Automatic selection of relevant levels in a hierarchy of fuzzy segmentations

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Abstract

In this paper we face the problem of the automatic selection of relevant levels in a hierarchy of fuzzy segmentations. Today there is a wide variety of techniques (multi-scale or hierarchical) to analyze an image with several levels of detail, but they don't consider that not all the segmentations resulting from this process are interesting for the user. Here we propose a technique based in measuring the homogeneity of the fuzzy segmentation in each level, and analyze its evolution along the hierarchy to select for the user a subset of relevant results.

Keywords: Fuzzy segmentation, Hierarchical segmentation, Relevant levels, Automatic Selection.

1 Introduction

In image segmentation framework it is well known that a given image can be analyzed under different detail levels: studying it in high detail the segmentation result will be a set of numerous, small, fine regions, while in a less detailed segmentation the results is a set with a few, big and coarse regions.

In some cases it is enough with setting the desired detail level [17], but every day more the leaning is to process the image with a multi-scale or hierarchical technique [20], that offer as result a segmentation of the image for each scale or level, respectively.

Today there is a number of multi-scale techniques, based on study the evolution of the contours through the different scales [20, 12, 21, 10], as well as hierarchical proposals, that start from an initial segmentation of the image and obtain each detail level merging regions from the previous one [22, 11, 8, 7]. The point shared by most of these methods is that they offer as a result for each level a different segmentation of the image, verifying the condition that the set of regions in that level is included in the set of regions of the next one.

Nonetheless these proposals offer the user the whole hierarchy, without considering that not all the levels are really interesting, and therefore the user must manually analyze the hierarchy, level by level, to select the relevant ones. In 2000 Tilton et al. [18] pointed this problem and presented a tool to make easier to the user the study the different levels of the hierarchy, but even though the problems related to supervised procedures, like subjectivity and time consumption, still remain.

It highlights the need of methods to automatically select results in segmentation hierarchies, solving hence problems like over-segmentation, or finding the level where a given region is better represented. This is a new and emerging topic, since the first proposal in this sense, was made in 2005 by Plaza

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et al. [14], who presented three region's descriptors to study how regions change from a hierarchical level to the next. However this proposal is specifically designed for hyperspectral image data sets, and is difficult to extend to general images.

Here we face the problem from a different point of view: we propose to establish the characteristics of an interesting result, i.e. a *relevant level*, select a criteria according to this idea, and analyze the observance of it along the hierarchy levels. In our proposal, like all the hierarchical approaches, we start from an initial segmentation of the image and build the hierarchy on it. As will be detailed later, we assume this segmentation is fuzzy, to incorporate the well known advantages of these techniques regarding the crisp ones [7, 12, 11, 8, 19], specially when there are blurred contours, color shades and light effects. This way, we obtain a hierarchy of fuzzy segmentations, from which we automatically select the relevant levels, with the method proposed here.

Considering it this paper is structured as follows: firstly, in section 2, we indicate how we obtain the hierarchy of fuzzy segmentations. Then, in section 3 we determine what we understand as *relevant levels* and propose a technique to automatically select them, showing in section 4 the results obtained with it. Finally we present our conclusions in section 5.

2 Hierarchy of Fuzzy Segmentations

Formalizing the above mentioned condition followed by most of hierarchical proposals [11, 2, 16], and extending it to the fuzzy case, we could say that:

Definition 2.1 *A Hierarchy of fuzzy segmentations for a given image $\mathcal{I}$, is a set, $H_{\widetilde{\Theta}}$, of fuzzy image segmentations, $H_{\widetilde{\Theta}} = \{\widetilde{\Theta}_1, \ldots, \widetilde{\Theta}_d\}$, such that the set of regions in the segmentation of a given level, $\widetilde{\Theta}_i$, is included in the next level segmentation, $\widetilde{\Theta}_{i+1}$, considering the inclusion in the sense of the less degree; i.e., given $\widetilde{R}_s^i \in \widetilde{\Theta}_i$, $\widetilde{R}_t^{i+1} \in \widetilde{\Theta}_{i+1}$,*

$$\mu_{\widetilde{R}_s^i}(p) \leq \mu_{\widetilde{R}_t^{i+1}}(p) \qquad (1)$$

$\forall p \in \mathcal{I}\ \forall i \in \{1 \ldots d-1\}$.

