Fuzzy Vehicle Routing Problem with Time Windows

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Abstract

We consider the Fuzzy Vehicle Routing Problem with Time Windows (FVRPTW) where the travel times are triangular fuzzy numbers. The Chance-Constrained Programming (CCP) methodology is used to handle uncertainty and specify a confidence level at which it is desired that the travel times to reach the customers fall into their time windows. We propose and analyze the application of GRASP metaheuristic to minimize the total distance traversed by the vehicles while the capacity constraints are satisfied and the service times fall within time windows at given confidence level.

Keywords: VRPTW, Fuzzy sets, GRASP.

1 Introduction

The efficient design of distribution strategies plays an important role in the success of logistic management because it improves service quality and reduces transportation costs. A standard objective in a distribution system for customers geographically dispersed customers is to determine the set of routes for the available vehicles which satisfies some constraints and minimize the total fleet operating cost.

Vehicle Routing Problems (VRP) are concerned with finding the best set of routes, beginning and ending at a depot, for a fleet of vehicles to serve customers with demands for some commodity [3] [28]. The Vehicle Routing Problem with Time Windows (VRPTW) considers time windows for each customer and the customers have to be served by a vehicle within their time windows [8] [19].

In many practical problems in transport and logistics it is necessary to take into account that the available knowledge about some data and parameters of the problem model is imprecise or uncertain. Therefore it is necessary to model the problems and to evaluate their solutions, bearing in mind parameters and variables that are characterized by their uncertainty.

There several possible sources of uncertainty in Vehicle Routing Problems; usually demands and travel times [2], [23], [24], [26], [27], [29]. Recent researches use hybrid algorithms that integrate stochastic simulation, fuzzy theory and an optimization procedure for solving these problems [10], [11], [16], [21], [33].

Chance-constrained Programming (CCP) is a methodology proposed by Charnes and Cooper [1] to specify levels of confidence for stochastic constraints within optimization problems. This approach can be used to determine if the obtained solutions of the problem with fuzzy information on the parameters are feasible, incorporating the required level of confidence for the model restrictions [12], [13], [14], [15], [17], [18].

Next section describes the use of triangular fuzzy numbers to model the uncertainty in travel times and the use of possibility and credibility measures associated with fuzzy variables to handle the uncertainty in the inequality constraints with these variables. Section 3 introduces the Fuzzy Vehicle Routing Problem with Time Windows (FVRPTW) obtained from the crisp VRPTW by considering that the travel times in the underlying communication network are fuzzy triangular variables. The travel times are usually fuzzy variables, for instance, since these times are very influenced by road and traffic conditions.

Bearing in mind these concepts we propose a Greedy Randomized Adaptive Search Procedure (GRASP) for solving the Fuzzy Vehicle Routing Problem with Time Windows. Finally, we describe the experiments with a small instance given in [33].where we apply the proposed procedure and we compare it with their results. Brief conclusions are also included to end the paper.

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2 Fuzzy travel time

An ordinary set can be described, among other ways, by using the characteristic function on an universe, in which 1 indicates membership and 0 nonmembership. However, in many cases, the membership is not clear when the sets are imprecisely described. In order to deal with them, Zadeh [30] introduced the concept of a fuzzy set given by a membership function from the universe to the real interval [0,1]. Fuzzy sets have been well developed and applied in a wide variety of real problems. As a fuzzy set of real numbers, the fuzzy variable was first introduced by Kaufmann [9]. For each $\alpha \in [0,1]$, the α-cut of a fuzzy set is the ordinary set of values where the membership is equal or greater than α. The support of a fuzzy set is the set of values whose membership is positive and its mode is the value with maximum membership. A fuzzy number is usually defined as a fuzzy set of real numbers whose α-cuts are closed intervals, with compact support and unique mode.

In real-life applications of routing problems, it is often the case that the exact travel time between two locations cannot be known in advance. However, based on previous experience, a user may have some knowledge about the travel time. In a model on a graph, the length of each edge or link can be represented by a fuzzy variable representing the time used to traverse it.

