Clan Theory and its application in the selection of financial products

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Abstract

When the decision maker invests in the banking organizations, him is faced with the need to choose between apparently different products but which, when all is said and done, are very similar. Every financial product is perceived by a set of attributes that are held in a degree or level. The new situation which we faced cannot be treated by means of the application of conventional models, since we were in the total uncertainty.

Often, the allocation of the degree or level becomes subjective, it is subject to valuation between two heights that we agree with the segment [0, 1]. Here we are going to incorporate in a new study decisional process to present the best opportunities to choose from among several different groups of products, those that possess the attributes previously established.

Keywords: Clan theory, Preferences, Financial Products

1 Introduction

The raising of financial means by businesses brings up a problem of decision as a consequence of the variety of financial products that the banks and other credit institutions place at the disposal of their eventual customers.With increasing frequency it can be seen that new products appear on the market under many different forms that, either real or apparent, have different characteristics. It should not be forgotten that the strong competition characterising the financial world obliges those offering payment means to a great effort of diversification and differentiation of products that permits them, on the one hand, to cover the widest range of possible users and, on the other, provoke a flaw by means of the presentation of different products with the object of get around the inexorable laws of the perfect market.

When the need arises for resorting to outside financing, executives in business find themselves faced with a certain number, obviously finite, of options offered by the market, from among which a selection must be made of the one that is best suited to the specific requirements of the business.

Evidently that for each business, and even for each specific situation, there will be a different valuation of each one of the characteristics of the financial products. Therefore, in certain cases, the speed of obtaining the financial means will be very important, on other occasions what is more important is the repayment period. In short, the decision maker will estimate for each circumstance an order of precedence of the characteristic that go to make up the products.

In this context two fundamental elements appear that make up the problem:

1) Differentiation in the characteristics of each one of the financial products on offer.

2) Different estimate, by the acquirer, of each of the characteristics relative to the rest, which provides an order of preference.

Evidently, the degree of preference for each one of the characteristics relative to the others may sometimes be determined by means of measurements[9,10], that is, with an objective nature, but on other occasions it will be necessary to resort to subjective numerical situations, that is by means of valuations.The same thing occurs when a comparison must be

L. Magdalena, M. Ojeda-Aciego, J.L. Verdegay (eds): Proceedings of IPMU'08, pp. 737–744
Torremolinos (Málaga), June 22–27, 2008

made, for each characteristic, of the degree of preference between one product and the rest.

The possible participation of objective data and subjective estimates makes it advisable to use management techniques that are valid for the field of uncertainty [3,15], taking into account the fact that the mathematics of certainty can be considered as a particular case of the mathematics of uncertainty, the schemes of which, of a «soft»nature, can also be applied to the case of crisp data of a «hard» nature.

On the other hand, the existence of relations between products, as well as the relations in the estimates of the different characteristics bring to mind the convenience of presenting this problem by means of subjective matrices, taking advantage of all the possibilities offered by matrix calculations.With all this an attempt is made to arrive at certain results that express the order of preference between different financial products to which a business may opt. The subjective nature of the estimated values should lead to certain conclusions that can be expressed by means of fuzzy sets.

2 Clan Theory

Clans of fuzzy sets are generalizations of Bolean algebras of sets [13]. We are going to introduce a formal theory of clans as a theory over **LΠ**$_\omega$ In addition, we relax the assumption that clans are crisp at the very beginning.

Definition 1. Let C be a constant standing for a fuzzy set of fuzzy sets. The theory of fuzzy clans is a theory with the following axioms [7]:

($C1$) $\emptyset \in T(E)$

($C2$) $(\forall A \in T(E))*(-A \in T(E))$

($C3$) $(\forall A, B \in T(E))*(A \cup_L B \in T(E))$

The last formula should be read as

$$(\forall A)(\forall B)((A \in T(E) \,\&_*\, B \in T(E)) \rightarrow_* (A \cup_L B \in T(E)))$$

The constant $T(E)$ is represented in models of theory of fuzzy clans by a fuzzy set of fuzzy sets which contains the empty set in the degree 1 and satisfice conditions given by ($C2$) and ($C3$).

