Advanced Image Compression on the Basis of Fuzzy Transforms

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Abstract

We show that on the basis of preserving monotonicity an improved algorithm of image compression and reconstruction can be proposed. The algorithm is based on partitioning the range of the function f_I which corresponds to an image I.

Keywords: Fuzzy partition, Fuzzy transform, Data compression.

1 Introduction

After the first successful series of publications [4, 3, 5], the technique of fuzzy transforms (F-transforms) was investigated on monotonous functions and functions fulfilling the Lipschitz condition. It was conjectured that each property (monotonicity and Lipschitz continuity) will be preserved via the direct and inverse F-transforms. The conjecture was confirmed [6] and the theory pushes the authors into applications in image processing.

Since F-transform has already proved its ability to be used in image compression and reconstruction (see [3, 1]), there is always a room to improve quality of compressed and reconstructed images. In this contribution, we will show that on the basis of preserving monotonicity an improved algorithm of image compression and reconstruction can be proposed. Its advantage (over the one proposed in [3]) consists in keeping sharpness of an original image.

The structure of the contribution is the following. In Section 2, the fuzzy partition of the universe is recalled. In Section 3, the technique of the direct and inverse fuzzy transform is introduced. In Section 4, we discuss the discrete fuzzy transform. In Section 5, we show that the property of monotonicity is invariant with respect to direct and inverse fuzzy transforms. In Section 6, principles of image compression by the fuzzy transform are reminded and a new advanced image compression method is proposed.

2 Fuzzy Partition of the Universe

The key idea of the technique proposed in this paper is a fuzzy partition of the universe into fuzzy subsets (factors, clusters, granules etc.). Let us recall the Zadeh's paper [8] where the notion of a granule has been introduced and used in fuzzy logic based human reasoning. In the theory of fuzzy transforms, we show that for a sufficient representation of a function it is sufficient to know the function's average values over fuzzy subsets from the fuzzy partition of its domain. Then, the function can be associated with a mapping from the set of fuzzy subsets to the set of its average values. In brief, this is an idea of direct fuzzy transform of a function.

We take an interval $[a, b]$ as a universe. That is, all (real-valued) functions considered in this contribution have this interval as a common domain.

Definition 1
Let $x_1 < \ldots < x_n$ *be fixed nodes within* $[a, b]$,

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such that $x_1 = a$, $x_n = b$ and $n \geq 3$. We say that fuzzy sets $A_1, \ldots, A_n$, identified with their membership functions $A_1(x), \ldots, A_n(x)$ defined on $[a, b]$, form a fuzzy partition *of $[a, b]$ if they fulfil the following conditions for $k = 1, \ldots, n$:*

1. *$A_k : [a, b] \longrightarrow [0, 1]$, $A_k(x_k) = 1$;*
2. *$A_k(x) = 0$ if $x \notin (x_{k-1}, x_{k+1})$ where for the uniformity of denotation, we put $x_0 = a$ and $x_{n+1} = b$;*
3. *$A_k(x)$ is continuous;*
4. *$A_k(x)$, $k = 2, \ldots, n$, strictly increases on $[x_{k-1}, x_k]$ and $A_k(x)$, $k = 1, \ldots, n-1$, strictly decreases on $[x_k, x_{k+1}]$;*
5. *for all $x \in [a, b]$, $\sum_{k=1}^{n} A_k(x) = 1$.*

The membership functions $A_1, \ldots, A_n$ are called basic functions.

Let us remark that basic functions are specified by a set of nodes $x_1 < \ldots < x_n$ and the properties 1–5. The shape of basic functions is not predetermined and therefore, it can be chosen additionally according to further requirements. As the example, we give a formal representation of triangular membership functions:

$$A_1(x) = \begin{cases} 1 - \frac{(x - x_1)}{h_1}, & x \in [x_1, x_2], \\ 0, & \text{otherwise}, \end{cases}$$

$$A_k(x) = \begin{cases} \frac{(x - x_{k-1})}{h_{k-1}}, & x \in [x_{k-1}, x_k], \\ 1 - \frac{(x - x_k)}{h_k}, & x \in [x_k, x_{k+1}], \\ 0, & \text{otherwise}, \end{cases}$$

$$A_n(x) = \begin{cases} \frac{(x - x_{n-1})}{h_{n-1}}, & x \in [x_{n-1}, x_n], \\ 0, & \text{otherwise}. \end{cases}$$

where $k = 2, \ldots n-1$, and $h_k = x_{k+1} - x_k$.

