On Properties of Type-1 OWA Operators in Aggregating Uncertain Information for Soft Decision Making

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Abstract

The type-1 OWA operator is a new aggregation operator that is used to directly aggregate fuzzy sets via an OWA mechanism. In this paper, an α-level type-1 OWA operator capable of aggregating the α-cuts of fuzzy sets is proposed. Based on the fuzzy set Representation Theorem, we indicate that a general type-1 OWA operator can be represented by its α-level type-1 OWA operators. This result is very useful in investigating the properties of general type-1 OWA operators. In this paper, we give the conditions under which the type-1 OWA operator possesses the properties that Yager's OWA operator holds: idempotency, monotony, compensativity, and commutativity. This provides a solid basis for the type-1 OWA operator to be applied to multi-expert decision making, multi-criteria decision making and multi-expert multi-criteria decision making.

Keywords: Type-1 OWA, OWA, α-level, Aggregation, Idempotency, Monotony, Compensativity, Commutativity

1 Introduction

One of the most important steps in dealing with multi-expert decision making (i.e. group decision making), multi-criteria decision making and multi-expert multi-criteria decision making is the aggregation operation. The objective of aggregation in decision making is to combine individual experts' preferences or criteria into an overall one in a proper way so that the final result of aggregation takes into account, in a given fashion, all the individual contributions [1]. Currently, at least 90 different families of aggregation operators have been studied [1, 2, 3, 4, 5, 6, 7]. Among the existing aggregation operators, the Ordered Weighted Averaging (OWA) operator proposed by Yager [8] is one of the most widely used and applied in many domains. However, the majority of the existing aggregation operators, including the OWA, focus exclusively on aggregating crisp numbers. As a matter of fact, inherent subjectivity, imprecision and vagueness in the articulation of opinions in real world decision applications make human experts exhibit remarkable capability to manipulate perceptions without any measurements. In these cases, the use of linguistic terms instead of precise numerical values is more adequate in dealing with vague or imprecise information or when experts have to express opinions on qualitative aspects that cannot be assessed by means of quantitative values [9, 10]. So techniques for aggregating linguistic information rather than crisp numerical values are in high demand, which has motivated us to suggest a new operator, called the type-1 OWA operator [11]. This new operator is able to directly aggregate linguistic terms expressed as fuzzy sets via an OWA mechanism. We have shown previously that

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some well-known existing aggregation operators, such as Yager's OWA operator and the join and meet operators of fuzzy sets, are special cases of the type-1 OWA operator [11]. To the best of our knowledge, aggregating fuzzy sets using the type-1 OWA operator has not been reported elsewhere in the literature.

In order to investigate the properties of type-1 OWA operators, in this paper we propose the α-level type-1 OWA operator, which is capable of aggregating the α-cuts of fuzzy sets. The importance of the α-level type-1 OWA operators is that they can be used to represent a general type-1 OWA, a proposition that we call the Representation Theorem of type-1 OWA operators in this paper. We also show that under certain conditions the type-1 OWA operator (like Yager's OWA operator) is still idempotent, monotonic, commutative, and compensative. This provides a solid basis for type-1 OWA operators to be applied to real world decision making.

The rest of the paper is organised as follows. Section 2 reviews the type-1 OWA operator. Section 3 provides the definition of the α-level type-1 OWA operator and the Representation Theorem of type-1 OWA operators. Section 4 addresses the partial order relations of fuzzy sets. The type-1 OWA operator properties are analysed in Section 5. Finally, Section 6 provides some discussion and conclusion.

2 Overview of the Type-1 OWA Operator

Let $F(\mathbb{X})$ be the power set of fuzzy sets defined on the domain of discourse $\mathbb{X}$. In the interests of aggregating linguistic information we define a type-1 OWA operator as follows.

