SMV-algebras and Kripke models: comparing the semantics

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Abstract

In this paper we compare the variety of SMV-algebras with the class of Kripke models introduced in [8, 6, 5] as semantics for the logics $FP(\text{Ł}_n, \text{Ł})$ and $FP(\text{Ł}, \text{Ł})$. The main result of this paper tells us that a formula φ written in the language of SMV-algebras is satisfiable in a Kripke model iff there exists a non-trivial SMV-algebra satisfying φ. This result is used also to provide results about the decidability and complexity for SMV-algebras.

Keywords: States, MV-algebras, SMV-algebras, Kripke models, complexity.

1 Introduction

States on MV-algebras were introduced by Mundici in [11] as averaging processes for formulas in Łukasiewicz logic. In [7] we introduce the class of *MV-algebras with an internal state* (SMV-algebras for short) to treat a state as an internal operator of an MV-algebra. (We shall recall the definition of SMV-algebras in the next section).

In order to treat states in a logical framework, in the last years various probabilistic logics have been introduced. Hájek (cf [8]) presents a fuzzy logic (called $FP(\text{Ł})$ in [8]) with a modality Pr (interpreted as *probably*) which is suitable for the treatment of probability over classical events. The axioms of this logic are suggested by the following semantic interpretation: the probability of an event φ is interpreted as the truth value of the modal formula $\Pr(\varphi)$ ("φ is probable"). Along these lines, Flaminio and Godo extend in [6] Hájek's original work introducing the logics $FP(\text{Ł}_n, \text{Ł})$ and $FP(\text{Ł}, \text{Ł})$, so to treat the probability of many-valued events.

A complete discussion on $FP(\text{Ł}, \text{Ł})$ falls out of the scope of this paper (we suggest the reader to consult [5, 6] for a complete treatment). Here we want just to recall that the class of its well-founded formulas includes all the formulas of Łukasiewicz logic (that are the non-modal formulas), and the class of modal formulas defined as follows: for each non-modal formula φ, $\Pr(\varphi)$ is a modal formula, the truth constant $\overline{0}$ is modal, finally these formulas are combined by means of the Łukasiewicz connectives.

A Kripke model for the logic $FP(\text{Ł}, \text{Ł})$ is a pair $K = (W, \mu)$ where W is a set of valuations of propositional variables of Łukasiewicz logic in $[0, 1]$ and $\mu : W \to [0, 1]$ has to satisfy the condition: $\sum_{w \in W} \mu(w) = 1$. Elements of W are also called *nodes* or *possible worlds*.

Given a Kripke model $K = (W, \mu)$ and a formula Φ of $FP(\text{Ł}, \text{Ł})$, the truth value $\|\Phi\|_{K,w}$ of Φ in K at the node w is inductively defined as follows:

- If Φ does not contain any occurrence of the modality Pr, then $\|\Phi\|_{K,w} = w(\Phi)$,
- If Φ is in the form $\Pr(\psi)$, then

$$\|\Phi\|_{K,w} = \textstyle\sum_{w \in W} w(\psi) \cdot \mu(w).$$

– Compound formulas are evaluated by truth-functionality by means of the *standard* interpretation of Łukasiewicz connectives (see Example 1.1 (1)).

A natural expectation is that $FP(\text{Ł},\text{Ł})$ may be complete with respect to the following notion of Kripke models[1].

The logic $FP(\text{Ł},\text{Ł})$ is not algebraizable in the sense of Blok-Pigozzi (cf [1]). Recall in fact that $\Pr(\varphi)$ is a well-founded formula only if φ is a non-modal formula, and hence φ does not contain any occurrence of Pr, therefore the algebraic counterpart of the operator Pr is a partial operation but not an operation.

SMV-algebras are the algebraic counterpart of a natural extension of the logic $FP(\text{Ł},\text{Ł})$. The differences with that logic are:

(a) The language is extended by the rule: $\Pr(\varphi)$ is a formula whenever φ is a formula, without the restriction that φ does not contain occurrence of Pr.

(b) The axioms of $FP(\text{Ł},\text{Ł})$ (cf [5, 6]) are extended to the formulas of the new language.

(c) The axiom schema $\Pr(\varphi) \leftrightarrow \varphi$ is added, whenever φ ranges over all formulas all of whose variables only occur under the scope of Pr (this axiom reflects the fact that such formulas represent real numbers which coincide with their probability).