In the literature can be found several proposals to obtain a hierarchy with these characteristics, based on a merging process applied to the initial fuzzy segmentation [7, 11]. These techniques must face three problems: the calculation of the initial segmentation, the selection of some criteria to decide which regions must be merged in each level, and how to perform this merging process. These three topics are briefly tackled in sections 2.1 to 2.3, respectively.

2.1 Initial Fuzzy Segmentation

In this paper we assume that we have an initial fuzzy segmentation of the image that can be obtained, as an example, applying the algorithm that we proposed in [3]. Due to the lack of space we refer to [15, 3] for a detailed explanation of our fuzzy path based segmentation technique.

This method offer as a result set of fuzzy regions, and information of the optimum path from every region seed to every pixel. This information is used to calculate a fuzzy similarity measure to obtain the hierarchy. To apply our proposal just as presented in this paper the single restriction is that the fuzzy segmentation must have been computed considering topological information like in [23, 13, 3]. Otherwise our proposal can also be applied, by adapting the similarity measure to the information provided by the fuzzy segmentation technique used.

2.2 Similarity Measure

Concerning the criteria to decide which regions should be merged in each level, it is on it where the main differences between actual proposals lays. In some cases this criteria evaluates the transition between regions (borders) [22], whereas in other cases they evaluate the similarity of the regions (usually through the distance between average colors)

[12]. However, as we pointed in [16] both criteria should be considered to make a good decision, since they offer significative and complementary information.

In [16] we proposed two measures: $fuzz(\widetilde{R}_i, \widetilde{R}_j)$ to evaluate the transition between two given fuzzy regions, $\widetilde{R}_i$ and $\widetilde{R}_j$, and $par(\widetilde{R}_i, \widetilde{R}_j)$ to measure their likeness. To combine those criteria and obtain a nested hierarchy of fuzzy segmentations we proposed in [16, 2] the use of a *similarity relation* between regions (reflexive, symmetric and max-min transitive) from whose α-cuts we obtain the hierarchy since, as it is well known, this type of relations verify that each α-cut is a crisp equivalence relation. In other words, each α-cut corresponds to a level in the hierarchy, a fuzzy segmentation of the image, where the regions with a similarity value higher or equal to α will be joined together.

To obtain this similarity relation, that we note as $Sim_{\widetilde{\Theta}}$, we propose a resemblance relation between fuzzy regions, $Res_{\widetilde{\Theta}}(\widetilde{R}_i, \widetilde{R}_j)$, computed as the minimum of $fuzz$ and par values of the regions compared. Then we apply any of the existing procedures to obtain a max-min transitive similarity relation from a resemblance relation, like the one proposed by Kandel et al. [1], whose efficiency is $O(m^3)$, since for each couple of regions must be checked wether their similarity is higher through a third region.

Given the initial fuzzy segmentation $\widetilde{\Theta}$ and once computed the similarity relation, $Sim_{\widetilde{\Theta}}$, with an efficiency order of $O(m^3)$ as detailed explained in [16], we can obtain a nested hierarchy of fuzzy segmentations, $H_{\widetilde{\Theta}} = \{\widetilde{\Theta}_1, \ldots, \widetilde{\Theta}_d\}$, as proposed in [2]. In that paper, the fuzzy segmentation, $\widetilde{\Theta}_i$, associated to each hierarchy detail level, i, is obtained from an α-cut in the similarity relation, that we will note as $\left(Sim_{\widetilde{\Theta}}\right)_{\alpha_i}$. This α-cut gives a crisp equivalence relation that generates the fuzzy segmentation in that level, $\widetilde{\Theta}_i = \{\widetilde{R}^i_1, \ldots, \widetilde{R}^i_{m_i}\}$, since each region of that segmentation is computed merging the regions in the same equivalence class, as explained in next section.