Proposition 1 These are provable formulae in the theory of fuzzy clans:

1. $V \in T(E)$

2. $(\forall A, B \in T(E))*(A \cap_L B \in T(E))$

3. $(\forall A, B \in T(E))*(A \cup_G B \in T(E))$

4. $(\forall A, B \in T(E))*(A \cap_G B \in T(E))$

Proof.

1. Putting together axioms ($C1$) and ($C2$) from Definition 3, we get immediately $V \in T(E)$.

2. Since the formula $A \cap_L B \in T(E)$ is provably equivalent to $-(-A \cup_L -B) \in T(E)$, it follows from ($C2$) and ($C3$) that the second formula is provable.

3. The expressión $A \cap_G B$ may be rewritten as $A \cap_L (-A \cup_L B)$

4. $A \cup_G B$ is the same as $-(-A \cap_G -B)$.

We can also prove:

$$(\forall A_1,\ldots,A_n \in T(E))*(A_1 \cup_L \ldots \cup_L A_n \in T(E)),$$
$$(\forall A_1,\ldots,A_n \in T(E))*(A_1 \cap_L \ldots \cap_L A_n \in T(E)),$$
$$(\forall A_1,\ldots,A_n \in T(E))*(A_1 \cup_G \ldots \cup_G A_n \in T(E)),$$
$$(\forall A_1,\ldots,A_n \in T(E))*(A_1 \cap_G \ldots \cap_G A_n \in T(E))$$

Since the previous formulae can be rewritten as implications each having the same conjunction

$$A_1 \in T(E) \,\&_* \ldots \&_*\, A_n \in T(E)$$

In the antecedent, the respective implications can be quite weak as, for instance, in case of Lukasiewicz conjunction $\&_L$. This difficulty can be simply overcome by dealing rather with crisp sets of fuzzy sets than fuzzy sets of fuzzy sets. This assumption is in fact the traditional approach adopted in the field of measures on clans of fuzzy sets [2]. Moreover, we avoid in this way any discussion inevitably related to a concept of 'fuzzy measurability' of elements belonging to fuzzy clans.

The theory of clans is an extension of theory of fuzzy clans by the axiom of crispness for $T(E)$:

($C4$) Crisp $T(E)$

Since clans were introduced to generalize Bolean algebras of sets, we can expect that any clan contains one.

We put

$$\mathbf{B}(T(E)) =_{df} \{A[A \in T(E) \,\&\, A \cup_L A = A\}$$

And call $\mathbf{B}(T(E))$ a Boolean skeleton of $T(E)$C. Observe that $\mathbf{B}(T(E)) \subseteq T(E)$ and $\mathbf{B}(T(E))$ is crisp from the definition. In any model of the theory of clans, the constant $\mathbf{B}(T(E))$ is represented by a Boolean álgebra of sets.

The next theorem enables to interpret algebras of sets as particular cases of clans.

Proposition 2. This formula is provable in the theory of clans:

$$(\forall A \in T(E))(\text{Crisp}(A)) \rightarrow T(E) = \mathbf{B}(T(E))$$

Proof. We want to show that $(T(E)) \subseteq \mathbf{B}(T(E))$ which is provably equivalent to $A \in T(E) \rightarrow A \in \mathbf{B}(T(E))$. It follows by modus ponens from the premises $A \in T(E)$ and $A \in T(E) \rightarrow$ Crisp (A) that Crisp (A). Since Crisp $(A) \rightarrow A \cup_L A = A$, we obtain immediately $A \in \mathbf{B}(T(E))$.

3 Application

Let us assume that in the financial market there are three products P_1, P_2 and P_3 with different characteristics [15], relative to:

C_1 = Price of the money;

C_2 = Repayment period;

C_3 = Possibilities for renewal;

C_4 = Fractioning of repayment;

C_5 = Speed in granting.

For each characteristic a property is considered. For C_1 «Inexpensive money»; for C_2 «Good repayment period»; for C_3 «Possibility of renewal»; for C_4 «Suitable for fractioning repayment»; for C_5 «Speed in granting».

For each one of these characteristics the following information is obtained.

For C_1:

– The price for P_1 is 20%;

– The price for P_2 is 22%;

– The price for P_3 is 18%.