In this paper we compare SMV-algebras and Kripke models, and our main result shows that a formula φ in the language of SMV-algebras is satisfiable in a Kripke model K iff φ holds in an SMV-algebra whose internal state is the integral of f_φ, the latter being a function associated to φ. Finally we use this result to provide results about the decidability and the computational complexity for SMV-algebras. We end discussing some future work.

[1]The problem of establishing completeness was left open by Flaminio and Godo in [6] and, as far as we know, no solution has been given yet

1.1 Preliminaries

An *MV-algebra*[2] is a system $(A, \oplus, ^*, 0)$, where $(A, \oplus, 0)$ is a commutative monoid with neutral element 0, and for each $x, y \in A$ the following equations hold: (i) $(x^*)^* = x$, (ii) $x \oplus 1 = 1$, where $1 = 0^*$, and (iii) $x \oplus (x \oplus y^*)^* = y \oplus (y \oplus x^*)^*$. The class of MV-algebras forms a variety which henceforth will be denoted by $\mathbb{MV}$.

In any MV-algebra one can define further operations as follows: $x \to y = (x^* \oplus y)$, $x \ominus y = (x \to y)^*$, $x \odot y = (x^* \oplus y^*)^*$, $x \leftrightarrow y = (x \to y) \odot (y \to x)$, $x \vee y = (x \to y) \to y$, and $x \wedge y = (x^* \vee y^*)^*$. Henceforth we shall use the following notation: for every $x \in A$ and every $n \in \mathbb{N}$, $nx = x \oplus \overset{n}{\ldots} \oplus x$, and $x^n = x \odot \overset{n}{\ldots} \odot x$.

Any MV-algebra A can be equipped with an order relation. As a matter of fact defining, for all $x, y \in A$, $x \leq y$ iff $x \to y = 1$. An MV-algebra is said *linearly ordered* (or an MV-chain) if the order $\leq$ is linear.

Example 1.1 *(1) The* standard *MV-algebra is the system* $[0,1]_{MV} = ([0,1], \oplus, ^*, 0)$ *where for each* $x, y \in [0,1]$, $x \oplus y = \min\{1, x + y\}$ *and* $x^* = 1 - x$.

(2) Fix a $k \in \mathbb{N}$ *and let* $F(k)$ *be the set of all McNaughton functions on* $[0,1]^k$, *(cf [4]). Then the algebra* $\mathcal{F}(k) = (F(k), \oplus, ^*, 0)$, *where* $\oplus$ *and* * *are the pointwise application of the operations defined as in the above example (1), and* 0 *is the function constantly equal to* 0, *is the free MV-algebra over* k *generators.*

A *state* s on an MV-algebra A is a map $s : A \to [0,1]$ satisfying the following two conditions: $s(0) = 0$, and for all $x, y \in A$, if $x \odot y = 0$, then $s(x \oplus y) = s(x) + s(y)$ (where in the left-side of the equalities, + denotes the usual sum between real numbers). A state s is said to be *faithful* if $s(x) = 0$ implies $x = 0$.

An *MV-algebra with an internal state* (SMV-algebra for short) is a pair (A, σ), where A is an MV-algebra and $\sigma : A \to A$ satisfies the following properties for all $x, y, z \in A$:

[2]We suggest the reader to consult [4] for a complete treatment of MV-algebras.

($\sigma 1$) $\sigma(0) = 0$,

($\sigma 2$) $\sigma(x^*) = (\sigma(x))^*$,

($\sigma 3$) $\sigma(x \oplus y) = \sigma(x) \oplus \sigma(y \ominus (x \odot y))$,

($\sigma 4$) $\sigma(\sigma(x) \oplus \sigma(y)) = \sigma(x) \oplus \sigma(y)$.

As in the case of states, we call an SMV-algebra *faithful* if it satisfies the quasi equation: $\sigma(x) = 0$ implies $x = 0$.