2.3 Hierarchical Union of Fuzzy Regions

Considering that each *alpha*-cut results in a fuzzy segmentation; i.e, a hierarchy level, and α can take any value in $[0, 1]$, we could obtain an infinite number of hierarchy levels but not all of them resulting if different segmentations. Hence, we propose in [16] to settle the hierarchy levels to the set of different values that α can take in $Sim_{\widetilde{\Theta}}$, as in definition 2.2.

Definition 2.2 *We define $\Lambda\left(Sim_{\widetilde{\Theta}}\right)$ as the set of all the possible different values of α in $Sim_{\widetilde{\Theta}}$:*

$$\Lambda\left(Sim_{\widetilde{\Theta}}\right) = \left\{Sim_{\widetilde{\Theta}}(\widetilde{R}_s, \widetilde{R}_t) | (\widetilde{R}_s, \widetilde{R}_t) \in \widetilde{\Theta} \times \widetilde{\Theta}\right\} \quad (2)$$

So $\Lambda\left(Sim_{\widetilde{\Theta}}\right) = \{\alpha_1, \ldots, \alpha_d\}$, with $\alpha_1 = 1$ and $\alpha_d = \min\left\{Sim_{\widetilde{\Theta}}\left(\widetilde{R}_s, \widetilde{R}_t\right) \mid \widetilde{R}_s, \widetilde{R}_t \in \widetilde{\Theta}\right\}$, with $\alpha_i > \alpha_{i+1}\ \forall i \in \{1, \ldots, d-1\}$.

For each $\alpha_i \in \Lambda(Sim_{\widetilde{\Theta}})$, we obtain a fuzzy segmentation $\widetilde{\Theta}_i$ from the crisp equivalence relation of that α-cut,

$$\widetilde{\Theta} / \left(Sim_{\widetilde{\Theta}}\right)_{\alpha_i} = \left\{C_1^{\widetilde{\Theta},\alpha_i}, \ldots, C_{m_i}^{\widetilde{\Theta},\alpha_i}\right\}$$

With it, the number of regions in each level is $m_i = |\widetilde{\Theta} / \left(Sim_{\widetilde{\Theta}}\right)_{\alpha_i}|$ where $\widetilde{\Theta} / \left(Sim_{\widetilde{\Theta}}\right)_{\alpha_i}$ is the quotient set. Each fuzzy region, $\widetilde{R}^i_k$, of this segmentation will be obtained through the merging of the fuzzy regions from $\widetilde{\Theta}$ that are in the same equivalence class, $C_k^{\widetilde{\Theta},\alpha_i}$ of the quotient set $\widetilde{\Theta} / \left(Sim_{\widetilde{\Theta}}\right)_{\alpha_i}$. To perform this merging process we compute the membership degree of the pixels to the new fuzzy as the maximum membership degree of the pixel to the regions to be merged, such as indicates the equation 3:

$$\mu_{\widetilde{R}^i_k}(p_j) = \max_{\widetilde{R}^i_s \in C_k^{\widetilde{\Theta},\alpha_i}} \left\{\mu_{\widetilde{R}^i_s}(p_j)\right\} \quad (3)$$

As an example, from the quotient set $\widetilde{\Theta} / \left(Sim_{\widetilde{\Theta}}\right)_{\alpha_1}$ we obtain the initial fuzzy segmentation $\widetilde{\Theta}_1$, whereas from the set

$\widetilde{\Theta}/\left(Sim_{\widetilde{\Theta}}\right)_{\alpha_d}$ we obtain the fuzzy segmentation $\widetilde{\Theta}_d$, with a single region corresponding to the whole image.