The financial director establishes as the descriptor for the concept «inexpensive money» the following normal fuzzy sub-set:

$D_1 = f(C_1) =$

P_1	P_2	P_3
0,9000	0,8181	1

For C_2:

– Payback period for P_1 is 5 years;

– Payback period for P_2 is 6 years;

– Payback period for P_3 is 4 years.

The descriptor of the concept «good payback period» for the business is:

$D_2 = f(C_2) =$

P_1	P_2	P_3
$0,8\hat{3}$	1	$0,\hat{6}$

For C_3:

- The «Possibilities for renewal» of P_1 are half those of P_2 and 1/3 those of P_3.

The following normal fuzzy sub-set is estimated as the descriptor of the concept of «Possibilities for renewal»:

$D_3 = f(C_3) =$

P_1	P_2	P_3
$0,\hat{3}$	$0,\hat{6}$	1

For C_4:

– Repayment of P_1 is quarterly;

– Repayment of P_2 is monthly;

– Repayment of P_3 is quarterly.

The business considers as the descriptor for «Fractioning of repayment» the following normal fuzzy sub-set:

$D_4 = f(C_4) =$

P_1	P_2	P_3
1	$0,\hat{3}$	1

For C_5:

- Renewal of P_1 will be three times faster and more fluid than P_2 and five times more than P_3.

The descriptor of the concept of «Speed in granting» is shown in the following normal fuzzy sub-set:

	P_1	P_2	P_3
$D_5 = f(C_5) =$	1	$0,\widehat{3}$	$0,2$

With this information we can arrive at a matrix formed by the descriptors placed as rows of the same. In this way the columns will represent the characteristics of each on of the products P_j, $j = 1,2,3$.

The following is the matrix of the descriptors (see Matrix 1):

Matrix 1

$[D] =$	P_1	P_2	P_3
C_1	0,9000	0,8181	1
C_2	$0,8\widehat{3}$	1	$0,\widehat{6}$
C_3	$0,\widehat{3}$	$0,\widehat{6}$	1
C_4	1	$0,\widehat{3}$	1
C_5	1	$0,\widehat{3}$	$0,2$

That allows us to find a fuzzy sub-set for each financial product. The result is:

	C_1	C_2	C_3	C_4	C_5
$P_1 =$	0,9000	$0,8\widehat{3}$	$0,\widehat{3}$	1	1

	C_1	C_2	C_3	C_4	C_5
$P_2 =$	0,8181	1	$0,\widehat{6}$	$0,\widehat{3}$	$0,\widehat{3}$

	C_1	C_2	C_3	C_4	C_5
$P_3 =$	1	$0,\widehat{6}$	1	1	$0,2$

Which brings to light the degree in which each product posses each one of the characteristics C_i , $i = 1,2,3,4,5$.

If we obtain the intersection of the descriptors $D_1 \cap D_2 \cap D_3 \cap D_4 \cap D_5$ the result will be:

	P_1	P_2	P_3
$D_1 \cap D_2 \cap D_3 \cap D_4 \cap D_5 =$	$0,\widehat{3}$	$0,\widehat{3}$	$0,2$

Which indicates that financial products P_1 and P_2, posses, at the very least in a degree of $0,\widehat{3}$ all the required characteristics, while P_3 posses them, at least, in a degree of $0,2$.

It is quite evident that the information we have received is very poor for taking the decision to select one or other of the three financial products. For this reason we are going to develop a new procedure that requires the incorporation of the concept of a «sub-set of thresholds».

4 The Result by means of the Clan Theory

For this we will accept the hypothesis according to which when the degree of the characteristic of a financial product does not reach the required level it is considered that this characteristic is not posses by it [4,7,8,9]. For showing this in a better way we will divide the process into the following sections:

1. A comparison is made of each μ_{D_j}, $j = 1,2,3$, of each descriptor D_i, $i = 1,2,3,4,5$ with the corresponding μ_{P^*}.

When $\mu_{D_j} < \mu_{P^*}$ assign a 0

When $\mu_{D_j} \geq \mu_{P^*}$ assign a 1.

