is rather small. Typical application examples are the selection of courses for students or the evaluation of applicants for a grant.

The category of aggregation operators defined based on preference relations like OWA, WOWA as well as fuzzy integrals and similar [4],[15],[16],[17],[20],[21] require a preference relation which for more complex decision problems is hard to obtain because it is defined on the power set of the set of the decision alternatives. If additionally the number of goals increases, obtaining the required preference relation becomes even harder.

Another category of approaches is based on fixed hierarchies of goals [19] or on the modeling of the decision situations as (probabilistic or possibilistic) graphs [12]. In case of hierarchies the complexity is reduced but the importance of the goals is hard-coded in the hierarchy. If the importance of the goals varies then the required number of hierarchies again grows exponentially [9]. In case of graphs the number of nodes is exponential with respect to the number of decision alternatives, anyhow.

As a conclusion of the discussion given above we have to say that the well known decision making and criteria aggregation approaches are limited with respect to the management of complex decision making problems. Furthermore, the aspect of complexity in the opinion of the author has not been sufficiently discussed and investigated in the related literature yet.

3 Human Decision Makers Manage Complexity

In complex decision situations human decision makers usually act during the decision making in the sense of a process. Consequently, decision making models should focus their attention more on decision processes and on what happens during such processes instead of referring only to the final output of the decision [18]. In the decision making processes human decision makers explicitly manage complexity of the decision making situation, especially in the context of multiple goals [7],[8]. One way of managing complexity is to concentrate on the question, which goals are positively or negatively affected by which decision alternatives and which goals interact with which other goals. In the case of contradicting goals the priorities of the goals are evaluated [8],[10].

4 Decision Making based on Interactions between Goals

In the following it is shown how an explicit modeling of interaction between decision goals that are defined as fuzzy sets of decision alternatives helps to manage complexity of the decision making and aggregation. This modeling of the decision making and aggregation process significantly differs from the related approaches and the way they manage complex decision situations. First the notion of positive and negative impact sets is introduced. Then different types of interaction between goals are defined. After this it is shown how interactions between goals are used in order to aggregate pairs of goals to the so called local decision sets. Then it is described how the local decision sets are used for the aggregation of a final decision set. The complexity of the different steps is discussed.

4.1 Positive and Negative Impact Sets

Before we define interactions between goals as fuzzy relations, we introduce the notion of the positive impact set and the negative impact set of a goal. A more detailed discussion can be found in [7],[8],[10] and [11].

Def. 1)

a) Let A be a non-empty and finite set of potential alternatives, G a non-empty and finite set of goals,

$A \cap G = \varnothing, a \in A, g \in G, \delta \in (0,1]$. For each goal g we define the two fuzzy sets S_g and D_g each from A into [0, 1] by:

1. Positive impact function of the goal g: $S_g(a):= \delta$, if a affects g positively with degree δ then $S_g(a)= \delta$, $S_g(a):= 0$ else.

2. Negative impact function of the goal g: $D_g(a):= \delta$, if a affects g negatively with degree δ then $D_g(a)= \delta$, $D_g(a):= 0$ else.

b) Let S_g and D_g be defined as in Def. 1a). S_g is called the positive impact set of g and D_g the negative impact set of g.

The set S_g contains alternatives with a positive impact on the goal g and δ is the degree of the positive impact. The set D_g contains alternatives with a negative impact on the goal g and δ is the degree of the negative impact.