Definition 1. *Given n linguistic weights $\{W_i\}_{i=1}^n \in F(\mathbb{U})$ in the form of fuzzy sets defined on the domain of discourse $\mathbb{U} = [0,1]$, let W represent the $\{W_i\}_{i=1}^n$. A type-1 OWA operator is a mapping Φ_W,*

$$\begin{array}{rcl}\Phi_W\colon F(\mathbb{X}) \times \cdots \times F(\mathbb{X}) & \longrightarrow & F(\mathbb{X}) \\ (A_1, \cdots, A_n) & \mapsto & G\end{array}$$

$$\mu_G(y) = \sup \Bigg\{ \mu_{W_1}(w_1) * \cdots * \mu_{W_n}(w_n) * \mu_{A_1}(a_1) * \cdots * \mu_{A_n}(a_n) \Bigg| \forall\, w_1, \cdots, w_n \in \mathbb{U} \; and \; \forall\, a_1, \cdots, a_n \in \mathbb{X},\ \sum_{k=1}^{n} \bar{w}_i a_{\sigma(i)} = y \Bigg\} \quad (1)$$

where $\bar{w}_i = w_i / \left(\sum_{i=1}^n w_i\right)$, whereas $*$ is a t-norm operator, $\sigma\colon \{1, \cdots, n\} \longrightarrow \{1, \cdots, n\}$ is a permutation function such that $a_{\sigma(i)} \geq a_{\sigma(i+1)}, \forall\, i \in \{1, \cdots, n-1\}$, i.e., $a_{\sigma(i)}$ is the ith greatest element in the set $\{a_1, \cdots, a_n\}$.

From the above definition, it can be seen that the aggregated result is still a fuzzy set defined on $\mathbb{X}$, $\Phi_W(A_1, \cdots, A_n) = G \in F(\mathbb{X})$. A direct approach described in the following can be used to perform the type-1 OWA operation given the linguistic weights $\{W_i\}_{i=1}^n$ in the form of fuzzy sets.

Step 1.

1. Select the domain of aggregated objects, $\mathbb{X}$.
2. Set up the parameters for the membership functions of linguistic weights $\{W_i\}_{i=1}^n$and aggregated objects $\{A_i\}_{i=1}^n$.

Step 2. $\forall y \in \mathbb{X}$.

1. For $(w_1, \cdots, w_n) \in [0,1]^n$, $(a_1, \cdots, a_n) \in \mathbb{X}^n$,
   - Normalise $(w_1, \cdots, w_n)$
   $$\bar{w}_i = w_i / \sum_{i=1}^{n} w_i$$
   - Perform Yager's OWA operation:
   $$\begin{array}{rcl}\bar{y} & = & \phi_{\bar{w}_1 \cdots \bar{w}_n}(a_1, \cdots, a_n) \\ & = & \displaystyle\sum_{k=1}^{n} \bar{w}_i a_{\sigma(i)}\end{array}$$
   - If $\bar{y} = y$, calculate the potential membership grade μ_0 of the point y belonging to the G, and record μ_0:

$$\mu_0 = \mu_{W_1}(w_1) * \cdots * \mu_{W_n}(w_n) * \mu_{A_1}(a_1) * \cdots * \mu_{A_n}(a_n)$$

- Go to ***Step 2***.1, and continue until all the weight vectors and aggregating points are selected.

2. Calculate the membership grade of the point y belonging to G as follows,

$$\mu_G(y) = max(\mu_0)$$

It is noted that in the calculation process, the domains $[0, 1]$ and $\mathbb{X}$ are discretised, and therefore only the points on the domains with non-zero membership grades are considered, so for each fuzzy set W_i (or A_i), the domain $\mathbb{U}$ (or $\mathbb{X}$) may be different. Also, the case $w_1 = w_2 = \cdots = w_n = 0$ is meaningless as it is required that $\sum_{i=1}^{n} \bar{w}_i = 1$. The following simple example illustrates the computation involved in aggregating two fuzzy sets by a type-1 OWA operator with two linguistic weights.