A little knowledge can be used to assign fuzzy intervals to represent the travel times, since precise distributions, would require a deeper knowledge of the instance and usually yield a complex calculus. If the knowledge can be used to specify values that appear to be more plausible than others, a natural extension is to use fuzzy numbers. The simplest models for these fuzzy variables are the triangular fuzzy numbers [5]. A triangular fuzzy variable is given by its support $[a_1,a_3]$ (the set of possible values) and its mode $a_2 \in [a_1,a_3]$ (the most plausible value). This triangular fuzzy number is denoted by $Tr(a_1,a_2,a_3)$ and has the following membership function.

$$Tr(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \dfrac{x-a_3}{a_2-a_3} & a_2 \le x \le a_3 \\ 0 & a_3 < x \end{cases}$$

The usual arithmetic operators for real numbers are extended to fuzzy numbers by the *Extension Principle* through the α-cuts [31]. However, the resulting numerical computations to operate with simple fuzzy numbers easily become intractable [4]. Therefore, for shake of simplicity and tractability of operations the results of the algebraic operations with triangular fuzzy numbers are approximated by triangular fuzzy numbers. So we only need to work with the three values defining the membership functions of triangular fuzzy numbers.

The sum of two triangular fuzzy numbers by the Extension Principle is also a triangular fuzzy number. This is not always true for maximum ($\vee$) and minimum ($\wedge$) of two fuzzy numbers. Then we use the following approximations:

$T(a_1,a_2,a_3) + T(b_1,b_2,b_3) = T(a_1+b_1,a_1+b_2,a_1+b_3)$
$T(a_1,a_2,a_3) \vee T(b_1,b_2,b_3) = T(a_1 \vee b_1,a_1 \vee b_2,a_1 \vee b_3)$
$T(a_1,a_2,a_3) \wedge T(b_1,b_2,b_3) = T(a_1 \wedge b_1,a_1 \wedge b_2,a_1 \wedge b_3)$

For every $\alpha \in [0,1]$, if $[x_\alpha,x^\alpha]$ is the α-cut of the maximum of two fuzzy numbers and $[z_\alpha,z^\alpha]$ is the α-cut of its triangular approximation then $x_\alpha \le z_\alpha$, and $z^\alpha \le x^\alpha$. Therefore the real maximum and its triangular approximations have the same mode and the same support.

Possibility theory was proposed by Zadeh [32], and developed by many researchers such as Dubois and Prade [6] to manage the uncertainty in fuzzy variables. Following Nahmias [20] we have the following definitions.

A possibility measure *Pos* on a nonempty universe Θ is a nonnegative function on its subsets such that $Pos(\emptyset) = 0$, $Pos(\Theta) = 1$ and $Pos(\cup_k A_k) = \sup_k Pos(A_k)$ for arbitrary subsets $A_k \subseteq \Theta$. A fuzzy variable ξ is defined as a function from the universe Θ to the real numbers whose membership function is related to the possibility *Pos* by $\mu(x) = Pos\{ \theta \in \Theta: \xi(\theta)=x \}$. From the possibility measure we get the necessity measure of every subset A defined by $Nec(A) = 1 - Pos(A^c)$ and the credibility measure defined by $Cr(A) = (Pos(A)+Nec(A))/2$.

If μ is the membership function of a fuzzy variable ξ then $Pos(\xi \le x) = \sup_{u \le x} \mu(u)$ and $Nec(\xi \le x) = 1 - \sup_{u > x} \mu(u)$. For a triangular fuzzy variable $\xi = Tr(a_1,a_2,a_3)$ we have the following equations.

$$Pos(\xi \le x) = \begin{cases} 1 & a_2 < x \\ \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ 0 & x < a_1 \end{cases}$$

$$Nec(\xi \le x) = \begin{cases} 1 & a_3 < x \\ \dfrac{x-a_3}{a_2-a_3} & a_2 \le x \le a_3 \\ 0 & x < a_2 \end{cases}$$

$$Cr(\xi \le x) = \begin{cases} 1 & a_3 < x \\ \dfrac{x-2a_3+a_2}{2(a_2-a_3)} & a_2 \le x \le a_3 \\ \dfrac{x-a_1}{2(a_2-a_1)} & a_1 \le x \le a_2 \\ 0 & x < a_1 \end{cases}$$

These formulas will be used to know the credibility that a solution satisfies the constraints.

3 Fuzzy VRP with Time Windows

We consider the Vehicle Routing Problem with Time Windows (VRPTW) and extend the usual model to include triangular fuzzy travel times.

The VRPTW is given by a set of k identical vehicles to serve a set of n customers within given time windows departing form a depot. Each vehicle goes by a route visiting a number of nodes satisfying their own demand.