Let a fuzzy partition of $[a, b]$ be given by fuzzy sets $A_1, \ldots, A_n$, $n \geq 3$, in the sense of Definition 1. We say that it is *uniform* if the nodes $x_1, \ldots, x_n$, $n \geq 3$, are equidistant. This means that $x_k = a + h(k-1)$, $k = 1, \ldots, n$, where $h = (b-a)/(n-1)$. Moreover, two additional properties are required:

6. $A_k(x_k - x) = A_k(x_k + x)$, for all $x \in [0, h]$, $k = 2, \ldots, n-1$,
7. $A_k(x) = A_{k-1}(x - h)$, for all $k = 2, \ldots, n-1$ and $x \in [x_k, x_{k+1}]$, and $A_{k+1}(x) = A_k(x - h)$, for all $k = 2, \ldots, n-1$ and $x \in [x_k, x_{k+1}]$.

In the case of a uniform partition, h is the length of each subinterval $[x_k, x_{k+1}]$, $k = 1, \ldots, n-1$. Moreover, the value of h is unambiguously determined by the number n of basic functions. We will further refer a uniform partition with subintervals of the length h as to a *h-uniform partition*.

3 Fuzzy transform

In this section, we recall ([7, 4]) the notion of fuzzy transform $\mathbf{F}_n[f]$ of a continuous function f on $[a, b]$. We will further use the short name F-transform instead of fuzzy transform. We will also recall some approximation properties of the both direct and inverse F-transforms.

3.1 Direct fuzzy transform

Let us fix an interval $[a, b]$ and nodes $x_1 < \ldots < x_n$, such that $x_1 = a$, $x_n = b$ and $n \geq 3$. Let $A_1, \ldots, A_n$ be some fixed basic functions which constitute a fuzzy partition of $[a, b]$.

Denote $C([a, b])$ the set of continuous functions on the interval $[a, b]$. The following definition (see also [4]) introduces the (direct) fuzzy transform of a function $f \in C([a, b])$.

Definition 2
Let $A_1, \ldots, A_n$ be basic functions which constitute a fuzzy partition of $[a, b]$ and f be any function from $C([a, b])$. We say that the n-tuple of real numbers $[F_1, \ldots, F_n]$ given by

$$F_k = \frac{\int_a^b f(x) A_k(x) dx}{\int_a^b A_k(x) dx}, \qquad k = 1, \ldots, n, \quad (1)$$

is the (direct integral) F-transform of f with respect to $A_1, \ldots, A_n$.

Denote the F-transform of a function f with respect to $A_1, \ldots, A_n$ by $\mathbf{F}_n[f]$. Then

$$\mathbf{F}_n[f] = [F_1, \ldots, F_n]. \qquad (2)$$

The elements $F_1, \dots, F_n$ are called *components of the F-transform.*

3.2 Inverse F-transform

The inverse F-transform is given by the inversion formula and approximates the original function in such a way that a universal convergence can be established. Moreover, the quality of approximation is given in the Theorem 1.

Definition 3 ([4])
Let $A_1, \dots, A_n$ be basic functions which form a fuzzy partition of $[a, b]$ and f be a function from $C([a, b])$. Let $\mathbf{F}_n[f] = [F_1, \dots, F_n]$ be the integral F-transform of f with respect to $A_1, \dots, A_n$. Then the function

$$f_{F,n}(x) = \sum_{k=1}^{n} F_k A_k(x) \tag{3}$$

is called the inverse F-transform.

The theorem and its corollary below show that the inverse F-transform $f_{F,n}$ can approximate the original continuous function f with an arbitrary precision. The proof can be found in [4].

Theorem 1
Let f be a continuous function on $[a, b]$. Then for any h-uniform fuzzy partition $A_1, \dots, A_n$ of $[a, b]$ (where $n \geq 3$ and $h = \frac{b-a}{n-1}$) and for all $x \in [a, b]$,

$$|f(t) - f_{F,n}(t)| \leq 2\omega(h, f) \tag{4}$$

where $f_{F,n}$ is the inverse F-transform of f with respect to the fuzzy partition $A_1, \dots, A_n$ and

$$\omega(h, f) = \max_{|\delta| \leq h} \max_{x \in [a, b-\delta]} |f(x + \delta) - f(x)|$$

is the modulus of continuity of f on $[a, b]$.

