Fuzzy information representation for decision aiding

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Abstract

In this paper we want to stress the relevance of decision aid procedures in complex decision making problems and claim for an extra effort in order to develop appropriate representation tools when fuzzy criteria or objectives are present. In particular, we point out how some painting algorithms may help decision makers to understand problems subject to fuzziness based upon a graphical first approach, like Statistics use to do. Moreover, we point out that although the standard communication tool with machines are either data or words, we should also consider certain families of graphics for such a role, mainly for the output.

Keywords: Multicriteria decision making, decision aid, criteria representation, fuzzy sets.

1 Introduction

When Zadeh [34] introduced fuzzy sets he was postulating the existence of certain *uncertainty* decision makers use to deal with which needed a more appropriate and efficient representation than the only existing model for uncertainty, i.e., probability theory. A big scientific dispute was declared with Bayesian approach to probability, since they claim fuzziness and any other kind of uncertainty can be consistently represented according to their model (see, e.g., [5, 6, 13, 22, 23, 27, 32] but also [8, 9, 20, 35] and a personal view in [26]).

No matter if we accept or not the axioms of the Bayesian approach, leading towards the existence of a unique representation of uncertainty, the true fact is that most students and even many researchers do not know about any alternative representation for uncertainty.

This situation is a consequence of a more important issue: most people do not know about any other formal alternative to binary logic which represents, together with observation, a key pilar of present Science [25].

We can recall here philosopher Francis Bacon when he wrote that *"those who have handled the sciences have been either empiricists or dogmatists. Empiricists are like ants, who only collect things and make use of them. Rationalists are like spiders, who weave webs out of their own bodies. But the bee has a middle policy: it extracts material from the flowers of the gardens and meadows, and digests and transforms it by its own powers."*

The problem we want to stress first in this paper is that experiments still need a logical structure behind, so it is not so clear that experiments are in one side and logic is on the other side. Experiments are usually built in order to answer certain questions provided within a particular logic, and the way we observe and understand reality depends on a logic previously defined.

Then we shall stress that any decision aid tool needs a protocol for communication with human being. Quite often this protocol is as-

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sumed within a linguistic framework, but we may be forgetting that part of the success of Statistics is based upon the great explanatory potential some simple graphic have.

2 Experiments and probability

Perhaps the main success of Statistics is quite concentrated in two fields mathematicians do not care too much about: the *design of experiments* and what is usually called *Descriptive Statistics*. The first one represents the first stage of any statistical study, and the second one appears once those experiments have been run.

Design of experiments tells how reality must be observed in order to maximize information, at the same time that cost is being reduced (this is actually done by means of a sometimes quite boring mathematical reckoning).

But Descriptive Statistics pretends a simplified global view of data, of course subject to manipulation and frequent errors. It is surprising how some complex situations are easily explained with a simple standard statistical graphic.

A key argument to be pointed out is how Probability Theory is fully consistent with binary logic: uncertainty is about observability of certain event, but not about the event itself (we may not know if such an event happened or not, but for sure either the event happens or the event does not happens). Hence, those experiments Statistics talk about are conceived and designed according to such a binary logic.

At this point, it is interesting to note that most introductory books to Probability Theory assume that the *experimental space* Ω has been someway given (see, for example, [11]), so events (that define a Boolean algebra according to binary operators *and*, *or* and *no*) can be represented in terms of subsets of that experimental space, which looks like a regular set but it is defined as a set of *possible results during an experiment* (so, it is not a *regular* set).

On the contrary, a consistent approach to Kolmogorov [21] can be based upon events, simply assuming the existence of a set representation (which can be assured as soon the structure of events is being assumed to follow the Boolean structure, see [33]).

But even in this case, an appropriate experiment should be built in such a way that we can have an answer to every question we make, *does event A holds when I get result ω during the experiment?* And according to the assumed binary logic, only one of two answers is allowed: *yes* or *no*.

Once binary logic is being assumed, events have to be crisp in the above sense. Defining a different kind of experiments, subject to an alternative non-binary logic, should be a main objective of fuzzy researchers in order to build the new Science being claimed by those *soft* sciences where most information is given in linguistic terms rather than Set Theory terms.

3 Complex decision making problems and decision aiding

Complex decision making problems are deeply related to the above discussion, once they are not confused with *difficult* decision making problems.

By a *difficult* decision making problem we mean here those problems that have been or can be modelled in terms of a classical optimization problem, where basic information fits into a real space: we may find of course that the optimal solution does not exist, perhaps because there is an inner conflict between criteria or objectives. But obtaining an informative output about the true situation hinges simply on our reckoning capacity. We just need faster algorithms.

By a complex decision making problem we mean here a problem where basic information is subject to deep modelling uncertainties. Criteria or objectives are poorly defined or simply not provided, and they have to be estimated from the available information, perhaps linguistic preferences. Any classical approximation to such this kind of problems is subject to an essential criticism, most often because a linguistic term is forced to fit into a crisp representation.

Complexity in our context refers to modelling, no matter if the proposed model is *difficult* or not to solve.