We assume that:

- Each vehicle has a container with a capacity limitation and the total loading of each vehicle cannot exceed its capacity.
- Each vehicle is assigned to only one route on which there may be more than one customer.
- Each customer will be visited by one and only one vehicle.
- Each route begins and ends at the depot.
- Each customer has its time window within which the delivery is permitted to start.
- The travel times between customers are assumed to be triangular fuzzy variables.

We consider the following indices and model parameters:

$i = 0$: is the depot index;

$i = 1, 2, ..., n$: are the customers indexes;

$k = 1, 2, ..., m$: are the vehicles;

$q[i]$: is the amount of demand of customer i; $i = 1, 2, ..., n$;

$C[k]$: is the capacity of vehicle k; $k = 1, 2, ..., m$;

$d[i,j]$: is the distance from customers (or depot) i to j; $i,j = 0, 1, ..., n$;

$T[i,j]$: is the triangular fuzzy travel time from customer i to j; $i,j = 0, 1, ..., n$;

$U[i,j]$: is the unloading time at customer i; $i = 1, 2, ..., n$;

$[a[i],b[i]]$: is the time window of customers i; $a[i]$ and $b[i]$ are the respective beginning and end of the time window; $i = 1, 2, ..., n$.

We describe the operational plan consisting of the routes for the k vehicles to serve the n customers by a single decision vectors x of size $n+m+1$ denoted by $x = (x_0, x_1, x_2, ..., x_{n+m})$ that is a rearrangement of $(0, 1, 2, ..., n+m)$ such that $x_0 = 0$ and $x_{n+m} > n$. Each element of the solution greater than n represent a vehicle at depot in such a way that $x_r = n+k$ represents the vehicle k at depot. The customers visited for this vehicle are those of the indexes that appeared in the solution from the previous vehicle (x_0 if this is the first vehicle). For instance, a solution of a problem with 9 customers and 3 vehicles can be:

$$x = [\ 0, 5, 3, 4, 7, 12, 8, 2, 10, 1, 9, 6, 11\ ]$$

That represents the three routes:

- $0 \to 5 \to 3 \to 4 \to 7 \to 0$-(12): vehicle 3
- (12)-$0 \to 8 \to 2 \to 0$-(10): vehicle 1
- (10)-$0 \to 1 \to 9 \to 6 \to 0$-(11): vehicle 2

In this way, the vector arrangement x ensure that that

- each vehicle will be used at most one time;
- all tours begin and end at the depot;
- each customer will be visited by one and only one vehicle; and
- there is no sub-tour.

The objective of the optimization problem to be minimized is the total traveled distance of the solution x that is computed by:

$$F = \sum_{r=0}^{n+m} d[x_r, x_{r+1}]$$

where for every $i > n$, $d[i,j] = d[j,i] = d[0,j]$, for all $j = 1,2, \ldots, n$.

The amount of demand served by each vehicle is computed by recurrent formula on the vector x. Take $Q[0] = 0$, and, for $r = 1, 2, \ldots, m+n$, apply: If $x_{r-1} > n$ then $Q[x_r] = q[x_r]$, otherwise

$$Q[x_r] = Q[x_{r-1}] + q[x_r].$$

For the first indexes $x_r > n$, the value computed for $Q[x_r]$ represents the load of the vehicle $x_r - n$. Therefore its capacity constraint is verified by

$$Q[x_r] \leq C[x_r - n].$$

The service time for each customer, denoted by $S[.]$ is obtained by similar recurrent formula on the vector x. We use the recursive formula:

$$S[x_r] = a[x_r] \vee (S[x_{r-1}] + U[x_{r-1}] + T[x_{r-1}, x_r])$$

with the following conventions. The initial values at the depot are $S[x_0] = 0$ and $U[x_0] = 0$. If x_r corresponds to a vehicle ($x_r > n$) the service time is set to 0 to compute the service time for the next customer x_{r+1} by the above recurrent formula. For these cases, the values of $U[x_r] = U[j]$ for $j > n$ are also null, so that the service time for the next customer x_{r+1} (that is the first customer of the next vehicle) is:

$$S[x_{r+1}] = a[x_{r+1}] \vee T[0,x_{r+1}].$$

A positive value for $U[j]$ for these cases ($j > n$) would correspond to some time-consuming preprocessing operation at the depot like the load of the vehicles. The time windows for these indexes ($[a[j],b[j]]$ for $j > n$) do not exist unless the vehicle have to return to the depot within given time interval.

