Non-classical logic for Elderly Care Management

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Abstract

Health decline among elderly is diverse and we indeed often speak of multiple diseases e.g. involving psychiatric and cardiac diseases. This disease orientation with a treatment add-on is, however, only one scenario in which care of elderly can be accurately and completely described. In the case of elderly it is important to place ability and its measurement in primary focus as measurable reductions in abilities trigger all processes involving solicitation and caring together with clinical estimation. Interventions are always joint concerns and also involve teamwork building upon a wide range of professional skills. Workflow modelling is important and, in particular, decision making within these processes along which patient abilities are declining. Decision support is always based on rules and in the end on underlying logic. We will show how coordination of professional skills and capacities require consideration of different logic platforms which need to be interrelated.

Keywords: Elderly care, functional and cognitive ability, general logic.

1 Introduction

General logics [8] is a suitable and flexible logic framework for representing information and rules managed by different professional groups at various health levels of decision oriented nursing and care together with facts and conclusions provided within clinical classifications. Health levels are characterized by degrees of specificity which again calls for having *morphic* transformations of rules and inference mechanisms between these levels.

From information standardization point of view we need to consider classifications for functioning, disability and health (ICF [10]), together e.g. with the International Classification of Diseases (ICD-10). ICF provides a standardization framework for health and disability where focus is on impact rather than cause. Note that disability is not just a health oriented dysfunction and ICF indeed assists e.g. in measuring environmental impact of patient abilities in daily, also from social point of view. ICD-10 on the other hand is mostly diagnosis oriented.

In this we will show how coordination of professional skills and capacities require consideration of different logic frameworks which need to be related by a formal treatment of transformations from one logic to another.

The paper builds upon [5, 3] and provides further insight into methodological and regional developments. In [5] we provided a logical analysis of a diagnostic manual dividing mental disorders into disorder types. The subdivision is based on criteria sets in form of rules and defining features in form of facts. The focus was on dementia and we showed and compared how particular underlying logic for encoding the dementia classifier could manage

heterogeneity of individuals sharing a diagnosis and many-valuedness of truth regarding diagnosis of boundary cases. Methodologies for many-valued were drawn mostly from [4] and [6].

This paper is organized as follows. Section 2 describes the categorical tools needed for our transformation purposes. In Section 3 we discuss scenarios based on which we formally present transformations from one logic to another. In Section 4 we discuss possibilities for regional development concerning elderly care management. Section 5 concludes the paper.

2 General logics

Notations in this section are drawn from [8, 7].

An *entailment system* is a triple $\mathscr{E} = (\mathtt{Sign}, Sen, \vdash)$ where

- $\mathtt{Sign}$ is a category of signatures,

- Sen is a functor $Sen : \mathtt{Sign} \to \mathtt{Set}$, and

- $\vdash$ is a family of binary relations consisting of
$$\vdash_\Sigma \; \subseteq \mathcal{P}Sen(\Sigma) \times Sen(\Sigma)$$
  for each signature $\Sigma \in \mathrm{Ob}(\mathtt{Sign})$ where $\vdash_\Sigma$ is called a Σ-*entailment*

subject to the condition that each $\vdash_\Sigma$

- is reflexive, that is, $\{\varphi\} \vdash_\Sigma \varphi$ for each $\varphi \in Sen(\Sigma)$;

- is monotone, that is, $\Gamma' \vdash_\Sigma \varphi$, for all $\Gamma' \subseteq \Gamma$ such that $\Gamma \vdash_\Sigma \varphi$;

- is transitive, that is, given sentences φ_i, $i \in I$, such that $\Gamma \vdash_\Sigma \varphi_i$ and, additionally, $\Gamma \cup \{\varphi_i \mid i \in I\} \vdash_\Sigma \psi$, then $\Gamma \vdash_\Sigma \psi$; and

- is an $\vdash$-*translation*, meaning that, if $\Gamma \vdash_\Sigma \varphi$ then for all $\mu \in \mathrm{Hom}_{\mathtt{Sign}}(\Sigma, \Sigma')$, it is the case that $\mathcal{P}Sen(\mu)(\Gamma) \vdash_{\Sigma'} Sen(\mu)(\varphi)$.