3 Relevant Levels

With the methodology summarized in previous section we obtain a hierarchy of fuzzy segmentations. However, as mentioned in the introduction, our aim is avoid the user the task of finding, the levels (segmentations) where all the fuzzy regions of an over-segmented region in the image have merged together. In this section we propose a methodology to automatically select *relevant levels* verifying this condition.

According to it, a *relevant level* would be the one where a set of fuzzy regions have been joined into a single fuzzy region that represents better the original over-segmented image region. However, it is difficult to know a priori which is the set of fuzzy regions from the initial segmentation that correspond to the same over-segmented region of the image. To solve this problem we propose to apply to the hierarchy the same idea used to perform the segmentation, where we look for sets of homogeneous pixels, resemblant and connected. Now we will look for sets of *homogeneous regions*, as any user would do.

Intuitively our proposal is based on studying the homogeneity along the hierarchy levels to find the levels where the homogeneity drastically decreases. It means that the new formed region is not homogeneous and therefore that the regions merged on it shouldn't have joined because they where homogeneous. Then the relevant levels would be those where that homogenous regions were firstly obtained.

Of course, we could also chose any level between the one where the region was firstly obtained and the previous to the homogeneity drastic decrease. However, we have preferred selecting the one were the region was firstly obtained for two reasons: On one hand, the lower levels in the hierarchy correspond to fuzzy segmentations with more homogeneous regions, while in higher levels we could find segmentations where fuzzy regions had joined into new regions without meaning regarding the objects in the image. On the other hand, it let us offer the user additional useful information: the homogeneity degree of the newly formed region.

In this proposal two topics must be solved: firstly, as will be seen in section 3.1, we need a measure of the homogeneity in a hierarchy level; i.e. the homogeneity of a segmentation. Secondly, we must study the evolution of the homogeneity along the hierarchy to find the relevant levels, and we face it in section 3.2.

3.1 Homogeneity of a Hierarchy Level Segmentation

The usual notion of *Homogeneity* is related to the resemblance of the elements in a set [9, 19, 4, 5]. Applying this idea to a level of the hierarchy, the homogeneity of the segmentation in a given level should be representative of the homogeneity of the fuzzy regions in that segmentation.

Considering the methodology followed to obtain the hierarchy, each new level is obtained from the union of less similar regions that those on the previous level. Hence, the homogeneity measure of hierarchy levels should be a decreasing function. In addition, in the first level of the hierarchy we should have completely homogeneous regions. These properties we consider interesting for the homogeneity measure are summarized as follows, as well as the range we consider useful for it:

Definition 3.1 *Given the fuzzy segmentation, $\widetilde{\Theta}_i$, of the hierarchy level i, we define its homogeneity as a function $\mathcal{H}omo : \widetilde{\Theta}_i \longrightarrow [0,1]$ verifying the following properties:*

- *If $\alpha_i = 1$, then $\mathcal{H}omo(\widetilde{\Theta}_i) = 1$.*
- *Monotonicity: If $\alpha_i > \alpha_j$ then $\mathcal{H}omo(\widetilde{\Theta}_i) > \mathcal{H}omo(\widetilde{\Theta}_j)$*

With the first property we indicate that the initial segmentation, the kernel of the hierarchy, has maximum homogeneity, while the second one means that as the value of α decreases, the homogeneity of the α-cut should be lower.

Here we propose the use of the value α of each hierarchy level as a measure of its homogeneity. Given the set of regions, $\widetilde{\Theta}_i$, in the fuzzy segmentation of the hierarchy level, i, we propose to measure the homogeneity of the segmentation in that level as the value α_i corresponding to that α-cut, as in equation 4:

$$\mathcal{H}omo(\widetilde{\Theta}_i) = \alpha_i \tag{4}$$

where $\alpha \in [0, 1]$. It is trivial to show that this measure verifies the properties above.