Note that since the times between customers $T[i,j]$ are triangular fuzzy numbers then also each the service time $S[x_r]$ is also triangular fuzzy number. The load times $U[x_r]$ and beginning of the time windows $a[x_r]$ are crisp number that are special case of triangular fuzzy numbers where the three defining numbers are equal. Then the credibility that, with the solution x, the customer x_r is served within its time windows is: $Cr(S[x_r] \leq b_r)$.

Therefore, at confidence value α, the solution x verifies the fuzzy constraint of service times within the corresponding time window if:

$$max_r\ Cr(S[x_r] \leq b_r). \geq \alpha.$$

4. GRASP

We propose a solution algorithm based on GRASP (Greedy Randomized Adaptive Search Procedure) for the FVRPTW. GRASP is a multi-start two-phase metaheuristic for combinatorial optimisation proposed by Feo and Resende [7] basically consisting of a construction phase and a local search improvement phase. It is a recently exploited method that combines the power of greedy heuristics, randomisation, and local search in an adaptive schema. The *solution construction* mechanism builds a solution step-by-step by adding a random new element from a candidate list (the restricted candidate list RCL) to the current partial solution. Subsequently, a *local search* phase is applied to try to improve the current solution. This two-phase process is iterative, continuing until the user termination condition such as the maximum allowed CPU time or the maximum number of iterations is reached.

In the construction phase of GRASP, the greedy criterion for updating the RCL can be *value-based* or size-based restricted candidate list is used [22]. The value-based RCL involves placing in the list only the candidate items having a greedy value not greater than a specified threshold, whose values should vary dynamically during the search process. In the other side, the size-based RCL involves placing in the list a specified number of candidate items, those having the greatest greedy values, the size of the RCL should also vary dynamically during the search process. The size or value of the threshold of the RCL needs to be tuned to get a good balance between intensification and diversification. Indeed, a small RCL results in a large intensification capability and a small diversification capability. This means that the resulting algorithm is very fast, but it can easily become trapped at a local optimum. Conversely, a large list size produces an algorithm with a large diversification capability, but a short intensification capability, because its behaviour approaches a completely random construction mechanism.

Constructive phase of GRASP for FVRPTW

```
1.      Initialization.
1.1.         x ← [0, 1, …, n, …, n+m].
1.2.         F ← 0 ;
1.3          r ← 0 ;
1.4          s ← n ;
1.5          M ← 1 ; /* Minimum credibility
2.      Iterations.
2.0          Repeat
2.1               s ← s+1 ;
2.2               Select j∈[s..n+m]
2.3               Interchange x[s] and x[j] ;
2.4               Q ← 0 ; S ← 0 ;
2.5               Repeat
2.6                    z ← x[r] ;
2.7                    S ← S + U[z] ;
2.8                    r ← r+1 ;
2.9                    Select i∈LCR[r..s-1]
2.10                   If Q + q[x[i]] < Cap[s] AND Cr(S+T[z,x[i]] > b[x[i]]) > ALPHA
2.11                   Then
2.12                        Interchange x[r] and x[i] ;
2.13                        S[x[r]] ← S + T[z,x[r]] ;
2.14                        [x[r]] ← a[x[r]]∨S[x[r]];
2.15                        Q ← Q + q[x[r]] ;
2.16                        F ← F + D[z,x[r]] ;
2.17                   Else      /* Complete vehicle
2.18                        F ← F + D[z,0] ;
2.19                        r ← r+1 ;
2.20                        Interchange x[r] and x[s] ;
2.21                        Goto 2.1.
2.22              Until r = s
2.23         Until s = n+m
```

Figure 1: Constructive phase of GRASP

The construction phase of our GRASP for the FVRPTW problem is specified in Figure 1. It starts with the original sequence from 0 to $n+m$ that is rearranged by choosing the components x_r, $r = 1, 2, \ldots, n+m$, one by one. After the initialization (1.1-1.5) the algorithm chooses the first vehicle and then starts to choose customers.

Once each vehicle is chosen (2.2), its partial load and used time are initialized (2.4), before to start the selection of its customers. The customers are randomly selected from the Restricted Candidate List LCR (2.9) until the capacity of the vehicle or the credibility limits do not allow choosing new customers (2.10). With the selected customer, the service time (2.13-14), the partial load (2.15), and the total distance (2.16) are updated.

When there is not new customer to add (2.17) the distance to the depot is added to the objective (2.18) and the vehicle is stored in x (2.19-20). The algorithm continues to select a new vehicle (2.21) if there are customers (2.22) and vehicles to choose.