Within such a modelling complexity in this paper we stress the relevance of two problems, respectively located at both sides of any decision aid procedure, in between such a procedure and decision makers:

- Estimation of criteria or objectives from preferences, for a better knowledge of decision maker's mind.
- Graphical representation of results, for a better knowledge of possible consequences of decision maker's alternatives.

4 Estimation of criteria or objectives from preferences

A typical complex situation is when the basic information is given in terms of preferences. Decision maker compares a set of given alternatives, which can be poorly defined (as pointed out in [25], the most important human alternatives are *strategic* and therefore poorly defined, so details are fixed in the very last moment).

Since in a complex problem it is always desirable that our model allows the possibility of creating new alternatives, initially not taken into account (see [28, 31]), understanding the space were these alternatives move is essential.

In this sense, we can try some kind of decomposition or representation in terms of some possible underlying criteria or objectives.

Pursuing this objective, an interesting approach has been recently proposed by some of the authors [18, 19] by generalizing the dimension theory restricted in [10] to partial order sets. Although more efficient algorithms and approaches are being investigated, we should stress that from this approach an alternative to Saaty's importance weighting [30] can be developed (see [15]).

The relevance of this approach is to suggest decision makers with some hints in order to understand decision maker's own mind. Having a hint about possible underlying criteria or objectives should help in the search of new alternatives.

In Statistics we find tools for reducing data dimension that can be complementary.

5 Producing graphical representations

Another key issue is the output decision maker will get from our decision aid tool.

First of all, we must acknowledge that no decision maker will accept *black boxes*, i.e., a machine telling decision makers what to do, unless the proposed model fully fits their mind.

The true objective in a complex decision making problem is to help decision makers to understand the problem they are facing to (see again [28, 31]).

We should then realize that too often mathematical models use to assume that input and output are the same kind of information. But this is not true in most cases, where we give simple information (the one we as decision makers have) and we expect some help about a problem we cannot face, at least directly. So, we are acknowledging that such a problem is more complex than the available information. Why then should we expect that the output will be similar to the input? If our data are given in terms of words, we should be trying a more complex representation framework for the output, rather than the linguistic one.

A natural more sophisticated framework for communication is the graphical framework (which may contain words, at least in their written form).

Of course, a certain restriction to some singular kind of graphics is needed, in order to build up a proper logic within a structured family of graphical symbols (see [24] for an interesting discussion within the language structure).

In this context, an interesting approach for classification is being developed by some of the authors in [17, 16] generalizing Ruspini's [29] fuzzy partitions [1, 2] in order to produce representative paintings where colors representing classes are subject to gradation. The final objective is to be able to produce similar graphics to those used in statistics, but allowing color gradation (first results are being applied in a remote sensing framework, see [14]). We find here obvious reckoning difficulties, so a great effort is being made in order to reduce computing time keeping meaningful outputs. Improvements are being tested against standard libraries of figures together with real remote sensing images. For example, the algorithm proposed in [17] can be significatively improved in time together with the number of classes under consideration, so pictures will be more easily manageable.

There is a lot of work ahead under this approach, deserving in our opinion more interest than too sophisticated tools difficult to be managed by regular decision makers.

A key characteristic for a revolutionary tool should be its simplicity.

6 Final comments

In this paper we stress the relevance of aiding tools for decision making, specifically multicriteria problems.

The development of aiding tools represents a key issue in complex decision making problems, where the key characteristics and their relationship use to be difficult to be captured, measured or described. The amount and the structure of data (initial or processed) may represent themselves serious difficulties for a direct intuition of the problem, not allowing neither an easy *a priori* modelling or an easy *a posteriori* explanation). In fact, quite often the deep objective problem in a multicriteria decision making is to find out those criteria which explain present preferences and will help future decisions. Needless to say, such criteria may not be representable in the real line but, on the contrary, they can be poorly defined [34].

In particular, two key decision aid tools are considered in this paper. On one hand, those procedures allowing a better understanding of decision maker's mind (pursuing an accurate, significative and suggesting representation model of the problem). On the other hand, those procedures allowing a simplified, significative and suggesting view of possible results, so that decision maker can understand the consequences of each decision and even find some hints about possible new alternatives.

Those two key procedures address the first and the last stage of any decision aid methodology, i.e., the communication between machine and decision maker. In this sense, we point out that meanwhile the linguistic support may be the standard way in which human beings give information, graphical information can be the standard way in which information is given back to the decision maker.

Certainly, there are additional important paradigms in complex decision making that have not being addressed in this paper. For example, aggregation operators do play a key role in any summarizing process (see, e.g., [12] and [4]). In fact, we usually get from the set of rough data certain significative indices, which will be quite often the true base for the posterior treatment or representation. Aggregation operators taking into account operative reckoning (see, e.g., [3, 7]) and the underlying structure of data should become a key issue in the next future (see, e.g., [24]).

But at the end, a picture can be more illustrative than many words, and sometimes we cannot control the affective pressure words may represent in decision maker's mind.

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