4.2 Interactions between Goals

Let now A be a finite non-empty set of alternatives. Let $\mathcal{P}(A)$ be the set of all fuzzy subsets of A. Let X, Y ∈ $\mathcal{P}(A)$, x and y the membership functions of X and Y respectively. Assume now that we have a binary fuzzy inclusion $I: \mathcal{P}(A) \times \mathcal{P}(A) \rightarrow [0,1]$ and a fuzzy non-inclusion $N: \mathcal{P}(A) \times \mathcal{P}(A) \rightarrow [0,1]$, such that $N(X,Y) := 1 - I(X,Y)$. In such a case the degree of inclusions and non-inclusions between the impact sets of two goals indicate the degree of the existence of interaction between these two goals. The higher the degree of inclusion between the positive impact sets of two goals, the more cooperative the interaction between them. The higher the degree of inclusion between the positive impact set of one goal and the negative impact set of the second, the more competitive the interaction. The non-inclusions are evaluated in a similar way. The higher the degree of non-inclusion between the positive impact sets of two goals, the less cooperative the interaction between them. The higher the degree of non-inclusion between the positive impact set of one goal and the negative impact set of the second, the less competitive the relationship. The pair (S_g, D_g) represents the whole known impact of alternatives on the goal g. Then S_g is the fuzzy set of alternatives which satisfy the goal g. D_g is the fuzzy set of alternatives which are rather not recommendable from the point of view of satisfying the goal g.

Based on the inclusion and non-inclusion between the impact sets of the goals as described above, 8 basic fuzzy types of interaction between goals are defined. The different types of interaction describe the spectrum from a high confluence between goals (analogy) to a strict competition (trade-off) [8].

Def. 2)
Let S_{g_1}, D_{g_1}, S_{g_2} and D_{g_2} be fuzzy sets given by the corresponding membership functions as defined in Def. 1). For simplicity we write S_1 instead of S_{g_1} etc.. Let $g_1, g_2 \in G$ where G is a set of goals. T is a t-norm.

The fuzzy types of interaction between two goals are defined as relations which are fuzzy subsets of $G \times G$ as follows:

1. g_1 *is independent of* g_2: <=> $T(N(S_1, S_2), N(S_1, D_2), N(S_2, D_1), N(D_1, D_2))$
2. g_1 *assists* g_2: <=> $T(I(S_1 S_2), N(S_1, D_2))$
3. g_1 *cooperates with* g_2: <=> $T(I(S_1, S_2), N(S_1, D_2), N(S_2, D_1))$
4. g_1 *is analogous to* g_2: <=> $T(I(S_1, S_2), N(S_1, D_2), N(S_2, D_1), I(D_1, D_2))$
5. g_1 *hinders* g_2: <=> $T(N(S_1, S_2), I(S_1, D_2))$
6. g_1 *competes with* g_2: <=> $T(N(S_1, S_2), I(S_1, D_2), I(S_2, D_1))$
7. g_1 *is in trade-off to* g_2: <=>

$$T(N(S_1, S_2), I(S_1, D_2), I(S_2, D_1), N(D_1, D_2))$$

8. g_1 *is unspecified dependent from* g_2*:* <=>

$$T(I(S_1, S_2), I(S_1, D_2), I(S_2, D_1), I(D_1, D_2))$$

The interactions between goals are crucial for an adequate orientation during the decision making process because they reflect the way the goals depend on each other and describe the pros and cons of the decision alternatives with respect to the goals. For example, for cooperative goals a conjunctive aggregation is appropriate. If the goals are rather competitive, then an aggregation based on an exclusive disjunction is appropriate. Note that the complexity of the calculation of every type of interaction between two goals is O(card(A) $*$ card(A)) = O($(\text{card}(A))^2$) [7].

4.3 Two Goals Aggregation based on the Type of their Interaction

The assumption, that cooperative types of interaction between goals imply conjunctive aggregation and conflicting types of interaction between goals rather lead to exclusive disjunctive aggregation, is easy to accept from the intuitive point of view. For a more detailed formal discussion see for instance [8],[10]. Knowing the type of interaction between two goals means to recognize for which goals rather a conjunctive aggregation is appropriate and for which goals rather a disjunctive or even exclusively disjunctive aggregation is appropriate. This knowledge then in connection with information about goal priorities is used in order to apply interaction dependent aggregation policies which describe the way of aggregation for each type of interaction. The aggregation policies define which kind of aggregation operation is the appropriate one for each pair of goals. The aggregation of two goals g_i and g_j leads to the so called local decision set $L_{i,j.}$. For each pair of goals there is a local decision set $L_{i,j.}$ $\in$ P(A), where A is the set of decision alternatives (see Def 1 a)) and P(A) the power set upon A. For conflicting goals, for instance, the following aggregation policy which deduces the appropriate decision set is given:

if (g_1 is in trade-off to g_2) and (g_1 is slightly more important than g_2) then $L_{1,2} := S_1 / D_2$.