For the improvement phase of GRASP, we can use the well known local searches for VRPTW based on swap moves, Or-Opt moves and r-opt moves (2-opt or 3-opt) [19]. For the sake of simplicity, we use in our first experiments the single swap moves consisting of interchanging two items in x. Note that, for each move, the objective value is easily computed in O(1) time but feasibility must be tested by the sequential applications of recurrent formula 2.13-2.14 with the resetting of Q and S each time a vehicle is encountered while condition 2.10 is tested.

Table 1: Demands (q) and distances (d) between customers

d	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	q
1	19.0																		200
2	17.5	6.0																	100
3	28.0	11.0	10.5																140
4	24.0	21.0	15.0	20.0															160
5	24.5	32.0	26.0	34.0	15.5														200
6	31.2	44.5	39.5	49.0	31.0	16.0													60
7	31.0	48.5	45.0	55.5	41.5	28.5	16.0												200
8	21.0	37.5	33.5	44.0	30.0	18.5	13.0	11.5											135
9	18.0	36.0	33.0	44.0	38.0	24.0	20.0	13.5	7.0										120
10	21.5	40.0	39.0	49.5	43.0	36.5	32.5	21.0	20.0	13.0									140
11	36.5	55.0	54.0	65.0	56.0	46.0	37.0	21.0	28.0	23.0	15.5								100
12	31.5	46.5	48.0	57.0	55.5	51.0	48.5	36.0	36.0	29.0	16.0	21.5							200
13	23.0	38.5	39.0	48.5	47.0	44.0	43.0	32.5	30.0	23.0	12.0	23.0	9.0						80
14	28.0	38.5	41.5	49.0	52.0	51.0	52.5	43.5	40.0	33.0	22.5	33.0	13.0	11.0					60
15	34.5	40.0	44.0	50.0	56.5	58.5	62.0	54.0	50.0	43.0	34.0	44.5	24.0	22.0	11.5				200
16	30.0	29.5	34.5	38.0	48.5	54.0	60.5	56.0	49.0	43.5	38.0	51.5	33.0	28.0	20.5	14.0			90
17	18.5	16.5	21.3	26.5	35.0	41.0	50.0	48.0	39.0	35.0	33.0	48.0	34.0	27.0	24.0	23.5	13.5		200
18	24.0	14.0	20.0	22.0	35.0	44.0	54.5	55.0	45.0	41.5	41.0	56.5	43.0	35.0	32.0	36.0	17.0	8.5	100

6. Experiments

For the experiments we use the example of Zheng and Liu [33]. It has 18 customers, labelled “1”, “2”, …, “18”, and a depot, labelled ‘0’. The amount of demand at each customer and the distances between customers (and the depot) are given in Table 1.

The travel times are triangular fuzzy numbers shown in Tables 2 and 3, with the time windows of each customer. The unloading time at each location is 15 minutes and the capacity of each of the four available vehicles is 1000. The considered confidence level α is 0.90.

Table 2. Time windows and fuzzy travel times between customers (part I)