The underlying reason of this proposal is that with the methodology we use to compute the hierarchy, the value α_i means that all the regions in the segmentation $\widetilde{\Theta}_i$ have been obtained merging regions from the previous level whose similarity is higher or equal to α_i. Hence, according to the notion of homogeneity, it is representative of the resemblance between the elements in that set of regions, and seems natural its use to compute the homogeneity of a hierarchy level segmentation.

3.2 Automatic Selection of Relevant Levels

In the intuition presented at the beginning of this section we indicated that a *relevant level* can be found looking for hierarchy levels where the homogeneity of the fuzzy segmentation drastically decreases regarding the previous ones. The idea of change or variation can be expressed in terms of the gradient of the homogeneity measure evolution along the α-cuts. In this paper, we calculate the gradient of the homogeneity measure along the set of α-cuts by means of the difference between the homogeneity of consecutive α-cuts. Anyway we point that there are images with a higher number of regions where a discrete derivative of a gaussian function [6] would be more suitable. The local minima of the gradient yield the place (α values) and magnitude of the drastic decreases.

Definition 3.2 *Given a nested hierarchy of fuzzy segmentations, $H_{\widetilde{\Theta}}$, we define the set of* minimum homogeneity α values*, noted $m\mathcal{H}\Lambda(H_{\widetilde{\Theta}})$, as the set of $\alpha_i \in \Lambda(Sim_{\widetilde{\Theta}})$ corresponding to local minima in the hierarchy levels homogeneity gradient.*

From definition 3.2 we obtain the set of *relevant α-values* of the hierarchy (relevant α-cuts), as those where the regions to be joined in the level of a minimum homogeneity α-value, were firstly obtained; i.e. the level at which where formed the regions whose similarity is a minimum homogeneity α-value. It is formalized as follows:

Definition 3.3 *Given a nested hierarchy of fuzzy segmentations, $H_{\widetilde{\Theta}}$, we define the set of its* relevant α-values*, $\mathcal{R}\Lambda(H_{\widetilde{\Theta}})$, as the set*

$$\begin{aligned} \mathcal{R}\Lambda(H_{\widetilde{\Theta}}) = \{\alpha_i | \exists C_r^{\widetilde{\Theta},\alpha_i}, C_s^{\widetilde{\Theta},\alpha_j}, \alpha_k \\ Sim_{\widetilde{\Theta}}(C_r^{\widetilde{\Theta},\alpha_i}, C_s^{\widetilde{\Theta},\alpha_j}) = \alpha_k\} \bigcup \{\alpha_1\} \end{aligned} \tag{5}$$

where $\alpha_1, \alpha_i, \alpha_j \in \Lambda(Sim_{\widetilde{\Theta}})$, $\alpha_k \in m\mathcal{H}\Lambda(H_{\widetilde{\Theta}})$, $\alpha_i, \alpha_j > \alpha_k$ and $\alpha_1 = 1$.

In definition 3.2 we have included the value $\alpha_1 = 1$ because we consider the kernel of the hierarchy as a relevant level, since it contains all the original fuzzy regions. Given this set of *relevant α-values* we define the set of *relevant levels in the hierarchy* as in definition 3.4:

Definition 3.4 *The set of* relevant levels *in a given hierarchy, $H_{\widetilde{\Theta}}$, noted as $\mathcal{RL}(H_{\widetilde{\Theta}})$, is defined as the set of α-cuts such that α is a relevant α-value, i.e.,*

$$\mathcal{RL}(H_{\widetilde{\Theta}}) = \left\{ \widetilde{\Theta} / \left(Sim_{\widetilde{\Theta}} \right)_{\alpha_i} | \alpha_i \in \mathcal{R}\Lambda(H_{\widetilde{\Theta}}) \right\} \tag{6}$$

In figure 1 (A) and (B) we can see the original image and the initial fuzzy segmentation, respectively. Figure 1 (C) shows the magnitude of the homogeneity along the hierarchy levels as a green line, whereas in figure 1 (D) we observe the gradient of the homogeneity in blue color, with the local minima marked with a pink square (corresponding to the *minimum homogeneity α values*). Finally in figure 1 (E) we have the relevant levels found with our proposal marked with a blue square on the homogeneity evolution graph.