In this way we have:

	P_1	P_2	P_3	
$D_1^{(0,9)} = f^{(0,9)}(C_1) =$	0,9000	0,8181	1	$= \{P_1, P_3\}$

	P_1	P_2	P_3	
$D_2^{(0,8\widehat{3})} = f^{(0,8\widehat{3})}(C_2) =$	$0,8\widehat{3}$	1	$0,\widehat{6}$	$= \{P_1, P_2\}$

	P_1	P_2	P_3	
$D_3^{(0,\widehat{6})} = f^{(0,\widehat{6})}(C_3) =$	$0,\widehat{3}$	$0,\widehat{6}$	1	$= \{P_2, P_3\}$

	P_1	P_2	P_3	
$D_4^{(0,\widehat{6})} = f^{(0,\widehat{6})}(C_4) =$	1	$0,\widehat{3}$	1	$= \{P_1, P_3\}$

	P_1	P_2	P_3	
$D_5^{(0,\widehat{3})} = f^{(0,\widehat{3})}(C_5) =$	1	$0,\widehat{3}$	$0,2$	$= \{P_1, P_2\}$

which gives rise to the «family»:

$$F = \{P_1, P_3\},\ \{P_1, P_2\},\ \{P_2, P_3\},\ \{P_1, P_3\},\ \{P_1, P_2\}$$

2. Matrix 2, is composed by including the descriptors obtained at the required level, with which we arrive at:

Matrix 2

	P_1	P_2	P_3
C_1	1	0	1
C_2	1	1	0
C_3	0	1	1
C_4	1	0	1
C_5	1	1	0

We could reach this same result by taking the matrix of descriptors $[D]$ and the sub-set of thresholds P^*, (see Matrix 3-4).

Matrix 3

$[D] =$

	P_1	P_2	P_3
C_1	1	0	1
C_2	1	1	0
C_3	0	1	1
C_4	1	0	1
C_5	1	1	0

Matrix 4

$P^* =$

C_1	0,9000
C_2	$0{,}8\overset{\frown}{3}$
C_3	$0{,}\overset{\frown}{6}$
C_4	$0{,}\overset{\frown}{6}$
C_5	$0{,}\overset{\frown}{3}$

And the elements of each row are compared with the corresponding fuzzy subset assigning a 1 when the values of the matrix are equal to or higher and a zero when they are lower. Thus in the first row, as $0{,}900 = 0{,}900$, (C_1, P_1) will be assigned a 1; as $0{,}8181 < 0{,}900$, (C_1, P_2) a 0; as $1 > 0{,}900$, (C_1, P_3) will be assigned a 1, (see Matrix 5).

Matrix 5

	P_1	P_2	P_3
C_1	1	0	1
C_2	1	1	0
C_3	0	1	1
C_4	1	0	1
C_5	1	1	0

3. From the «family» obtained a determination is made of those products that have and those that do not have the five characteristics in the required degree. The result is:

$f^{(0,9)}(C_1) = \{P_1, P_3\}$ $\qquad f^{(0,9)}(\overline{C}_1) = \{P_2\}$

$f^{(0,8\overset{\frown}{3})}(C_2) = \{P_1, P_2\}$ $\qquad f^{(0,8\overset{\frown}{3})}(\overline{C}_2) = \{P_3\}$

$f^{(0,\overset{\frown}{6})}(C_3) = \{P_2, P_3\}$ $\qquad f^{(0,\overset{\frown}{6})}(\overline{C}_3) = \{P_1\}$

$f^{(0,\overset{\frown}{6})}(C_4) = \{P_1, P_3\}$ $\qquad f^{(0,\overset{\frown}{6})}(\overline{C}_4) = \{P_2\}$

$f^{(0,\overset{\frown}{3})}(C_5) = \{P_1, P_2\}$ $\qquad f^{(0,\overset{\frown}{3})}(\overline{C}_5) = \{P_3\}$