Example 1.2 *(1) Let A be any MV-algebra and σ be the identity on A. Then (A, σ) is an SMV-algebra.*

(2) Let A be the algebra of all continuous and piecewise linear functions with real coefficients from $[0, 1]^k$ into $[0, 1]$. Then A, with the pointwise application of MV-algebraic $\oplus$ and *, forms an MV-algebra. Now let for $f \in A$, $\sigma(f)$ be the function from $[0, 1]^k$ to $[0, 1]$ which is constantly equal to

$$\int_{[0,1]^k} f(x)\, dx.$$

Then (A, σ) is an SMV-algebra. As we have shown in [7], (A, σ) is simple, therefore it is subdirectly irreducible, but it is not totally ordered. Although rather general, this algebra is faithful: it satisfies the quasi equation $\sigma(x) = 0$ implies $x = 0$, which is not valid in general.

2 Tensor SMV-algebras

In [7] we compare the notions of SMV-algebra and of state on an MV-algebra. In particular we have shown how, starting from an SMV-algebra (A, σ), one can define a state s on the MV-algebra A and vice-versa. Clearly, in order to define an SMV-algebra starting from a state s on an MV-algebra A, we need to *internalize* the state s in a new MV-algebra T containing both A and $[0, 1]_{MV}$ as sub MV-algebras. The MV-algebra T has been defined by means of the so called *MV-algebraic tensor product construction* (cf. [12]). Recall that the tensor product $A_1 \otimes A_2$ of two MV-algebras A_1 and A_2 is an MV-algebra (unique up to isomorphism) such that there is a universal bimorphism β from the cartesian product $A_1 \times A_2$ into $A_1 \otimes A_2$ (see [12] Definition 2.1 for the concept of bimorphism). Universal means that for any (other) bimorphism $\beta' : A_1 \times A_2 \to B$ (B being an MV-algebra) there is a unique homomorphism $\lambda : A_1 \otimes A_2 \to B$ such that $\beta' = \lambda \circ \beta$. Henceforth, for $a_1 \in A_1$ and $a_2 \in A_2$, we denote $\beta(a_1, a_2)$ by $a_1 \otimes a_2$.

It is possible to show that, for each pair of MV-algebras A_1 and A_2, both A_1 and A_2 are sub MV-algebras of $A_1 \otimes A_2$ (the reader may consult [7, 12] for further details).

Let us now turn back to our starting assumption: let $s : A \to [0, 1]$ be a state (in the sense of Mundici) on the MV-algebra A. Then consider the MV-algebra $T = [0, 1]_{MV} \otimes A$, together with the unary operator $\sigma : T \to T$ so defined: for each $\alpha \otimes a \in T$,

$$\sigma(\alpha \otimes a) = s(a) \cdot \alpha \otimes 1.$$

Notice that σ actually maps T into T, and hence σ is internal. Moreover the following holds:

Theorem 2.1 ([7]) *Let s, T and σ be defined as above. Then σ is well-defined, and (T, σ) is an SMV-algebra.*

Notation 2.2 *SMV-algebras of the form (T, σ), where T and σ are defined as above, will be used in the remaining of the present paper. Hence we shall henceforth call them* tensor SMV-algebras.

2.1 Application to coherence

In [7] we show how the coherence of a rational assessment over finitely many formulas of Łukasiewicz logic, can be equationally characterized in the theory of SMV-algebras. First of all recall that an assessment

$$P(\varphi_1) = \frac{n_1}{m_1}, \ldots, P(\varphi_t) = \frac{n_t}{m_t},$$

where $\varphi_1, \ldots, \varphi_n$ are fuzzy events represented by formulas of Łukasiewicz logic, and $\frac{n_1}{m_1}, \ldots, \frac{n_t}{m_t} \in [0, 1] \cap \mathbb{Q}$, is said *coherent* if there exists a state s on the Lindenbaum algebra $\mathcal{F}(k)$ of Łukasiewicz logic ($x_1, \ldots, x_k$ being the of variables occurring in the formulas φ_i's) such that $s([\varphi_i]) = P(\varphi_i)$ (where $[\varphi_i]$ denotes the equivalence class of φ_i modulo provable equivalence).

Let $y_1, \ldots, y_t$ be fresh variables, and consider for each $i = 1, \ldots, t$, the equations:

$$\varepsilon_i : (m_i - 1)y_i = y_i^*, \text{ and } \delta_i : \sigma(\varphi_i) = n_i y_i.$$

In [7] (see Theorem 6.1) we proved the following:

Theorem 2.3 *Let $\chi : P(\varphi_i) = \frac{n_i}{m_i}$ be a rational assessment over the Łukasiewicz formulas $\varphi_1, \ldots, \varphi_t$. Then the following are equivalent:*

(a) χ is coherent.