If $\Gamma \vdash_\Sigma \varphi$, then we say that Γ is the set of *axioms* and φ is *derivable from* Γ or, alternatively, that φ is a *logical consequence of* Γ. Rules are presented in theories.

A ($\mathscr{E}$-)*theory* for an entailment system $\mathscr{E} = (\mathtt{Sign}, Sen, \vdash)$ is a pair $T = (\Sigma, \Gamma)$ such that $\Sigma \in \mathrm{Ob}(\mathtt{Sign})$ is a signature, and $\Gamma \subseteq Sen(\Sigma)$ is a set of *axioms*.

Let $\mathscr{E} = (\mathtt{Sign}, Sen, \vdash)$ be an entailment system. Then the category of $\mathscr{E}$-theories, denoted $\mathtt{Th}_\mathscr{E}$, or $\mathtt{Th}$ for short, is such that $\mathrm{Ob}(\mathtt{Th})$ is the set of all $\mathscr{E}$-theories and each *theory morphism* $(\Sigma, \Gamma) \xrightarrow{\mu_{\mathtt{Th}}} (\Sigma', \Gamma')$ with $\Sigma, \Sigma' \in \mathrm{Ob}(\mathtt{Sign})$, $\Gamma \subseteq Sen(\Sigma)$, $\Gamma' \subseteq Sen(\Sigma')$, consists of a signature morphism $\Sigma \xrightarrow{\mu} \Sigma'$ such that
$$\Gamma' \vdash_{\Sigma'} Sen(\mu)(\varphi)$$
holds for each $\varphi \in \Gamma$. If $\mu_{\mathtt{Th}}$ also fulfills the condition that $\mathcal{P}Sen(\mu)(\Gamma) \subseteq \Gamma'$, that is, axioms are mapped to axioms, then it is said to be *axiom-preserving*. We let $\mathtt{Th_0}$ denote the subcategory of $\mathtt{Th}$ containing the same objects but only axiom-preserving morphisms.

Mappings between entailment systems can now be introduced. Let $\mathcal{S} : \mathtt{Th} \to \mathtt{Sign}$ and $\mathcal{A} : \mathtt{Th} \to \mathtt{Set}$ be the forgetful functors taking a theory to its underlying signature and set of axioms, respectively. That is, for some theory, $T = (\Sigma, \Gamma) \in \mathrm{Ob}(\mathtt{Th})$, we have $\mathcal{S}(T) = \Sigma$ and $\mathcal{A}(T) = \Gamma$. Further, let $\mathcal{T} : \mathtt{Sign} \to \mathtt{Th}$ be the functor which for some signature Σ is such that $\mathcal{T}(\Sigma) = (\Sigma, \varnothing)$, i.e. given a signature, $\mathcal{T}$ yields the theory containing that signature but holds no axioms.

Let $\mathscr{E} = (\mathtt{Sign}, Sen, \vdash)$ and $\mathscr{E}' = (\mathtt{Sign}', Sen', \vdash')$ be two entailment systems. Then a map from $\mathscr{E}$ to $\mathscr{E}'$ is a pair $(\Phi, \alpha) : \mathscr{E} \to \mathscr{E}'$ where
$$\mathtt{Th_0} \xrightarrow{\Phi} \mathtt{Th_0'}$$
is a functor from the category of $\mathscr{E}$-theories to the category of $\mathscr{E}'$-theories, and
$$Sen \xrightarrow{\alpha} Sen' \circ \mathcal{S}' \circ \Phi \circ \mathcal{T}$$
is a natural transformation such that

(i) Φ maps theory signatures with no regard to axioms, that is, $\mathcal{S}'\Phi = \mathcal{S}'\Phi\mathcal{T}\mathcal{S}$;

(ii) the theories $\Phi(\Sigma, \Gamma)$ and $\Phi(\Sigma, \varnothing)$ are such that
$$(\mathcal{A}\Phi(\Sigma, \Gamma))^\bullet = (\mathcal{A}'\Phi(\Sigma, \varnothing) \cup \mathcal{P}\alpha_\Sigma(\Gamma))^\bullet$$