Example 1. *Given the linguistic weights on* $\mathbb{U} = [0, 1]$

$$W_1 = 0.2/0.1 + 1.0/0.8,$$

$$W_2 = 1.0/0.1 + 0.2/0.8,$$

The aggregation of the following two fuzzy sets on the domain $\mathbb{X} = \{1, 2, 3\}$

$$A_1 = 0.4/1 + 0.6/2 + 1.0/3,$$

$$A_2 = 0.6/1 + 1.0/2 + 0.4/3.$$

is carried out as follows:

1. *Computation of* $\mu_G(1)$*:*

   (a) *Find all the combinations* (w_1, w_2, a_1, a_2) *of weighting points in* $\mathbb{U}$ *and aggregating points in* $\mathbb{X}$ *making* $\bar{y} = \phi_{\bar{w}_1\bar{w}_2}(a_1, a_2) = 1$, *which includes*

   $$(0.1, 0.1, 1, 1),$$

   $$(0.1, 0.8, 1, 1),$$

   $$(0.8, 0.1, 1, 1),$$

   and

   $$(0.8, 0.8, 1, 1).$$

   (b) *Calculate all the potential degrees of membership of "1" belonging to G, which are*

   $$min(0.2, 1.0, 0.4, 0.6),$$

   $$min(0.2, 0.2, 0.4, 0.6),$$

   $$min(1.0, 1.0, 0.4, 0.6),$$

   and

   $$min(1.0, 0.2, 0.4, 0.6),$$

   i.e., 0.2*,* 0.2*,* 0.4*, and* 0.2 *respectively.*

   (c) *Calculate the degree of membership* $\mu_G(1)$ *as,*

   $$\mu_G(1) = max(0.2, 0.2, 0.4, 0.2) = 0.4.$$

2. *Similarly, we obtain* $\mu_G(2) = 0.6$ *and* $\mu_G(3) = 0.2$.

The aggregating result obtained by the type-1 OWA operator with t-norm min *is a fuzzy set on* $\mathbb{X}$,

$$G = 0.4/1 + 0.6/2 + 0.2/3.$$

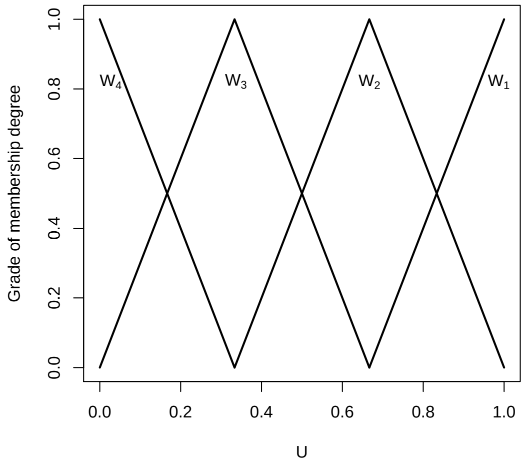
Figure 1 provides a visualisation of the application of the type-1 OWA operator with t-norm *min* to aggregating fuzzy sets: Figure 1a shows four weights in the form of fuzzy sets and four aggregated fuzzy sets, and the aggregation result by this type-1 OWA operator are depicted in Figure 1b.

3 Partial Order Relations of Fuzzy Sets based on Join and Meet

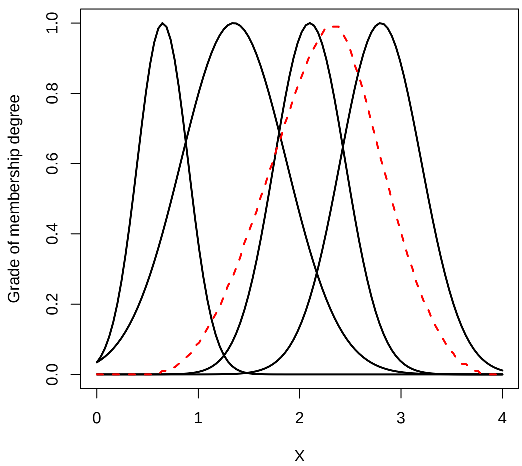
In order to clearly address the properties of type-1 OWA operators, the concept of a partial order relation of fuzzy sets [15] is needed to establish.

