How to improve the overall industrial performance in a multi-criteria context

Jacky Montmain, Sofiane Sahraoui
EMA- LGI2P-Site EERIE
Parc scientifique Georges Besse, 30035,
Nîmes, France
fisrtname.lastname@ema.fr

Abstract

This paper deals with the implementation of monitoring and control strategies to improve industrial performances. Industrial performances are nowadays defined in terms of numerous and multi-level criteria to be synthesized for overall improvement purposes In our approach, the overall performance of the company is the aggregated value of its partial performances related to each criterion. The aggregation model is performed with a Choquet integral. Two improvement strategies in this multi-criteria context are then envisaged. The contribution of a criterion to the Choquet aggregated performance has a key role in both strategies that are thus compared and justified through the monitoring of criteria contributions. A case study illustrates the propositions of the paper.

Keywords: industrial performance, performance improvement, contribution of a criterion, efficiency, multi-criteria decision-making.

1 Problem statement

To deal with the complexity of the current industrial context, new diagnosis and control strategies intended to bring about continuous improvement have to include both the multi-criteria performance expression aspects and the modeling of their relationships [1][2][3]. The so-called Performance Measurement Systems (PMS's), which are instruments to support decision-making [4], fulfill that purpose. A PMS is made of a set of performance expressions to be consistently organized w.r.t. the objectives of the company. Besides, aggregation models allow to define overall performances w.r.t. the different elementary objectives of the company [1]. They enable to highlight the priorities in the decision-maker's strategy. The aggregated performance model captures the company's strategy: in our approach, the aggregation model is performed with a Choquet integral that enables both tackling with relative importance of criteria and interactions among them [5].

Definition and design attempts for overall performance have already been considered in [1][4]. This work is focused on decision-support tools that could help managers to better plan performances improvements w.r.t. to the company strategy to reach a goal while minimizing costs. We have already proposed optimization techniques to fit this scheme and characterize an efficient improvement [5]. The paper focuses here on a related issue: how to define a relevant step by step procedure to reach the previously computed goal. Two different logics are semantically analyzed and justified. One is based upon statistical considerations w.r.t. the most likely profitable contributions of criteria to the overall performance. The idea is to determine the criteria on which the company should improve first statistically in order to improve as much as possible its overall performance. This study was initially proposed by Labreuche in [7] when no quantitative goal is targeted. The second one is related to the concept of efficient improvement when a quantitative goal is set [8][9]. The principle is to define a step by step locally efficient improvement in a multi-criteria context until the expected goal is reached. The corresponding iterative procedure is provided in both cases (algorithms A_1 and A_2). In both cases, the contribution of a criterion to the overall performance improvement plays a key role. As a consequence, a monitoring functionality of criteria' contributions in time is provided for a more quantitative and theoretical comparison of both improvement logics. The corresponding algorithm is provided.

This paper is organized as follows. Section II briefly recalls the characteristics of the industrial performance expressions. The Choquet integral is proposed as a solution for handling the interacting multi-criteria aspects of industrial performance. The aggregation viewpoint compels us to redefine what efficiency means. Performance improvement problems are modeled as optimization problems. Then, section III qualitatively analyzes two semantically different logics based upon criteria contributions to perform an efficient improvement. Section IV proposes a more quantitative and theoretical comparison. Finally, a case study illustrates all these notions.

2 The aggregative model of overall performance

2.1 The PMS notations

A performance expression is associated with a given objective and can be defined as a satisfaction degree.

L. Magdalena, M. Ojeda-Aciego, J.L. Verdegay (eds): Proceedings of IPMU'08, pp. 630–637
Torremolinos (Málaga), June 22–27, 2008

In practice, elementary performances are returned by the so-called performance indicators. They result from the straightforward comparison between the objectives (obtained by the breakdown of the overall considered objective) and the reached measurements. Hence, the performance expressions can be formalized by the following mapping [10]:

$$\mathrm{P}: O \times M \rightarrow E$$
$$(o,m) \rightarrow \mathrm{P}(o,m) = P$$

O, *M* and *E* are respectively the universes of discourse of the set of objectives *o*, the set of measures *m* and the normalized performance $P \in [0,1]$.