In case of very similar goals (analogous goals) the priority information even is not necessary:

if (g_1 is analogous to g_2) then $L_{1,2} := S_1 \cap S_2$ because $S_1 \cap S_2$ surely satisfies both goals.

In general, for every type of interaction there is a fixed set of aggregation policies. In this way for every pair of goals g_i and g_j, i,j $\in$ {1,…,4} decision sets are aggregated. The importance of goals is expressed by the so called priorities. A priority of a goal g_i is a real number $P_i \in [0,1]$. The comparison of the priorities is modeled based on the linear ordering of the real interval [0,1]. The statements like g_i *slightly more important than* g_j are defined as linguistic labels that simply express the extend of the difference between P_i and P_j.

4.4 Multiple Goal Aggregation as Final Aggregation based on the Local Decision Sets

The next step of the aggregation process is the final aggregation. The final aggregation is performed based on a sorting procedure of all local decision sets $L_{i,j}$. Again the priority information is used to build a semi-linear hierarchy of the local decision sets by sorting them. The sorting process sorts the local decision sets with respect to the priorities of the goals. Subsequently an intersection set of all local decision sets is built. If this intersection set is empty then the intersection of all local decision sets except the last one in the hierarchy is built. If the resulting intersection set again is empty then the second last local decision set is excluded from the intersection process. The process iterates until the intersection is not empty (or more generally speaking until its fuzzy cardinality is big enough with respect to a given threshold). The first nonempty intersection in the iteration process is the final decision set and the membership values of this set give a ranking of the decision alternatives that is the result of the aggregation process (for more details see [7]).

4.5 Complexity Analysis of the Aggregation Process

As already discussed in section 4.2 the complexity of the calculation of every type of interaction between two goals is O(card(A) $*$

card(A)) = $O((card(A))^2)$. Since the local decision sets are aggregated for every two goals the complexity of this step is $O((card(G))^2)$. In [7] it is shown that the relevant complexity of the final aggregation step is $O((card(A))^2 * (card(G))^2)$. The information required for the description of both the positive and the negative impact functions is $O(card(A) * card(G))$. The priority information required for the calculation of the local decision sets is the same as the priority information needed for the sorting process of the local decision sets required for the final aggregation step. This priority information is linear with respect to card(G) because for each goal a priority information is needed. Summarizing the different complexities of the different steps of the aggregation process and the complexity of the input information required by the aggregation process we obtain the relevant average complexity of $O((card(A))^2 * (card(G))^2)$ [7].

4.6 Priority Information Less Complex than Preference Information

The priority information required by the aggregation process based on interactions between goals is required for each single goal and is simply a value between 0 and 1. It is not a preference information in the sense of a preference relation as required by preference relation based aggregation approaches (for instance Choquet integral [17]). There, the preference relation expresses the preference between different subsets of decision alternatives. This means that the complexity of the priority information required by the model based on interactions between goals is linear with respect to the number of goals (card(G)), where G is the finite set of decision goals. In contrast to this a preference relation in context with fuzzy integral based modeling is defined by the power set P(A) of the set A of the decision alternatives A. This means that the complexity of the information required is exponential with respect to the cardinality of the set of decision alternatives A. Since usually card(G) is significantly lower than $2^{card(A),}$ we observe a significant reduction of the complexity of the input information required by the aggregation model based on interactions between goals compared to classical aggregation operators which work based on preference relations as input.

Since the local decision sets $L_{i,j}$ are elements of the power sets of the decision alternatives A, the calculation and the sorting process of the local decision sets induce a preference relation on the power set P(A) of the set of the decision alternatives A (now in the sense of the aggregation operators like OWA, WOWA and others). This is, in opinion of the author, a very interesting collateral observation which in future may help to investigate on the challenging topic of how to close the gap between the decision making based on interactions between goals and the existing literature on aggregation operators based on preference relations and their yet limited way of dealing with interactions between goals (see also section 5 of this paper).