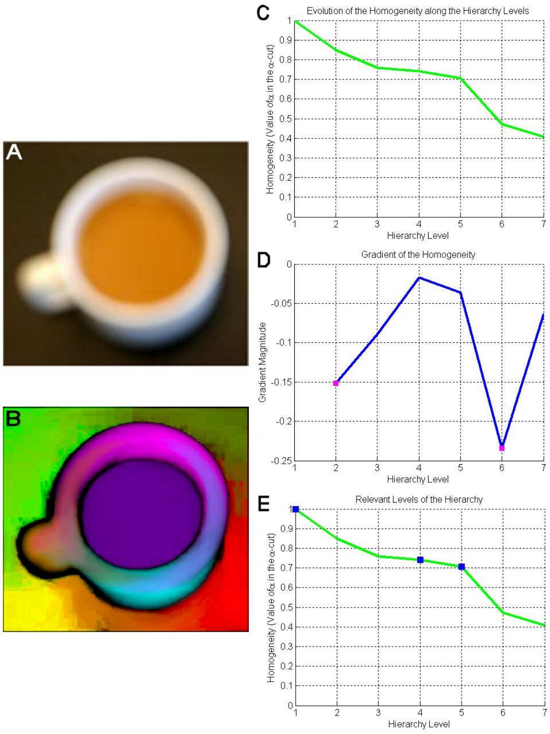


Figure 1: Process to automatically select relevant levels of the hierarchy $H_{\tilde{\Theta}}$. A: Original Image. B: Initial Fuzzy Segmentation. C: Evolution of the Homogeneity along the Hierarchy. D: Gradient of the Homogeneity with the local minima marked with a pink square (corresponding to the minimum homogeneity α-values). E: Relevant Levels Selected, marked with a blue square.

We would like to remark an additional advantage or our proposal, since it let as establish a relevance ordering between the levels selected, according to the magnitude of the gradient in the level from which they were found. It offers the user the added feature of having a preference ordering that can be helpful in case of having to prioritize or choose just some of the relevant levels.

4 Results

In this section we show the results obtained applying our proposal to the image in figure 1 (A). In figure 2 we show the hierarchy obtained applying the technique in section 2.

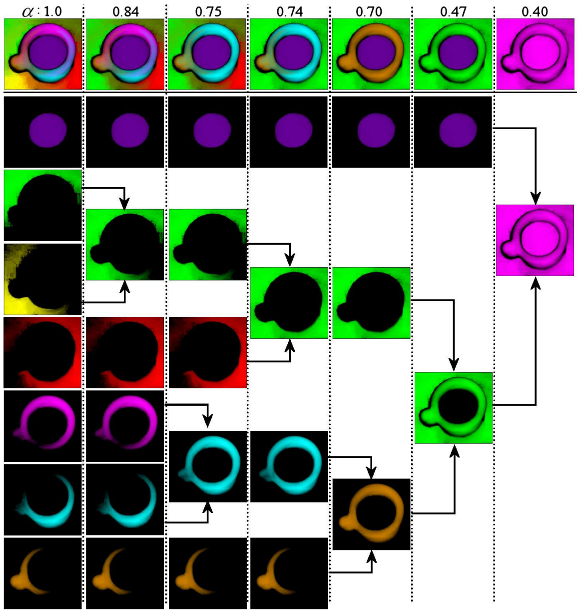


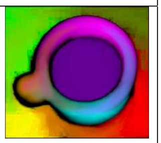
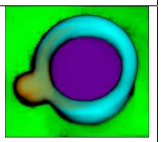
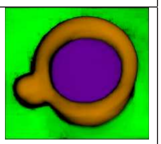
Figure 2: Hierarchy of fuzzy segmentations, $H_{\tilde{\Theta}}$, for image in figure 1. Each column is the hierarchy level obtained with the α-cut for the α value on top of each column. The first row is the fuzzy segmentation for that level, and images from rows 2 to 8 are the fuzzy regions in that segmentation.