It is known that the set $\mathbb{R}$ of real numbers is linearly ordered and $(\mathbb{R}, \min, \max)$ is a lattice where min and max represent the minimum and maximum operators respectively, and for any $x, y \in \mathbb{R}$, a partial ordering relation $\geq$ $(\leq)$ is defined as

$$x \geq y \iff x \vee y = x \iff x \wedge y = y \tag{2}$$



(a) Linguistic weights used in a type-1 OWA Operation



(b) Dashed line-aggregation result; Solid lines-aggregated fuzzy sets

Figure 1: Example of application of a type-1 OWA operator with t-norm min

On the other hand, in soft decision making, given two linguistic variables A and B with the truth-values $v(A)$ and $v(B)$, in which $v(A)$ and $v(B)$ are two fuzzy sets, Zadeh defined two basic operators [13] that are able to perform the aggregations of "A *and* B" and "A *or* B" by calculating the truth-values $v(A \textit{ and } B)$ and $v(A \textit{ or } B)$ separately. In 1976 the two operators were named as *meet* and *join* [14]. Very interestingly, based on the extension principle *meet*($\sqcap$) and *join* ($\sqcup$) are the generalisations of the lattice min and max operators to type-1 fuzzy sets respectively. Hence $A \sqcap B$ and $A \sqcup B$ can be referred to as the fuzzy minimum and fuzzy maximum of fuzzy sets A and B. $(F(\mathbb{X}), \sqcap, \sqcup)$ forms a distributive lattice [15] describing a partial ordering relation of fuzzy sets A and B as follows:

Definition 2. *Given any two fuzzy numbers A and B, an order relation $\succcurlyeq$ is defined as*

$$\begin{aligned} A \succcurlyeq B \quad &\Longleftrightarrow \quad A \sqcup B = A \\ &\Longleftrightarrow \quad A \sqcap B = B \end{aligned} \tag{3}$$

On the other hand, Dubois and Prade first suggested the following order relation [16], and afterwards it was exactly formulated by Ramik and Riminek [17].

Definition 3. *Given any two fuzzy numbers A and B, an ordering relation $\tilde{\geq}$ is defined as*

$$A \tilde{\geq} B \iff A_{\alpha+} \geq B_{\alpha+} \textit{ and } A_{\alpha-} \geq B_{\alpha-}$$

where $\forall \alpha \in [0, 1], A_\alpha = [A_{\alpha-}, A_{\alpha+}]$ and $B_\alpha = [B_{\alpha-}, B_{\alpha+}]$ are the α-cuts of A and B, respectively.

The order relation $\tilde{\geq}$ satisfies the axioms of partial order relation on $F(\mathbb{X})$ [17], and it is called the fuzzy max order. Interestingly, according to the following Lemma 1 [17], the partial order relations $\tilde{\geq}$ and $\succcurlyeq$ on fuzzy sets are equivalent.

Lemma 1. *Given any two fuzzy numbers A and B, the following three conditions are equivalent:*

1. $A \tilde{\geq} B$
2. $A \sqcup B = A$
3. $A \sqcap B = B$

4 Definition of α-Level Type-1 OWA Aggregation and Representation Theorem

The above examples show that operations on set values required in the application of a type-1 OWA operator are completely different from the ones on crisp numbers required by Yager's OWA operator. In order to analyse the properties of the type-1 OWA operator, we further define a new operator, called an α-level type-1 OWA operator. This operator is guided by the α-level cuts of linguistic weights and allows the aggregation of the α-level cuts of any number of fuzzy sets.

Definition 4. *Let $W = \{W^i\}_{i=1}^n \subset F(\mathbb{U})$ be a set of fuzzy linguistic weights defined on the domain of discourse $\mathbb{U} = [0,1]$, and $\alpha \in [0,1]$. The α-level type-1 OWA operator with α-cuts $\{W_\alpha^i\}_{i=1}^n$ is the operator that aggregates the α-cuts of the fuzzy sets $\{A^1, \cdots, A^n\} \in F(\mathbb{X})$ as follows:*

$$\Phi_\alpha^W\left(A_\alpha^1, \cdots, A_\alpha^n\right) = \left\{ \frac{\sum_{i=1}^n w_i a_{\sigma(i)}}{\sum_{i=1}^n w_i} \middle| w_i \in W_\alpha^i, a_i \in A_\alpha^i, i = 1, \cdots, n \right\} \tag{4}$$

where σ is a permutation function such that $a_{\sigma(i)} \geq a_{\sigma(i+1)}, \forall i = 1, \cdots, n-1$, $W_\alpha^i = \{w | \mu_{W^i}(w) \geq \alpha\}$, and $A_\alpha^i = \{x | \mu_{A^i}(x) \geq \alpha\}$.