Corollary 1
Let the assumptions of Theorem 1 be fulfilled. Then the sequence of inverse F-transforms $\{f_{F,n}\}$ uniformly converges to f.

4 Discrete F-Transform

Let us specially consider the discrete case, when an original function f is known (may be computed) only at some points $p_1, \dots, p_l \in [a, b]$ where $p_1 < \dots < p_l$. Let a fuzzy partition of $[a, b]$ be given by fuzzy sets $A_1, \dots, A_n$, $n \geq 3$, $n < l$, in the sense of Definition 1. The nodes $x_1, \dots, x_n$ are not necessarily among the points $p_1, \dots, p_l$. We assume that the set $P = \{p_1, \dots, p_l\}$ is *sufficiently dense with respect to the fixed partition*, i.e.

$$(\forall k)(\exists j) A_k(p_j) > 0. \tag{5}$$

Then the (direct) discrete F-transform of f is introduced as follows.

Definition 4
Let a function f be given at points $p_1, \dots, p_l \in [a, b]$ and $A_1, \dots, A_n$, $n < l$, be basic functions which constitute a fuzzy partition of $[a, b]$. We say that the n-tuple of real numbers $[F_1, \dots, F_n]$ is the (direct) discrete F-transform of f with respect to $A_1, \dots, A_n$ if

$$F_k = \frac{\sum_{j=1}^{l} f(p_j) A_k(p_j)}{\sum_{j=1}^{l} A_k(p_j)}. \tag{6}$$

In the discrete case, we define the inverse F-transform by the same inversion formula (3) and consider that function only at points where the original function is given.

5 Preserving Monotonicity

In this section we will consider the class $M[a, b]$ of bounded monotonous functions, defined on $[a, b]$, and show that for any uniform fuzzy partition of $[a, b]$ the inverse F-transform of a function from $M[a, b]$ belongs to the same class.

Let $M[a, b]$ be a class of bounded monotonous functions defined on $[a, b]$. Then functions from $M[a, b]$ are integrable on $[a, b]$ and their direct and inverse F-transforms exist. Let us choose some natural number $n \geq 3$ and consider a h-uniform partition of $[a, b]$ by n basic functions $A_1, \dots, A_n$.

Lemma 1
Let $f \in M[a,b]$ be a monotonically increasing function and $A_1, \ldots, A_n$ establish a h-uniform partition of $[a,b]$. Let $F_1, \ldots, F_n$ be the F-transform components of f with respect to $A_1, \ldots, A_n$. Then for each $k = 2, \ldots, n-2$, $F_k \le F_{k+1}$ so that $F_2 \le F_3 \cdots \le F_{n-1}$.

PROOF: Let k be one of $2, \ldots, n-2$. Then we can write

$$h(F_{k+1} - F_k) = \int_{x_k}^{x_{k+2}} f(x)A_{k+1}(x)dx - \int_{x_{k-1}}^{x_{k+1}} f(x)A_k(x)dx = \int_{x_k}^{x_{k+2}} f(x)A_{k+1}(x)dx - \int_{x_k}^{x_{k+2}} f(y-h)A_{k+1}(y)dy = \int_{x_k}^{x_{k+2}} (f(x) - f(x-h))A_{k+1}(x)dx \ge 0.$$

In the equalities above, we have used the following property of a h-uniform partition: $A_k(y-h) = A_{k+1}(y)$ where $y \in [x_k, x_{k+2}]$. □

Remark 1
Let the assumptions of Lemma 1 hold true. It may happen that the F-transform components F_1 and F_n of f break monotonicity between all components, i.e. that $F_1 \le F_2$ and/or $F_{n-1} \le F_n$ do not hold. This is caused by the different way of computing F_1 and F_n in comparison with $F_2, \ldots, F_{n-1}$. In order to remove this difference we will extend the function f to a function f^{ex}, A_1 to A_1^{ex} and A_n to A_n^{ex} so that the extended functions are defined as follows on domains $[a-h, b+h]$, $[a-h, a+h]$ and $[b-h, b+h]$ respectively:

$$f^{ex}(x) = \begin{cases} f(a-x) = 2f(a) - f(a+x), & \text{if } x \in [0,h], \\ f(x), & \text{if } x \in [a,b], \\ f(b+x) = 2f(b) - f(b-x), & \text{if } x \in [0,h], \end{cases}$$

$$A_1^{ex}(x) = \begin{cases} A_1(a-x) = A_1(a+x), & \text{if } x \in [0,h], \\ A_1(x), & \text{if } x \in [a, a+h], \end{cases}$$

$$A_n^{ex}(x) = \begin{cases} A_n(x), & \text{if } x \in [b-h, b], \\ A_n(b+x) = A_n(b-x), & \text{if } x \in [0,h]. \end{cases}$$