(b) The equations ε_i, and δ_i (for $i = 1, \ldots, t$) are satisfied in some non-trivial SMV-algebra.

3 SMV-algebras and Kripke-models

Kripke models allow to interpret SMV-terms as follows: let φ be a term in the language of SMV-algebras, let $K = (W, \mu)$ be a Kripke model (recall Section 1), and let $w \in W$. Then the truth-value of φ in K at the node w ($\|\varphi\|_{K,w}$) is inductively defined as follows:

(i) $\|x\|_{K,w} = w(x)$ for each variable x, and $\|0\|_{K,w} = 0$

(ii) $\|\sigma(\psi)\|_{K,w} = \sum_{w \in W} w(\psi) \cdot \mu(w)$,

(iii) $\|\psi_1 \oplus \psi_2\|_{K,w} = \min\{1, \|\psi_1\|_{K,w} + \|\psi_2\|_{K,w}\}$,

(iv) $\|\psi^*\|_{K,w} = 1 - \|\psi\|_{K,w}$.

Notice that the truth-value of a term falling in the scope of σ is independent on the chosen world w, hence we shall henceforth simply write $\|\sigma(\psi)\|_K$ instead of $\|\sigma(\psi)\|_{K,w}$. Moreover compound formulas are evaluated by truth-functionality (Caution: in evaluating a compound formula, the subformulas of the form $\sigma(\varphi)$ where φ is σ-free has to be evaluated as an atomic formula, and hence as in (ii), without a previous evaluation of φ in a fixed world w). The following lemma, whose proof can be easily obtained from [13] (proof of Theorem 2.1), will be useful to prove the main results of this section.

Lemma 3.1 *Let $\varphi_1, \ldots, \varphi_n$ be Łukasiewicz formulas in the variables $x_1, \ldots, x_k$, and let $\chi : \varphi_i \mapsto \beta_i$ $(i = 1, \ldots, n)$ an assessment. Then χ is coherent iff there is a finite set W of valuations from $\mathcal{F}(k)$ into $[0, 1]$, and a map μ from W into $[0, 1]$, such that $\sum_{w \in W} \mu(w) = 1$, and $\sum_{w \in W} w(\varphi_i)\mu(w) = \beta_i$.*

The next theorem is the main result of this section.

Theorem 3.2 *Let φ be a term in the language of SMV-algebras. Then the following are equivalent:*

(i) There is a Kripke-model (W, μ) such that $(W, \mu) \models \varphi$.

(ii) There is a tensor SMV-algebra (T, σ) such that $(T, \sigma) \models \varphi$.

(iii) There is an SMV-algebra (A, σ) such that $(A, \sigma) \models \varphi$.

Proof. The direction (ii)⇒(iii) is obvious.

(i)⇒(ii): Let $K = (W, \mu)$ be a Kripke-model satisfying φ, i.e. $\|\varphi\|_K = 1$. Let A be the MV-algebra of all functions from W into $[0, 1]$, that is $A = ([0, 1]^W, \oplus, ^*, 0)$ where $\oplus$ and * are defined pointwise and 0 denotes the function constantly equal to 0. Let now $s : A \to [0, 1]$ be so defined: for each $f \in A$,

$$s(f) = \sum_{w \in W} f(w)\mu(w).$$

Clearly s is a state on A. Let hence (T, σ) be the tensor SMV-algebra obtained by putting $T = [0, 1]_{MV} \otimes A$, and for each $\alpha \otimes f \in T$, $\sigma(\alpha \otimes f) = \alpha s(f)$. As we know by Theorem 2.1, (T, σ) is an SMV-algebra. Hence there remains to be shown that $(T, \sigma) \models \varphi$. Now interpret every MV-term ψ in the function $f_\psi : w \in W \mapsto w(\psi)$. Every SMV-term of the form $\sigma(\gamma)$ is then interpreted in $s(f_\gamma)$

Thus $(T, \sigma) \models \varphi$. In fact:

- If $\varphi = x$, then by hypothesis there is a $w \in W$ such that $\|x\|_{K,w} = 1$. Hence $f_x(w) = 1$.
- If $\varphi = \sigma(\psi)$, then $s(f_\psi) = \|\sigma(\psi)\|_K = 1$.