When the linguistic weights and the aggregated objects are fuzzy numbers, the α-level type-1 OWA operator produces closed intervals. Then a natural question to be answered is: what is the relationship between $\Phi_\alpha^W\left(A_\alpha^1, \cdots, A_\alpha^n\right)$ and the α-cuts of $\Phi_W\left(A^1, \cdots, A^n\right)$, i.e., $\left(\Phi_W\left(A^1, \cdots, A^n\right)\right)_\alpha = G_\alpha$? The following Theorem 1 and Corollary 1 address their relationships. Due to limited space we omit their proofs which will appear in a journal article currently being prepared.

Theorem 1. *For any fuzzy sets $A^1, \cdots, A^n$ and $\alpha \in [0,1]$,*

$$\left(\Phi_W\left(A^1, \cdots, A^n\right)\right)_\alpha \subseteq \Phi_\alpha^W\left(A_\alpha^1, \cdots, A_\alpha^n\right).$$

Corollary 1. *If Φ_W is a type-1 OWA operator with min t-norm, then for any fuzzy sets $A^1, \cdots, A^n$ and $\alpha \in [0,1]$,*

$$\left(\Phi_W\left(A^1, \cdots, A^n\right)\right)_\alpha = \Phi_\alpha^W\left(A_\alpha^1, \cdots, A_\alpha^n\right)$$

According to the Representation Theorem of fuzzy sets [12], the available α-cut sets can be used to construct a fuzzy set, so

$$G = \bigcup_{0 \leq \alpha \leq 1} \alpha \left(\Phi_W\left(A^1, \cdots, A^n\right)\right)_\alpha \tag{5}$$

Its membership function is

$$\mu_G(y) = \bigvee_{\alpha:\ y \in \left(\Phi_W\left(A^1, \cdots, A^n\right)\right)_\alpha} \alpha \tag{6}$$

Then, from Theorem 1 and Corollary 1, we have the following theorem, which we call the *Representation Theorem of type-1 OWA operators*:

Theorem 2. *Let Φ_W be a type-1 OWA operator defined by $W = \{W^i\}_{i=1}^n \subset F(\mathbb{U})$, and Φ_α^W be the corresponding α-level type-1 OWA operator defined by the α-cuts of weights in the W. For any fuzzy sets $A^1, \cdots, A^n$, let $G = \Phi_W\left(A^1, \cdots, A^n\right)$, and M be the following fuzzy set*

$$M \triangleq \bigcup_{0 \leq \alpha \leq 1} \alpha \Phi_\alpha^W\left(A_\alpha^1, \cdots, A_\alpha^n\right) \tag{7}$$

Then we have:

1. *$G \subseteq M$*
2. *$G = M$ when the t-norm used in the type-1 OWA aggregation is the minimum operator.*

5 Properties of Type-1 OWA Operators

It is known that Yager's OWA operators are idempotent, monotonic, compensative and commutative [8]. In this Section, we investigate whether the properties of idempotency, monotonicity, compensativity and commutativity remain for type-1 OWA operators. Due to limited space we omit the proofs of these properties and will supply these in a future article.

Firstly, it is clear that Yager's OWA operators, t-norms and sup operators are commutative, so the type-1 OWA operator is commutative as well.

Theorem 3. *For any type-1 OWA operator* Φ_W*, and* $\forall A^1, \cdots, A^n \in F(\mathbb{X})$

$$\Phi_W\left(A^1, \cdots, A^n\right) = \Phi_W\left(A^{p(1)}, \cdots, A^{p(n)}\right)$$

where p *is a permutation function* $p\colon \{1, \cdots, n\} \longrightarrow \{1, \cdots, n\}$.