Note that f^{ex} monotonically increases on $[a-h, b+h]$. If we replace the F-transform components F_1 and F_n by

$$F_1^{ex} = \frac{1}{h}\int_{a-h}^{a+h} f(x)A_1^{ex}(x)dx,$$
$$F_n^{ex} = \frac{1}{h}\int_{b-h}^{b+h} f(x)A_n^{ex}(x)dx$$

respectively then the inverse F-transform will be changed to

$$f_{F,n}^{ex}(x) = F_1^{ex}A_1^{ex}(x) + \sum_{k=2}^{n-1}(F_kA_k(x)) + F_n^{ex}A_n^{ex}(x).$$

It is worth to notice that the inverse F-transforms of f and f^{ex} coincide on $[x_2, x_{n-1}]$.

Let the partition of $[a-h, b+h]$ be given by $A_1^{ex}, A_2, \ldots, A_{n-1}, A_n^{ex}$ and $F_1^{ex}, F_2, \ldots, F_{n-1}, F_n^{ex}$ be the F-transform components of f^{ex} with respect to $A_1^{ex}, A_2, \ldots, A_{n-1}, A_n^{ex}$. Then by the proof of Lemma 1, $F_1^{ex} \le F_2 \le \cdots \le F_{n-1} \le F_n^{ex}$.

Theorem 2
Let $f \in M[a,b]$ be a monotonically increasing function and $A_1, \ldots, A_n$ establish a h-uniform partition of $[a,b]$. Then the inverse F-transform $f_{F,n}$ of f with respect to $A_1, \ldots, A_n$ monotonically increases on the interval $[a+h, b-h]$.

PROOF: Let $F_1, \ldots, F_n$ be the F-transform components of f with respect to $A_1, \ldots, A_n$ and $f_{F,n}$ be the inverse F-transform. Let $x, y \in [a+h, b-h]$ be such that $x < y$. We will prove that $f_{F,n}(x) \le f_{F,n}(y)$. The proof will be based on Lemma 1 and split in a number of cases according to positions of x, y with respect to the nodes $x_2, \ldots, x_{n-1}$ of the partition. Below, we will consider only one possible case and refer to [6] for all others.

Assume that for some $k = 2, \ldots, n-2$, $x, y \in$

$[x_k, x_{k+1}]$. Then

$$\begin{aligned} f_{F,n}(y) - f_{F,n}(x) = \\ \sum_{j=1}^{n} F_j A_j(y) - \sum_{j=1}^{n} F_j A_j(x) = \\ F_k(A_k(y) - A_k(x)) + \\ F_{k+1}(A_{k+1}(y) - A_{k+1}(x)) = \\ F_k(A_k(y) - A_k(x)) + \\ F_{k+1}(1 - A_k(y) - 1 + A_k(x)) = \\ (A_k(x) - A_k(y))(F_{k+1} - F_k) \geq 0. \end{aligned}$$

The last inequality is due to the fact that A_k monotonically decreases on $[x_k, x_{k+1}]$ and $F_k \leq F_{k+1}$. □

Corollary 2
Let $f \in M[a, b]$ be a monotonically increasing function and $A_1, \ldots, A_n$ establish a h-uniform partition of $[a, b]$. Let f^{ex} extend f (see Remark 1) and the partition of $[a - h, b + h]$ be given by $A_1^{ex}, A_2, \ldots, A_{n-1}, A_n^{ex}$ (see Remark 1). Then the inverse F-transform $f_{F,n}^{ex}$ of f^{ex} monotonically increases on the interval $[a, b]$.

6 Application to Image Compression

A method of lossy image compression and reconstruction on the basis of fuzzy transforms has been proposed in [4] and then analyzed and compared with other compression techniques in [3, 1].

Below, after reminding the principles of image compression and reconstruction by direct and inverse F-transforms, we will explain how preserving monotonicity can improve quality of the reconstructed image.