(iii) for each $\Sigma \in \text{Ob}(\texttt{Sign})$,

$$\Gamma \vdash_\Sigma \varphi \implies$$

$$\mathcal{P}\alpha_\Sigma(\Gamma) \cup \mathcal{A}'\Phi(\Sigma, \varnothing) \vdash'_{\mathcal{S}'\Phi(\Sigma,\varnothing)} \alpha_\Sigma(\varphi).$$

We say that the map (Φ, α) is *conservative* if instead of (iii), the stronger condition

$$\Gamma \vdash_\Sigma \varphi \iff$$

$$\mathcal{P}\alpha_\Sigma(\Gamma) \cup \mathcal{A}\Phi(\Sigma, \varnothing) \vdash'_{\mathcal{S}\Phi(\Sigma,\varnothing)} \alpha_\Sigma(\varphi)$$

holds.

We may in an intuitive sense split an entailment system into two separate entailment systems by choosing appropriate signature morphisms. This split will also give rise to a map between the two newly constructed entailment systems. The following proposition makes this claim precise.

Proposition 1. [3] Let $\mathscr{E} = (\texttt{Sign}, sen, \vdash)$ be an entailment system. Additionally, let $\mathscr{E}_1 = (\texttt{Sign}_1, Sen_1, \vdash^1)$ and $\mathscr{E}_2 = (\texttt{Sign}_2, Sen_2, \vdash^2)$ be such that $\texttt{Sign}_1$ and $\texttt{Sign}_2$ are subcategories of $\texttt{Sign}$ and Sen_1, Sen_2, $\vdash^1$, and $\vdash^2$ are the restrictions of sen and $\vdash$ to the signatures of their respective signature categories. Further, place on $\texttt{Sign}_1$ and $\texttt{Sign}_2$ the restriction that they may contain only identity morphisms and, finally, for each $\Sigma_1 \in \text{Ob}(\texttt{Sign}_1)$, choose a signature morphism $\mu \in \text{Hom}_{\texttt{Sign}}(\Sigma_1, \Sigma_2)$, $\Sigma_2 \in \text{Ob}(\texttt{Sign}_2)$.

Under these conditions $\mathscr{E}_1$ and $\mathscr{E}_2$ are entailment systems and $(\Phi, \alpha) : \mathscr{E}_1 \to \mathscr{E}_2$ as given below is a map between them. The functor $\Phi : \texttt{Th}_0^1 \to \texttt{Th}_0^2$ takes $\mathscr{E}_1$-theories to $\mathscr{E}_2$-theories and is defined for each $\mathscr{E}_1$-theory (Σ, Γ) by

$$\Phi(\Sigma, \Gamma) = (\mu(\Sigma), \mathcal{P}Sen(\mu)(\Gamma))$$
$$\Phi(\text{id}_{(\Sigma,\Gamma)}) = id_{\Phi(\Sigma,\Gamma)}$$

and $\alpha : Sen_1 \to Sen_2 \circ \mathcal{S}_2 \circ \Phi \circ \mathcal{T}_1$ is a natural transformation similarly defined

$$\alpha_\Sigma = Sen(\mu).$$

3 Transformations between classifications

Disease classification is a diagnosis process which typically involve the problem of differential diagnosis, which in the case of dementia could be to distinguish between dementias of Alzheimer's and Lewy body type. The effect of harmful symptoms for these dementia types, especially if diagnosis at an early clinical stage of reduced cognitive ability, can be reduced by pharmacologic treatment, thus prolonging the time during which the patient has better abilities in their daily activities. This in turn means reducing the time during which the patient, due to severely reduced abilities, requires the societally expensive services of nursing homes or even hospital wards.

In the scenario of early detection it is important to observe the situations where cognitive problems are encountered and by whom these observations are made. Clearly, the very first observations of cognitive decline are made by relatives (if not self-detected by the patient) who would seek advice firstly from nurses and primary care doctors within their local health care centres. Representatives in social and nursing areas obviously will not perform formal diagnosis but a good qualified guess can be key to further steps involving accurate diagnosis with possible pharmacologic interventions. In this scenario it is then critical to investigate respective information types and rule representations for respective professional group providing qualified guesses for further referrals and final differential diagnosis of the dementia. Note that a primary care doctor also provides such a qualified guess, or suspicion, where a neurological investigation will provide sufficient information for providing a final diagnosis[1].