Each Column of this figure corresponds to a hierarchy level; i.e. a fuzzy segmentation obtained from the α-cut with the α value indicated on top of the column. The image in the first row represent the fuzzy segmentation, obtained from the fuzzy regions in that level, in rows 2 to 8.

In table 1 we indicate the hierarchy levels found relevant for each local minima. The first column references the local minima in figure 1 (D) and the second column indicates the corresponding minimum homogeneity α-value. In the third column we show the relevant α values obtained from these local minima as well as the hierarchy level (in forth column) and the resulting fuzzy segmentation (α-cut).

As can be seen the first level in the hierarchy is always included because of its significance, but in this case in addition has been found to be relevant from the first local minima. The second local minima has given raise to two relevant levels: the forth, where the fuzzy re-

Table 1: Relevant levels in the Hierarchy.

Local Min.	$m\mathcal{H}\Lambda(H_{\widetilde{\Theta}})$	$\mathcal{R}\Lambda(H_{\widetilde{\Theta}})$	**Level** i	$\widetilde{\Theta}i$
1	0.84	1	1	
2	0.47	0.74	4	
2	0.47	0.70	5	

gion corresponding to the background of the image has been obtained, and the fifth one, where all the regions of the over-segmented cup have been merged into a single region representing the whole cup, as can be observed in the hierarchy of figure 2.

5 Conclusions

In this paper we have proposed a technique to automatically select, from a hierarchy of fuzzy segmentations, a subset of results that can be relevant for the user. To this purpose, we have firstly summarized a method to obtain the hierarchy of fuzzy segmentations from a given path-based initial segmentation and a similarity measure between fuzzy regions.

Then we have proposed a measure to represent criteria of the user in the process to select relevant levels, assuming the user looks for homogeneous regions. Therefore we have proposed an homogeneity measure that indicates the homogeneity degree of a given segmentation. With this homogeneity measure, we have proposed a technique to automatically select the levels were have been obtained homogeneous regions from the union of those in previous levels, such that if any other region would join them it would make the drastically decrease the homogeneity of the corresponding segmentation.

Results obtained show that our technique works reasonably well in most of the cases: the levels found as relevant correspond to levels where all the fuzzy regions of an over-segmented area in the image have merged together, offering additional information as the degree of homogeneity of that region and a criteria to order by preference the relevant results selected. In addition, our proposal can be apply to general images (not only to an specific type), and sets a frame to adapt the technique to different purposes just changing the homogeneity measure.

References

[1] Lawrence Yelowitz Abraham Kandel. Fuzzy chains. *IEEE Trans. on System, Man and Cybernetics*, SMC-4(5):472–475, 1974.

[2] J. Chamorro-Martinez, D. Sanchez, B. Prados-Suarez, E. Galan-Perales, and M.A. Vila. A hierarchical approach to fuzzy segmentation of colour images. In *12th IEEE Int. Conf. on Fuzzy Systems, 2003.*, volume 2, pages 966–971 vol.2, 2003.

[3] J. Chamorro-Martínez, D. Sánchez, B. Prados-Suarez, and E. Galán-Perales. Fuzzy homogeneity measures for path-based colour image segmentation. *International Journal of Intelligent Systems Technologies and Applications*, 1(3/4):301–318, 2006.

[4] H. D. Cheng and J. Li. Fuzzy homogeneity and scale-space approach to color image segmentation. *Pattern Recognition*, 36(7):1545–1562, July 2003.

[5] Heng-Da Cheng and Ying Sun. A hierarchical approach to color image segmentation using homogeneity. *Image Processing, IEEE Trans. on*, 9(12):2071–2082, 2000.

[6] C.Russ. *The Image Processing Handbook*. CRC Press, 1994.