6.1 Principles of Image Compression by the F-Transform

Let an image I of the size $N \times M$ pixels be represented by a function of two variables (a fuzzy relation) $f_I : \mathbb{N} \times \mathbb{N} \longrightarrow [0, 1]$ partially defined at nodes $(i, j) \in [1, N] \times [1, M]$. The value $f_I(i, j)$ represents an intensity range of each pixel. A compression of the image I is represented by the $n \times m$ matrix $\mathbf{F}_{nm}[f_I]$ of the discrete F-transform components of f_I:

$$\mathbf{F}_{nm}[f_I] = \begin{pmatrix} F_{11} & \ldots & F_{1m} \\ \vdots & \vdots & \vdots \\ F_{n1} & \ldots & F_{nm} \end{pmatrix}$$

where

$$F_{kl} = \frac{\sum_{j=1}^{M} \sum_{i=1}^{N} f_I(i, j) A_k(i) B_l(j)}{\sum_{j=1}^{M} \sum_{i=1}^{N} A_k(i) B_l(j)},$$

$k = 1, \ldots, n$; $l = 1, \ldots, m$. Basic functions $A_1, \ldots, A_n$ and $B_1, \ldots, B_m$ establish fuzzy partitions of $[1, N]$ and $[1, M]$ respectively, and $n < N$, $m < M$. The value $\rho = (nm)/(NM)$ is called the *compression ratio*.

A reconstruction of the image I (function f_I) is given by the inverse F-transform of f_I adapted to the domain $[1, N] \times [1, M]$:

$$f_{F,nm}(i, j) = \sum_{k=1}^{n} \sum_{l=1}^{m} F_{kl} A_k(i) B_l(j).$$

Two quality indices PSNR (Peak Signal to Noise Ratio) and RMSE (Root Mean Square Error) of the reconstructed image are given by

$$PSNR = 20 \lg \frac{255}{RMSE}$$

where

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} \sum_{j=1}^{M} (f_I(i, j) - f_{F,nm}(i, j))^2}{NM}}.$$

A comparison of PSNR and RMSE for three techniques: F-transform, fuzzy relation compression [2] and JPEG has been attempted in [1]. It has been shown that the technique of F-transform is better than the technique of fuzzy relation compression, but it is worse than the technique of JPEG. However, a complexity of the F-transform based compression is (in many cases) less than the complexity of JPEG.

6.2 Advanced Image Compression

The following standard trick allows to improve the effectiveness of a compression/reconstruction technique: divide a whole image into blocks and apply the technique to each block separately. Instead of this, we propose another trick which takes advantage of preserving monotonicity. The details and some examples are discussed below.

Aiming to increase the quality of F-transform based compression and reconstruction, we propose to divide the image I into a finite number of layers by partitioning the range of its corresponding representing function f_I. Each layer will be compressed and then reconstructed with the help of F-transform. The resulting inverse F-transforms of layers will be then combined into one function which represents the final reconstructed image of I. The detailed description will be given below for the fixed number (four) of layers.

Step 1. Partition the interval $[0, 1]$ into four subintervals: $[0, .25]$, $[.25, .5]$, $[.5, .75]$ and $[.75, 1]$.

Step 2. "Divide" the function f_I into four auxiliary functions (layers) $f_I^l(i, j)$, $l = 1, 2, 3, 4$, so that they are as follows:

$$f_I^l(i,j) = \begin{cases} f_I(i,j) & \text{if } f_I(i,j) \in S^l, \\ 1 - 0.25l & \text{otherwise.} \end{cases}$$

and $S^l = \{y \mid 1 - 0.25l \leq y \leq 1 - 0.25(l-1), \}$.

Step 3. Compress each function $f_I^l(i, j)$, $l = 1, 2, 3, 4$, and then reconstruct each of them with the help of direct and inverse F-transforms, respectively.

Step 4. Combine the resulting inverse F-transforms $f^1_{F,nm}(i, j), \dots, f^4_{F,nm}(i, j)$ into one function $f^f_{F,nm}(i, j)$ which represents the final reconstructed image of I.

Let us stress that the ranges of the compressed and reconstructed functions will be preserved due to the preserving monotonicity. Therefore, the combination of the four inverse F-transforms $f^1_{F,nm}(i, j), \dots, f^4_{F,nm}(i, j)$ into one does not lead to a collision.