The following Theorem establishes that the type-1 OWA operators with minimum t-norm ($\wedge$) and linguistic weights modelled by fuzzy numbers are idempotent

Theorem 4. *For type-1 OWA operators* Φ_W *with minimum t-norm* ($\wedge$) *and linguistic weights modelled by fuzzy numbers, and* $\forall A \in F(\mathbb{X})$*, we have*

$$\Phi_W(A, \cdots, A) = A$$

In what follows we investigate when monotonicity is verified by type-1 OWA operators. Firstly, we propose the following definition of *α-equivalently ordered* for two groups of fuzzy numbers:

Definition 5. *Let* $\left\{A^i\right\}_{i=1}^n \subset F(\mathbb{X})$ *and* $\left\{B^i\right\}_{i=1}^n \subset F(\mathbb{X})$ *be two groups of fuzzy numbers. If for each* $\alpha \in [0,1]$*,*

$$A_{\alpha+}^{\sigma(1)} \geq A_{\alpha+}^{\sigma(2)} \geq \cdots \geq A_{\alpha+}^{\sigma(n)} \Longrightarrow B_{\alpha+}^{\sigma(1)} \geq B_{\alpha+}^{\sigma(2)} \geq \cdots \geq B_{\alpha+}^{\sigma(n)}$$

$$A_{\alpha-}^{\eta(1)} \geq A_{\alpha-}^{\eta(2)} \geq \cdots \geq A_{\alpha-}^{\eta(n)} \Longrightarrow B_{\alpha-}^{\eta(1)} \geq B_{\alpha-}^{\eta(2)} \geq \cdots \geq B_{\alpha-}^{\eta(n)}$$

where σ *and* η *are two permutations of* $\{1, \cdots, n\}$*, then* $\left\{B^i\right\}_{i=1}^n$ *is said to be α-equivalently ordered with* $\left\{A^i\right\}_{i=1}^n$.

For example, the set of fuzzy sets $\{B^1, B^2, B^3\}$ shown in Figure 2 is *α-equivalently ordered* with $\{A^1, A^2, A^3\}$. However, the set of fuzzy sets $\{B^1, B^2, B^3\}$ shown in Figure 3 is not *α-equivalently ordered* with $\{A^1, A^2, A^3\}$; it can be seen that at the $\alpha = 0.2$ level, the permutation σ is $\sigma = [3,\ 2,\ 1]$, but $B_{0.2+}^3 \geq B_{0.2+}^1 \geq B_{0.2+}^2$.

The following result provides the conditions under which a type-1 OWA operator verifies monotonicity:

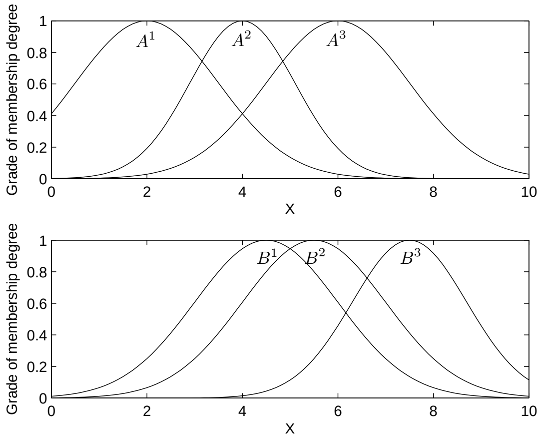


Figure 2: *α-equivalently ordered* fuzzy sets B^1, B^2, B^3 (bottom: from left to right) with A^1, A^2, A^3 (up: from left to right)

Theorem 5. *Assume* Φ_W *is a type-1 OWA operator in which min t-norm is used. Let* $\left\{A^i\right\}_{i=1}^n \subset F(\mathbb{X})$ *be the aggregated fuzzy number objects, and* $\left\{B^i\right\}_{i=1}^n \subset F(\mathbb{X})$ *be the second group of aggregated fuzzy number objects. If for each* i*,*

$$A^i \succcurlyeq B^i$$

and $\left\{B^i\right\}_{i=1}^n$ *is α-equivalently ordered with* $\left\{A^i\right\}_{i=1}^n$*, then*

$$\Phi_W\left(A^1, \cdots, A^n\right) \succcurlyeq \Phi_W\left(B^1, \cdots, B^n\right)$$

We shall now introduce two special type-1 OWA operators induced by singleton weights.