[7] Abraham Duarte, Ángel Sánchez, Felipe Fernández, and Antonio Sanz. *Region*

Merging for Severe Oversegmented Images Using a Hierarchical Social Metaheuristic, volume 3449, pages 345–355. January 2005.

[8] M. Jeon, M. Alexander, W. Pedrycz, and N. Pizzi. Unsupervised hierarchical image segmentation with level set and additive operator splitting. *Pattern Recognition Letters*, 26(10):1461–1469, July 2005.

[9] Bo-Yeong Kang, Dae-Won Kim, and Qing Li. Spatial homogeneity-based fuzzy c-means algorithm for image segmentation. *LNCS*, 3613:462–469, jul 2005.

[10] Daehee Kim, Yo-Sung Ho, and B.S. Manjunath. Color image segmentation using anisotropic diffusion and agglomerative hierarchical clustering. *LNCS*, 2532:759–766, January 2002.

[11] J. Maeda, S. Saga, and Y. Suzuki. Rough segmentation of natural color images using fuzzy-based hierarchical algorithm. In *Circuits and Systems, 2004. MWSCAS '04. The 2004 47th Midwest Symposium on*, volume 1, pages I–217–20 vol.1, 2004.

[12] S. Makrogiannis, G. Economou, S. Fotopoulos, and N.-G. Bourbakis. Segmentation of color images using multiscale clustering and graph theoretic region synthesis. *IEEE Trans. on Systems, Man & Cybernetics, Part A Systems & Humans*, 35(2):224–38, 2005.

[13] S. Philipp-Foliguet, M.B. Vieira, and A. De Albuquerque Araujo. Segmentation into fuzzy regions using topographic distance. In *Computer Graphics and Image Processing, 2001 Proceedings of XIV Brazilian Symposium on*, pages 282–288, 2001.

[14] Antonio J. Plaza and James C. Tilton. Automated selection of results in hierarchical segmentations of remotely sensed hyperspectral images. *IEEE IGARSS'05*, pages 4946–4949, 2005.

[15] B. Prados-Suarez, J. Chamorro-Martínez, D. Sánchez, and J. Abad. Region-based fit of colour homogeneity measures for fuzzy image segmentation. *Fuzzy Sets and Systems*, 158(3):215–229, February 2007.

[16] B. Prados-Suárez. *Desarrollo de Modelos para la Segmentación Difusa de Imágenes en Color.* PhD thesis, University of Granada, 2006.

[17] W. Shitong and W. Min. A new detection algorithm based on fuzzy cellular neural networks for white blood cell detection. *Information Technology in Biomedicine, IEEE Trans. on*, 10(1):5–10, 2006.

[18] J.C. Tilton. Analysis of hierarchically related image segmentations. In *Advances in Techniques for Analysis of Remotely Sensed Data, 2003 IEEE Workshop on*, pages 60–69, 2003.

[19] D. Van der Weken, M. Nachtegael, V. De Witte, S. Schulte, and E.E. Kerre. A survey on the use and the construction of fuzzy similarity measures in image processing. In *IEEE Inter. Conf. CIMSA 2005.*, pages 187–192, 2005.

[20] I. Vanhamel, H. Sahli, and I. Pratikakis. Nonlinear multiscale graph theory based segmentation of color images. In *18th Int. Conf.on Pattern Recognition, 2006.*, volume 2, pages 407–411, 2006.

[21] Iris Vanhamel, Ioannis Pratikakis, and Hichem Sahli. *Hierarchical Segmentation Using Dynamics of Multiscale Color Gradient Watersheds*, volume 2106, pages 371–. January 2001.

[22] S. Wesolkowski and P. Fieguth. Hierarchical region mean-based image segmentation. In *Computer and Robot Vision, 2006. The 3rd Canadian Conference on*, pages 30–30, 2006.

[23] Ying Zhuge, Jayaram K. Udupa, and Punam K. Saha. Vectorial scale-based fuzzy-connected image segmentation. *Computer Vision and Image Understanding*, 101(3):177–193, March 2006.