- If either $\varphi = \psi_1 \oplus \psi_2$, or $\varphi = \psi^*$, then the claim easily follows.

(iii)⇒(i): Let (A, σ) be any SMV-algebra, and let e be an SMV-evaluation into (A, σ) such that $e(\varphi) = 1$. Let moreover $h : A \to [0,1]_{MV}$ be a homomorphism. Then $h \circ e$ is a $[0,1]$-evaluation satisfying φ. Moreover, if $\sigma(\psi_1), \ldots, \sigma(\psi_n)$ are all the φ subformulas beginning by σ, then the assessment

$$\chi : \psi_i \mapsto h(e(\sigma(\psi_i))) \text{ (for } i = 1, \ldots, n)$$

is coherent (actually it is easy to see that the composition $h \circ e \circ \sigma$ is a state on the Lindenbaum algebra $\mathcal{F}(k)$ generated by the variables $x_1, \ldots, x_k$ occurring in the ψ_i). Hence, by Lemma 3.1, there are a finite set $W = \{w_1, \ldots, w_m \mid w_i : \mathcal{F}(k) \to [0,1]\}$, and a $\mu : W \to [0,1]$ such that:

$$\sum_{w \in W} \mu(w) = 1, \text{ and}$$
$$\sum_{w \in W} w(\psi_i)\mu(w) = h(e(\sigma(\psi_i))) \text{ for all } i = 1, \ldots, n.$$

Let now $W' = W \cup \{h \circ e\}$, and put $\mu(h \circ e) = 0$. Then (W', μ) is a Kripke-model satisfying φ. In fact:

(1) If φ has no occurrences of σ, then $\|\varphi\|_{K,h\circ e} = (h \circ e)(\varphi) = 1$.

(2) If $\varphi = \sigma(\psi)$, then $\|\varphi\|_K = \sum_{w \in W'} w(\psi)\mu(w) = h(e(\sigma(\psi))) = 1$.

(3) If φ contains subformulas $\gamma_1, \ldots, \gamma_l$ not falling in the scope of σ, and it also contains subformulas $\sigma(\psi_1), \ldots, \sigma(\psi_n)$, then evaluate the γ_i as in (1), and the $\sigma(\psi_j)$ as is (2). Finally $\|\varphi\|_{K,h\circ e} = h(e(\varphi)) = 1$.

This ends the proof of the theorem.

■

3.1 Complexity issues

Now we are going to apply Lemma 3.1 and Theorem 3.2 to provide some results about the complexity of the satisfiability problem in the variety $\mathbb{SMV}$ of SMV-algebras.

First of all we need to fix some notation: let Φ be a formula in the language of SMV-algebras, and let $\psi_1, \ldots, \psi_l$ all the subformulas of Φ (Φ included). Now we can translate the satisfiability of Φ in a Kripke-model, by means of the satisfiability of a first order formula of field theory, in the field of reals[3]. We need the following famous result from linear programming:

Lemma 3.3 ([3]) *If a system of k linear equalities and/or inequalities has a (non-negative) solution, then it has a non-negative solution with at most k positive entries.*

The translation works as follows: for each Φ subformula ψ_i, let us enlarge the language of fields by a fresh variable x_{ψ_i}. For each ψ_i, ψ_j consider the formulas:

(A_{ij}) $[(x_{\psi_i} + x_{\psi_j} \geq 1) \to (x_{\psi_i \oplus \psi_j} = 1)] \wedge [(x_{\psi_i} + x_{\psi_j} < 1) \to (x_{\psi_i \oplus \psi_j} = x_{\psi_i} + x_{\psi_j})]$

and

(B_i) $x_{\neg\psi_i} = 1 - x_{\psi_i}$.

Moreover, if $\sigma(\gamma_1), \ldots, \sigma(\gamma_k)$ are all the Φ subformulas beginning by σ, we have to guarantee that the evaluation of the variables $x_{\sigma(\gamma_1)}, \ldots, x_{\sigma(\gamma_k)}$ is coherent. Due to Lemma 3.1, and Lemma 3.3, this can be expressed by the following formula in the language of fields:

$$(C)\ \exists z_1, \ldots, z_k, y_{11}, y_{12}, \ldots, y_{kk} \left[\left(\sum_{t=1}^{k} z_t = 1 \right) \wedge \right.$$
$$\left. \wedge \left(\bigwedge_{s=1}^{k} \sum_{r=1}^{k} y_{sr} z_r = x_{\sigma(\psi_{js})} \right) \right].$$

Recalling Lemma 3.1, the variable z_i (for each $i = 1, \ldots, k$) stands for the value $\mu(w_i)$, while the variables y_{lt} express the evaluations $w_l(\gamma_t)$ (for each $0 \leq l, t \leq k$). Notice that, due to Lemma 3.3, we have assumed the variables y_{lt} to be $2k$ in all, because we are considering just those evaluations of the k variables, which are not constantly zero.