4.7 Complexity Comparison with Respect to Related Approaches

As discussed in Section 2 the well known related approaches are limited when the decision situations become more complex. They are only applicable to linear decision problems and/or the complexity of the input information required is exponential with respect to the number of decision alternatives and/or the complexity of the computation is exponential and explodes if the number of decision alternatives and/or goals increases. Decision making systems based on decision trees are limited if the priorities of the decision goals dynamically change from decision situation to decision situation. Decision graph based systems are limited because of the complexity of the required graph when the number of goals and/or decision alternatives increases. The consequence of this limitations is that these models are rather limited as solutions for real world problems which usually are nonlinear and/or the number of goals and/or decision alternatives increases. In contrast to this, the presented aggregation process based on interactions between goals is less limited because the complexity of the required input information and the complexity of the aggregation process is not higher than $O((card(A))^2 *(card(G))^2)$. The complexity is polynomial with degree 2.

5 Two Challenges for Future Work

The aggregation policies as described in section 4.3 may be considered as rules for selecting appropriate aggregation operators with respect to each particular type of interaction [8]. One challenging topic of future research may be to investigate if, and if so then in which way, the information about the types of interaction can be integrated into the research about aggregation operators. Probably the answer could be that instead of applying one single aggregation operator for a particular decision problem we should think of having families of aggregation operators and selection strategies [1] based on interactions between the goals that will help to guide the aggregation process by selecting the appropriate aggregation operators. Please note that in the process of decision making based on interactions between decision goals the local decision sets $L_{i,j.}$ are so to speak consistently aggregated with respect to the type of interaction between the goals g_i *and* g_j, $i,j \in \{1,\ldots,n\}$ and with respect to the goal priorities for every pair of goals g_i *and* g_j.

The sorting procedure as described in section 4.4 used for the calculation of the aggregation of the final decision set is guided by the priorities of all the goals and ranks the local decision sets $L_{i,j}$ by the goal priorities. The higher the priority of each pair of goals g_i,g_j the closer the corresponding $L_{i,j}$ to the top of the ranking. The local decision sets $L_{i,j.}$ are subsets of A. Therefore the ranking generated by the sorting procedure induces a kind of fuzzy preference relation in the sense of preference relation based decision making. It can be supposed that the induced preference relation possesses a certain kind of relaxed consistency in the sense of [14]. This can be supposed because of both the properties of the fuzzy interactions between the goals [7],[10] and the way of the fuzzy aggregation based on these interactions as described in the sections 4.3 and 4.4. Please note that the interactions between the goals help to fuzzy-consistently apply the information about the priorities of the goals. The priorities themselves are evaluated subject to the linear order upon the real interval [0,1]. The observation that decision making based on interactions between goals may induce a preference relation with a supposed certain kind of relaxed consistency may possibly be the answer how to close the gap between the decision making based on interactions between goals and the decision making based on preference relations as known from existing literature. To investigate to which extend the observation and the supposition hold and what do they depend on is a second challenging topic of future work.

6 Conclusions

In this paper it has been analyzed that many decision making approaches are limited with respect to the management of complexity of the decision situation. The consequence of this limitation is that these models are rather not applicable for many relevant real world problems. In contrast to this, the presented decision making model based on interactions between goals is less limited because the degree of complexity of both the input information required and the aggregation process is not higher than polynomial. Two challenging topics are identified as the result of the research on interaction between goals. The first one is the suggestion that the types of interactions between goals may be used for the definition of selection strategies based on interactions between the goals that may help to guide the aggregation process by selecting appropriate aggregation operators. The second challenge is based on the observation that interactions between goals can be used to induce preference relations on the set of decision alternatives. This can help to close the gap between the decision making based on interactions between goals and the existing literature on aggregation operators based on preference relations and their yet limited way of dealing with interactions between goals.

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