Many-valuedness of truth provides another scenario for logic transformations since it is common that information viewed is considered not accurate enough or knowledge entailed is understood as just vaguely true. De-

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[1]It is important to note that not even autopsy will provide higher diagnostic accuracy than just above 80%

spite this many-valuedness clinicians still have to reach conclusions and perform decision making before proceeding with further intervention steps. Information loss must be minimized so that the remaining two-valued situation still is the best and most accurate representation of the vague situation.

Important in both these scenario is to guarantee consistency when information and knowledge is mapped between ontological domains as understood and used by these professional groups. In what follows, we will provide a more formal description related to these scenarios. For a more detailed treatment, see [3].

Let $\mathscr{E} = (\mathtt{Sign}, sen, \vdash)$ denote an entailment systems, where $\mathrm{Ob}(\mathtt{Sign}) = \{(S, \Omega)\}$. The entailment system in our first example may intuitively be seen as an equational logic basically covering boolean terms. Thus we use $S = \{\mathtt{BOOL}\}$ and $\Sigma = \{\mathtt{false}, \mathtt{true}, \mathtt{OR}\}$ with $\mathtt{false}, \mathtt{true} :\to \mathtt{BOOL}$ and $\mathtt{OR} : \mathtt{BOOL} \times \mathtt{BOOL} \to \mathtt{BOOL}$.

The Sen^{Eq} functor for equational logic is given by

$$Sen^{Eq}(\Sigma) =$$

$$\{(X, t, t') \mid t, t' \in T_{\Sigma, s}X, s \in S,$$

and X a family of variables for $\Sigma\}$

for any signature $\Sigma = (S, \Omega) \in \mathrm{Ob}(\mathtt{Sign}^{Eq})$, where

$$Sen^{Eq}(\mu)((X, t, t')) = (\mu(X), \mu(t), \mu(t'))$$

for some signature morphism $\mu : \Sigma \to \Sigma'$.

Delirium, as distinct from dementia and mild cognitive impairment (MCI), can be used as first example to illuminate the utility of logic transformations.

The equations for $\mathtt{OR}$ are as usual. Part of the theory $T_1 = (\Sigma_1, \Gamma_1)$, where $\Sigma_1 = (S_1, \Omega_1)$ The DSM-IV guideline [2] for delirium is the following

```
op false true : -> BOOL .
op OR : BOOL BOOL -> BOOL .
var ... X_episodic : BOOL
var X_semantic : BOOL
var X_shortterm ... : BOOL .
```

```
eqn OR ... .
eqn CogDisDSMdelirium =
  ...
  OR
    {
     X_episodic
     X_semantic
     X_shortterm
    }
... .
eqn X_episodic = false .
eqn X_semantic = true .
eqn X_shortterm = true .
```

Clearly,

```
X_episodic = false .
X_semantic = true .
X_shortterm = true .
```

is specific (electronic) patient data, and

```
eqn OR ... .
eqn CogDisDSMdelirium =
  ...
  OR
    {
     X_episodic
     X_semantic
     X_shortterm
    }
... .
```

is the set of axioms for the theory. Note that these set of rules can be used in home care in order to produce qualified guesses for further informal referrals as well as within more formal clinical classification task when producing differential diagnosis involving questions about delirium.

In a more elaborate entailment system, also equationally based, we will see more sorts and operators. We may include the sort $\mathtt{qualICF}$ to enable representation of qualification values according to the classification of function [10]. We now use $S = \{\mathtt{BOOL}, \mathtt{qualICF}\}$ and $\Omega = \{\mathtt{false}, \mathtt{true}, \mathtt{OR}, 0, 1, 2, 3, 4, \mathtt{a}\}$ with $\mathtt{false}, \mathtt{true} :\to \mathtt{BOOL}$ and $\mathtt{OR} : \mathtt{BOOL} \times \mathtt{BOOL} \to \mathtt{BOOL}$, with $0, 1, 2, 3, 4 :\to \mathtt{qualICF}$ and $\mathtt{a} : \mathtt{qualICF} \to \mathtt{BOOL}$. The operator $\mathtt{a}$ obviously converts many-valued qualifications to