[3]Notice that the idea of such a translation is not new, see for instance [9, 10] for details

Theorem 3.4 *The problem of deciding whether an SMV-formula Φ is satisfiable in some SMV-algebra (A, σ) is in PSPACE.*

Proof. Let as above $\psi_1, \ldots, \psi_l$ be all the subformulas of Φ, and, among all the ψ_j, let $\sigma(\gamma_1), \ldots, \sigma(\gamma_k)$ be all those beginning by σ. Let now Φ^F be the formula:

$$\left(\bigwedge_{i,j=1}^{l} (A_{ij})\right) \wedge \left(\bigwedge_{i=1}^{l} (B_i)\right) \wedge (C) \wedge (x_\Phi = 1).$$

(A_{ij}), (B_i) and (C) being as above.

Claim 3.5 *Φ^F is satisfiable in the field of real numbers iff Φ is satisfiable in some Kripke model $K = (W, \mu)$.*

Proof. (of Claim 3.5). ($\Rightarrow$) If η is an evaluation on the field of reals such that $\eta(\Phi^F) = 1$, then by (A_{ij}) and (B_i) we have a $[0,1]_{MV}$-evaluation e of Φ subformulas. Moreover (C) and Lemma 3.1 tells us that the evaluation of formulas $\sigma(\gamma_1), \ldots, \sigma(\gamma_k)$ is coherent. Thus define a Kripke model K as follows:

$$W = \{w_1, \ldots, w_k \mid w_i(\gamma_t) = \eta(y_{it})\} \cup \{e\},$$

and $\mu(w_i) = \eta(z_i)$ and $\mu(e) = 0$.

One can prove by induction on the complexity of a formula Ψ, that there is a $w \in W$ such that $\eta(\Psi^F) = \|\Psi\|_{K,w}$, hence $(W, \mu) \models \varphi$.

($\Leftarrow$) Let $K = (W, \mu)$ be a Kripke model such that $(W, \mu) \models \Phi$, then there is an evaluation $\eta \in W$ such that $\eta(\Phi^F) = \|\psi_i\|_{K,\eta}$. Actually η can be regarded as an evaluation on the real field such that $\eta(\Phi^F) = 1$. Thus the claim follows.

■

Claim 3.5 together with Theorem 3.2 says us that Φ^F is satisfiable in the field of reals iff Φ is satisfiable in some SMV-algebra (A, σ).

Now Φ^F is an existential formula in the language of reals, and the main theorem of [2] is to the effect that satisfiability of existential formulas of field theory in the field of reals is in PSPACE. This settles our claim. ■

We now immediately obtain:

Corollary 3.6 *Let $\chi : P(\varphi_i) = \frac{n_i}{m_i}$ be a rational assessment over Łukasiewicz formulas $\varphi_1, \ldots, \varphi_n$. Then the problem of testing the coherence of χ is in PSPACE.*

Proof. The proof can be easily obtained by combining Theorem 2.3, Theorem 3.4 and observing that the total length of equations which in Theorem 2.3 characterize the coherence of χ is polynomial in n. ■

4 Conclusion and future work

The main result of the present paper states that, regarding satisfiability, SMV-algebras are complete with respect to Kripke models. Using this result we have shown that the satisfiability problem for SMV-algebras is in PSPACE.

In our future work we plan to investigate the following problems:

(a) Is the variety $\mathbb{SMV}$ of SMV-algebras generated by tensor SMV-algebras? Does Theorem 3.2 still hold with tautologies in place of satisfiable formulas?

(b) Is the satisfiability problem for SMV-algebras NP-complete?

A positive answer to the first question would settle the problem posed in [6], of proving that $FP(Ł, Ł)$ is complete with respect to Kripke models of the form (W, μ).

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