We claim that the quality (measured by PSNR and/or RMSE) of F-transform based compression and reconstruction of each layer $f_I^l(i, j)$, $l = 1, 2, 3, 4$, is better than the quality of the reconstructed function $f_{F,nm}$. The justification of this claim is based on Theorem 1. Indeed, the above mentioned quality is measured with the help of difference between the original image and its reconstruction. By (4), this difference is estimated from above by the "modulus of continuity" of the original image. The latter is represented by a discrete function which may have big "jumps". On the other hand, the layers of the image have restrictive "jumps" and by this, the quality of their reconstructions is better. Let us illustrate this explanation by two pictures (Figure 1 and Figure 2), both show inverse F-transforms of two functions $5*sign(x)$ and $max(5 * sign(x), 3)$ defined on the interval $[-2, 2]$. Both functions have "jumps" at the point $x = 0$, however, the "jump" of the second function $max(5 * sign(x), 3)$ is smaller. Therefore, the inverse F-transform of $max(5 * sign(x), 3)$ on the interval $(0, 2]$ is closer to the original function. This is justified by values of two different inverse F-transforms at point $x = .04$.

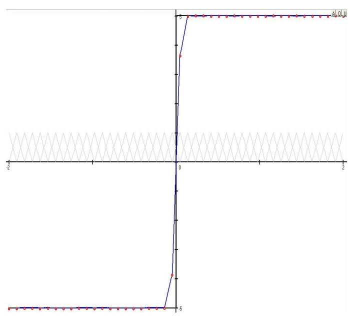

Figure 1: The functions $5 * sign(x)$ and its inverse F-transform. At point $x = .04$, $5 * sign(x) = 1$, and the value of the inverse F-transform is 3.65.

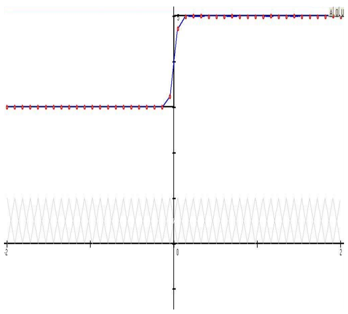

Figure 2: The functions $max(5 * sign(x), 3)$ and its inverse F-transform. At point $x = .04$, $max(5 * sign(x), 3) = 1$, and the value of the inverse F-transform is 4.74.

Let us illustrate the proposed technique on the picture "Cameraman" which is taken from the Corel Galery. Below, we show four pictures with the original image (Fig. 3), its inverse F-transform applied to the whole image (Fig. 4), four layers (Fig. 5) and finally, their combination into one reconstructed image (Fig. 6).

Figure 3: Original image "Cameraman"

Figure 4: Reconstructed image "Cameraman" after applying F-transform.

7 Conclusion

The theory of fuzzy transforms proved to be successful in application to image compression and reconstruction. However, the respective reconstruction is lossy. The aim of this contribution is to show that the quality of reconstruction can be improved if we take into account the monotonicity property.

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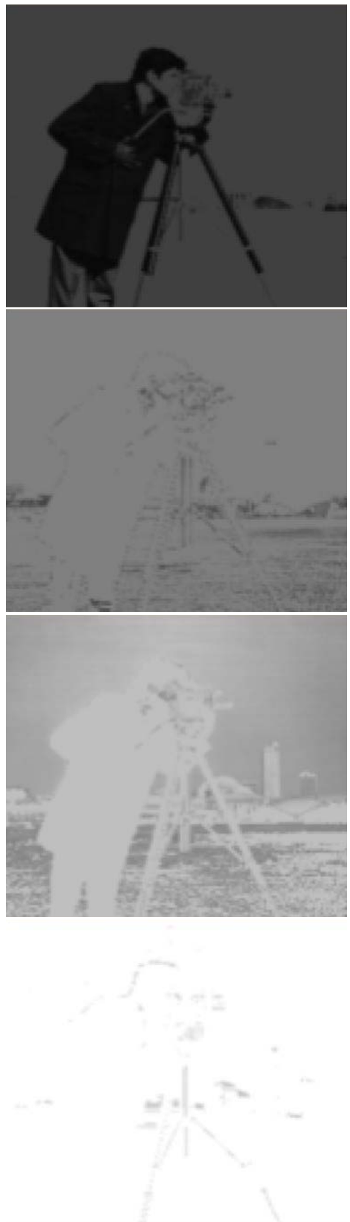

Figure 5: The four layers of "Cameraman".

Figure 6: Reconstructed image "Cameraman" after combining four layers of the inverse F-transform.