4. We now move on to find the mini-terms or «atoms» by means of the intersection of the common sub-sets found in the previous section[1]. We arrive at:

$$f(C_1) \cap f(C_2) \cap f(C_3) \cap f(C_4) \cap f(C_5) = \varnothing$$

$$f(C_1) \cap f(C_2) \cap f(C_3) \cap f(C_4) \cap f(\overline{C}_5) = \varnothing$$

$$f(C_1) \cap f(C_2) \cap f(C_3) \cap f(\overline{C}_4) \cap f(C_5) = \varnothing$$

$$f(C_1) \cap f(C_2) \cap f(\overline{C}_3) \cap f(C_4) \cap f(C_5) = \{P_1\}$$

$$f(C_1) \cap f(\overline{C}_2) \cap f(C_3) \cap f(C_4) \cap f(C_5) = \varnothing$$

$$f(\overline{C}_1) \cap f(C_2) \cap f(C_3) \cap f(C_4) \cap f(C_5) = \varnothing$$

[1] With the object of avoiding a too complex nomenclature we will not indicate the super-index in functions .

$f(C_1)\cap f(C_2)\cap f(C_3)\cap f(\overline{C}_4)\cap f(\overline{C}_5)=\varnothing$

$f(C_1)\cap f(C_2)\cap f(\overline{C}_3)\cap f(C_4)\cap f(\overline{C}_5)=\varnothing$

$f(C_1)\cap f(\overline{C}_2)\cap f(C_3)\cap f(C_4)\cap f(\overline{C}_5)=\{P_3\}$

$f(\overline{C}_1)\cap f(C_2)\cap f(C_3)\cap f(C_4)\cap f(\overline{C}_5)=\varnothing$

$f(C_1)\cap f(C_2)\cap f(\overline{C}_3)\cap f(\overline{C}_4)\cap f(C_5)=\varnothing$

$f(C_1)\cap f(\overline{C}_2)\cap f(C_3)\cap f(\overline{C}_4)\cap f(C_5)=\varnothing$

$f(\overline{C}_1)\cap f(C_2)\cap f(C_3)\cap f(\overline{C}_4)\cap f(C_5)=\{P_2\}$

$f(C_1)\cap f(\overline{C}_2)\cap f(\overline{C}_3)\cap f(C_4)\cap f(C_5)=\varnothing$

$f(\overline{C}_1)\cap f(C_2)\cap f(\overline{C}_3)\cap f(C_4)\cap f(C_5)=\varnothing$

$f(\overline{C}_1)\cap f(\overline{C}_2)\cap f(C_3)\cap f(C_4)\cap f(C_5)=\varnothing$

We interrupt the process at this point, since there are not three $f(\overline{C}_1)$ that have a same P_j and, therefore, the result of the intersection is, from here on, the void set.

The mini-terms or atoms that are not void then are $\{P_1\}$, $\{P_2\}$, $\{P_3\}$.These terms or atoms that are not void can also be obtained from Matrix 6.

Matrix 6

	P_1	P_2	P_3
C_1	1	0	1
C_2	1	1	0
C_3	0	1	1
C_4	1	0	1
C_5	1	1	0

By successively changing the rows in order to include the $\overline{C}_1$.

In this way we arrive at Matrix 7-11.

Matrix 7

	P_1	P_2	P_3
C_1	1	0	1
C_2	1	1	0
C_3	0	1	1
$\overline{C}_4$	0	1	0
C_5	1	1	0

Matrix 8

	P_1	P_2	P_3
C_1	1	0	1
C_2	1	1	0
$\overline{C}_3$	1	0	0
C_4	1	0	1
C_5	1	1	0

Matrix 9

	P_1	P_2	P_3
$\overline{C}_1$	0	1	0
C_2	1	1	0
C_3	0	1	1
$\overline{C}_4$	0	1	0
C_5	1	1	0

Matrix 10

	P_1	P_2	P_3
C_1	1	0	1
$\overline{C}_2$	0	0	1
C_3	0	1	1
C_4	1	0	1
$\overline{C}_5$	0	0	1

Matrix 11

	P_1	P_2	P_3
C_1	1	0	1
C_2	1	1	0
C_3	0	1	1
C_4	1	0	1
$\overline{C}_5$	0	0	1

The columns that have a 1 in all their elements give rise to the mini-terms, which in this case are $\{P_1\}$, $\{P_3\}$, $\{P_2\}$ also arrived at by the previous procedure.