Definition 6. $\Phi_{\overline{W}}$ *is a type-1 OWA operator associated with singleton weights:* $W^1 = \dot{1}$, $W^i = \dot{0}$ $(i \neq 1)$*, i.e,*

$$\mu_{W^1}(w) = \begin{cases} 1 & w = 1 \\ 0 & others \end{cases} \quad (8)$$

$$(i \neq 1)\ \ \mu_{W^i}(w) = \begin{cases} 1 & w = 0 \\ 0 & others \end{cases} \quad (9)$$

And $\Phi_{\underline{W}}$ *is a type-1 OWA operator associated with singleton weights:* $W^n = \dot{1}; W^i = \dot{0}$ $(i \neq n)$.

Theorem 6. *Any type-1 OWA operator* Φ_W *with minimum t-norm lies between the* $\Phi_{\underline{W}}$

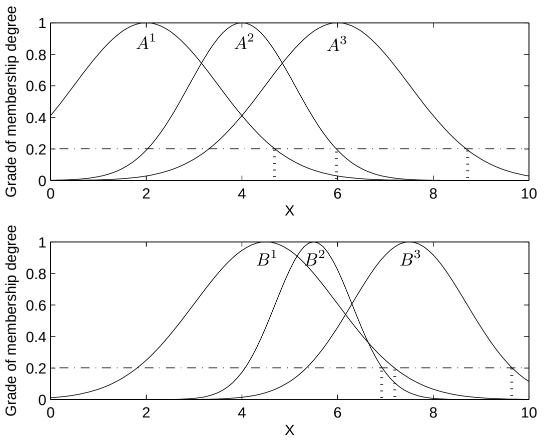


Figure 3: Fuzzy sets B^1, B^2, B^3 (bottom: from left to right) not α-*equivalently ordered* with A^1, A^2, A^3 (up: from left to right)

and $\Phi_{\overline{W}}$ *in aggregating fuzzy number objects:*

$$\begin{aligned} &\Phi_{\overline{W}}\ \left(A^1, \cdots, A^n\right) \succcurlyeq\ \Phi_W\ \left(A^1, \cdots, A^n\right) \\ &\succcurlyeq\ \Phi_{\underline{W}}\ \left(A^1, \cdots, A^n\right) \end{aligned} \tag{10}$$

On the other hand, we have proved that [11]

$$\Phi_{\overline{W}}\left(A^1, \cdots, A^n\right) = A^1 \sqcup \cdots \sqcup A^n$$

$$\Phi_{\underline{W}}\left(A^1, \cdots, A^n\right) = A^1 \sqcap \cdots \sqcap A^n$$

Considering that the *join* and *meet* of fuzzy numbers are the generalisation of the maximum and minimum of crisp values, Theorem 6 indicates that type-1 OWA operators with minimum t-norm are compensative.

6 Conclusions and Discussions

This paper has proposed the α-level type-1 OWA operator in order to aggregate the α-cuts of fuzzy sets and has indicated that a type-1 OWA operator can be decomposed into a series of the α-level type-1 OWA operators. Then this paper has described that, under certain conditions, the type-1 OWA operator is idempotent, monotonic, compensative and commutative. These properties provide a solid basis for the type-1 OWA operator to be further applied to multi-expert decision making, multi-criteria decision making and multi-expert multi-criteria decision making. Future articles will provide the missing proofs from this article and explore the roles of the type-1 OWA operator in a medical application. Other interesting topics inspired by the proposed new type of OWA operator include how to aggregate type-2 fuzzy sets [18, 19] and non-stationary fuzzy sets [20, 21] via OWA mechanism; the possibility of applying the type-1 OWAs to merging similar fuzzy sets for improving fuzzy model interpretability/transparancy and parsimony [22, 23], etc..

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