T	Window	0	1	2	3	4	5	6	7	8
1	[09:00,15:50]	(25,50,75)								
2	[09:20,15:30]	(5,10,15)	(20,40,60)							
3	[09:40,14:40]	(25,50,75)	(5,10,15)	(20,40,60)						
4	[09:20,14:30]	(7,15,23)	(25,50,75)	(7,15,23)	(22,45,68)					
5	[09:00,15:20]	(25,50,75)	(17,35,53)	(17,35,53)	(15,30,45)	(17,35,53)				
6	[09:00,14:20]	(25,50,75)	(7,15,23)	(20,40,60)	(2,5,8)	(22,45,68)	(15,30,45)			
7	[09:30,14:10]	(12,25,38)	(20,40,60)	(15,30,45)	(17,35,53)	(7,15,23)	(12,25,38)	(17,35,53)		
8	[09:00,15:30]	(7,15,23)	(20,40,60)	(5,10,15)	(22,45,68)	(10,20,30)	(17,35,53)	(20,40,60)	(17,35,53)	
9	[09:10,15:50]	(25,50,75)	(7,15,23)	(22,45,68)	(5,10,15)	(22,45,68)	(15,30,45)	(5,10,15)	(20,40,60)	(20,40,60)
10	[09:40,13:20]	(10,20,30)	(22,45,68)	(12,25,38)	(22,45,68)	(7,15,23)	(15,30,45)	(20,40,60)	(5,10,15)	(12,25,38)
11	[09:10,14:20]	(25,50,75)	(5,10,15)	(17,35,53)	(15,30,45)	(17,35,53)	(5,10,15)	(15,30,45)	(5,10,15)	(17,35,53)
12	[09:00,15:20]	(27,55,83)	(17,35,53)	(17,35,53)	(15,30,45)	(17,35,53)	(2,5,8)	(15,30,45)	(7,15,23)	(17,35,53)
13	[09:20,16:30]	(5,10,15)	(20,40,60)	(5,10,15)	(20,40,60)	(7,15,23)	(15,30,45)	(17,35,53)	(17,35,53)	(5,10,15)
14	[09:20,16:00]	(25,50,75)	(5,10,15)	(20,40,60)	(2,5,8)	(22,45,68)	(15,30,45)	(2,5,8)	(17,35,53)	(17,35,53)
15	[09:20,14:30]	(22,45,68)	(5,10,15)	(20,40,60)	(5,10,15)	(22,45,68)	(15,30,45)	(5,10,15)	(17,35,53)	(17,35,53)
16	[09:00,15:10]	(7,15,23)	(22,45,68)	(7,15,23)	(22,45,68)	(10,20,30)	(15,30,45)	(22,45,68)	(17,35,53)	(10,20,30)
17	[09:00,16:20]	(15,30,45)	(20,40,60)	(12,25,38)	(20,40,60)	(10,20,30)	(12,25,38)	(17,35,53)	(2,5,8)	(12,25,38)
18	[09:00,16:20]	(25,50,75)	(5,10,15)	(22,45,68)	(5,10,15)	(25,50,75)	(15,30,45)	(7,15,23)	(17,35,53)	(20,40,60)

The best solution obtained by Zheng and Liu [33] with a hybrid algorithm that combines a GA with fuzzy simulations (with 10,000 cycles in simulation, 5000 generations in GA) in 10 hours with a PC with 4 processors at 2 Ghz consists in the following four routes:

R1: 0→16→17→18→5→0
R2: 0→10→12→13→14→15→8→0
R3: 0→1→2→3→0
R4: 0→9→6→7→11→4→0

The total distance is F = 457.5 and the loads of the vehicles are 590, 815, 440 and 640.

We obtain with our GRASP a solution with only three routes that have loads 795, 930, and 760 and total distance travelled F = 365.5 in only 1 minute in a PC at 4.30 Ghz. The three routes are:

R1: $0 \to 17 \to 18 \to 16 \to 15 \to 14 \to 12 \to 13 \to 0$

R2: $0 \to 2 \to 1 \to 3 \to 4 \to 6 \to 8 \to 0$

R3: $0 \to 10 \to 9 \to 11 \to 7 \to 5 \to 0$

Table 3. Time windows and fuzzy travel times between customers (part II)

T	9	10	11	12	13	14	15	16	17
10	(22,45,68)								
11	(17,35,53)	(15,30,45)							
12	(17,35,53)	(12,25,38)	(7,15,23)						
13	(20,40,60)	(20,40,60)	(17,35,53)	(17,35,53)					
14	(5,10,15)	(20,40,60)	(15,30,45)	(15,30,45)	(20,40,60)				
15	(2,5,8)	(20,40,60)	(15,30,45)	(20,40,60)	(20,40,60)	(2,5,8)			
16	(22,45,68)	(15,30,45)	(17,35,53)	(7,15,23)	(7,15,23)	(22,45,68)	(22,45,68)		
17	(20,40,60)	(5,10,15)	(12,25,38)	(12,25,38)	(12,25,38)	(17,35,53)	(17,35,53)	(12,25,38)	
18	(7,15,23)	(20,40,60)	(15,30,45)	(20,40,60)	(20,40,60)	(7,15,23)	(7,15,23)	(20,40,60)	(20,40,60)

Conclusions

Fuzzy Logic systems have been used for manage the uncertainty in real logistic and transportation systems [25]. We consider the Fuzzy VRPTW considering uncertainty in the travel times that are modelled by triangular fuzzy numbers. The uncertainty in the constraints is managed with the Chance-Constrained Programming approach. A single experiment shows that our GRASP algorithm gets in very short time better solutions than the costly fuzzy simulation process proposed by Zheng and Liu [33].

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