5. Clan $K(F)$ is obtained produced by the «family» by taking the atoms and all their possible unions to which f will be added:

$$K(F)=\{\varnothing, \{P_1\}, \{P_2\}, \{P_3\}, \{P_1,P_2\}, \{P_1,P_3\}, \{P_2,P_3\}, \{P_1,P_2,P_3\}\}$$

It can be seen that this clan is a Boole sub-lattice, see Figure 1.

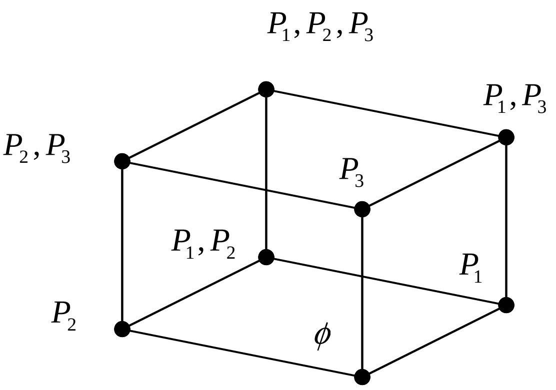

Figure 1: Boole sublattice

The non-void atoms have been represented by $\sum$.

By doing this process a wide range of information is obtained all of which can be most useful for taking decisions relative to the most suitable financial product for the interests of the business [6,9].

Thus, it will be seen that product P_1 has all the required characteristics except for the possibilities of renewal. Product P_2 does not have a good price nor adequate fractioning of repayments. On the other hand, product P_3 is not suitable for the business because of the repayment period set and also relative to the time necessary for granting the credit. All of this can easily be deduced due to the zeros that appear in the previous Matrix.

Even in a case as simple as the one we have shown, the decision does not have to be the only one and the financial product chosen will depend on the importance that the executives of the business assign to each of the characteristics.This is a new element that undoubtedly takes part in the selection of a financial product and which, due to its interest, should be explicitly taken into account on drawing up a model for the selection of financial products. But this will be the object for treatment in later sections.

Another type of information, perhaps less useful for this particular problem, but not exempt of interest, refers to the determination of the product or products that have some characteristics and not others, for which certain «keys» are established. Thus, for example, if we are looking for key: (low price «and» possibilities for renewal «and» suitable fractioning of payback) «and/or» (good repayment period «and» speed in granting), we arrive at:

$$(D_1 \cap D_3 \cap D_4) \cup (D_2 \cap D_5)$$

$$= (\{P_1,P_3\} \cap \{P_2,P_3\} \cap \{P_1,P_3\} \cup \{P_1,P_2\} \cap \{P_1,P_2\}$$

$$= \{P_3\} \cup \{P_1,P_2\}$$

$$= \{P_1,P_2,P_3\}$$

The result here is that this key is held by all three products. This can be verified by seeing if the values of the membership function of P_1, P_2 and P_3 are equal to or higher than the values corresponding to the threshold sub-set P^*.

One could also consider keys such as the following: (low price «and» suitable payback «and » no speed in granting) «and/or» (possibilities for renewal «and» good repayment period «and» good price), In this case we would have:

$$(D_1 \cap D_4 \cap \overline{D}_5) \cup (D_3 \cap \overline{D}_2 \cap D_1)$$

$$= (\{P_1,P_3\} \cap \{P_1,P_3\} \cap \{P_3\} \cup \{P_2,P_3\} \cap \{P_3\} \cap \{P_1,P_3\}$$

$$= \{P_3\} \cup \{P_3\}$$

$$= \{P_3\}$$

As we can see, the financial product P_3 complies with this particular key.

A large number of keys can be composed that can provide useful information for the financial institutions themselves who set up and sell the products as well as for businesses which are the eventual end users.

5. Conclusions

Information is one of the fundamental elements for taking decisions in a modern economic system. The financial environment is not, evidently, an exception. For this reason, we have developed a scheme which, under certain conditions, permits for treating data in a very wide way, giving rise to extraordinarily useful information in order to be able to decide on the suitability of taking a determined financial product. Normally, the customers wish to obtain the greater short term yield without repairing in the characteristics of the product that is being offered. The results obtained in some cases have been different depending on the method used. Then, we leave the option to the decision maker to choose the most suitable system according to his necessities.

Note that other potential applications could be developed such as human resource management, strategic management and product management.

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