boolean values. Theory $T = (\Sigma, \Gamma)$, where $\Sigma_2 = (S_2, \Omega_2)$, comes to include the following.

```
op false true : -> BOOL .
op OR : BOOL BOOL -> BOOL .
op 0 1 2 3 4 : qualICF .
op a : qualICF -> BOOL .
var ... X_b14411 : BOOL .
var X_b14412 : BOOL .
var X_b1440 ... : BOOL .
eqn OR ... .
eqn a 0 = false .
eqn a 1 = true .
eqn a 2 = true .
eqn a 3 = true .
eqn a 4 = true .
eqn CogDisDSMdelirium =
  ...
  OR
    {
     *** ICF codes:
     *** longterm episodic memory
     a X_b14411
     *** longterm semantic memory
     a X_b14412
     *** shortterm memory
     a X_b1440
    }
  ...
eqn X_b14411 = 0 .
eqn X_b14412 = 1 .
eqn X_b1440 = 3 .
```

Note that in this entailment system $\texttt{a 0} = \texttt{false} \in sen((S, \Omega))$.

In a mapping between the entirely boolean entailment system to the many-valued based system, the signature morphism μ is given by $\mu(\texttt{BOOL}) = \texttt{BOOL}$, and $\mu(\texttt{false}) = \texttt{false}, \mu(\texttt{true}) = \texttt{true}, \mu(\texttt{OR}) = \texttt{OR}$. The entailment morphism is trivial and basically embeds knowledge without changing granularity.

The embedding from the many-valued system to the boolean system, however, is non-trivial. In this case we need an 'identity' $\texttt{i} : \texttt{BOOL} \rightarrow \texttt{BOOL}$ as an operator in Ω_1, i.e. having

```
eqn i false = false .
```

```
eqn i true = true .
```

as its equations in Γ of the boolean based system.

Let now the signature morphism μ be given by $\mu(\texttt{BOOL}) = \texttt{BOOL}$ and $\mu(\texttt{qualICF}) = \texttt{BOOL}$, with $\mu(\texttt{false}) = \texttt{false}, \mu(\texttt{true}) = \texttt{true}, \mu(\texttt{OR}) = \texttt{OR}, \mu(0) = \texttt{false}, \mu(1) = \texttt{true}, \mu(2) = \texttt{true}, \mu(3) = \texttt{true}, \mu(4) = \texttt{true}$ and $\mu(\texttt{a}) = \texttt{i}$.

We then see how

$$sen(\mu)(\texttt{X\_b14411} = \texttt{0}) =$$
$$(\texttt{X\_episodic} = \texttt{false})$$
$$sen(\mu)(\texttt{aX\_b14411} = \texttt{false}) =$$
$$(\texttt{X\_episodic} = \texttt{false})$$

which gives some important indications on converters needed within a many-valued logic framework.

Note that diagnosis encoding is non-trivial as dementia can be encoded (at least) by DSM-IV ([2]) and ICD-10 ([11]). This is, howver, outside the scope of this paper. See [3] for considerations on diagnosis encoding.

4 Elderly care management from a regional development point of view

In this paper the region in focus is Southwest of Finland, with its 5 regions for its 56 municipalities, and a population of 450 000 residents. The Hospital District of Southwest Finland, e.g. with the University Hospital of Turku, is a federation covering 56 municipalities. Within the hospital district, there are 26 municipal health centres. The methodologies in this paper concerns especially region Salo, consisting of 10 municipalities with a population of 55 000 residents. Salo is now merging into one municipality starting from 2009. The structure and organization of residential and nursing/dementia homes in the region, and the coherence of respective care and nursing processes within these homes, provides excellent foundations for various developments within elderly care in the region.

Decision-making in so called *Investigate-Assess-Place* groups covers a wide range of professional skills, and aims at identifying optimal care and forms of living for elderly in their various stages of decline in cognition and functioning abilities. The information available must be understood in the same way by all relevant professions and professional groups, and decisions must be based on commonalities with respect some form of logical structures for rules underlying all decision-making.

5 Conclusions

The necessity to manage granularity of knowledge becomes evident when aiming at correctness preservations in transformations between decision support representations at different care and health levels. Future work will continue to focus of diversity in decision making and in the end also including optimality requirements for placement of patients within best forms of living. This will require very accurate assessment methodologies that are accepted as well-founded at health levels of municipal and health care decision making.

Acknowledgements

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