Let us note *C* the set of the *n* criteria implied by the PMS. The aggregation of the performances can be expressed as an operation that synthesizes the elementary performances into an overall expression:

$$Ag : [0,1]^n \rightarrow E \qquad \textbf{(1)}$$
$$(P_1, P_2, ..., P_n) \rightarrow P_{overall} = Ag(P_1, P_2, ..., P_n)$$

In our approach, the aggregative model Ag for the PMS is a Choquet integral. It enables tackling both with relative importance of criteria and interactions among them. This choice is not discussed here (see [5][8] for further justifications). Let us simply note that another viewpoint is proposed in [3] where a goal is directly modeled as a fuzzy set upon the measure scale. That is another way to tackle the commensurability issue. Furthermore, interactions considered in our approach are not the ones tackled in [3] where action plans may impact conjointly several performances indicators. Our approach focuses only on the preference decision-maker model at the performance level: the corresponding action plans have then to be designed. Thus, eventual interactions between improvement actions are not considered here as is the case in [3].

P^k is the aggregated performance of the partial performances profile $\vec{P}^k = (P_1^k, ..., P_n^k)$.

$$P^k = C_\mu(\vec{P}^k) = C_\mu(P_1^k, P_2^k, ..., P_n^k) = \sum_{i=1}^{n} (P_{(i)}^k - P_{(i-1)}^k).\mu(A_{(i)}^k) \qquad \textbf{(2)}$$

where $\mu : P(C) \rightarrow [0,1]$ is a fuzzy measure, (.) indicates a permutation such that the partial performances $P_{(.)}^k \in [0,1]$ are ranked $0 \le P_{(1)}^k \le \ldots \le P_{(n)}^k \le 1$ and $A_{(i)}^k = \{c_{(i)}, .., c_{(n)}\}$. It can be rewritten:

$$C_\mu(P_1^k, .., P_n^k) = \sum_{i=1}^{n} (P_{(i)}^k - P_{(i-1)}^k).\mu(A_{(i)}^k) = \sum_{i=1}^{n} \Delta\mu_{(i)}^k.P_{(i)}^k \qquad \textbf{(3)}$$

where $\mu_{(i)}^k = \mu(A_{(i)}^k)$, $\mu_{(n+1)}^k = 0$ and $\Delta\mu_{(i)}^k = \mu_{(i)}^k - \mu_{(i+1)}^k$.

Note that a simplex $H_{(.)} = \{P \in [0,1]^n / 0 \le P_{(1)} \le \ldots \le P_{(n)} \le 1\}$ is associated to each profile $\vec{P}^k$: it corresponds to the ranking (.) where the Choquet integral has a linear expression. This remark is of importance to tackle optimization problems implying a Choquet integral in terms of linear programming in simplex regions.

2.2 Efficiency in the aggregated framework

The problem is to help the decision-makers in their improvement analysis by considering the way the overall performance could be relevantly improved. The problem is to design a strategy that leads to the required overall performance improvement with a minimal increase w.r.t. each elementary performance, *i.e.* a minimal additional cost related to each of them. The answers to this question are intuitive when the aggregation operator is linear, e.g. with the weighted average mean (*WAM*). It is thornier with a Choquet integral.

This notion of optimal improvement is directly related to the concept of efficient improvement. Indeed, the notion of efficiency both implies the objective to be reached and the allocation of resources associated to an improvement: an improvement is efficient if any restrictive modification of its allocated resources necessarily entails a decrease of the overall performance. In our aggregated framework of PMS, the efficient improvement search can thus be formalized as the following optimization problem.

Let us note $\vec{P}^I = (P_1^I, P_2^I, ..., P_n^I)$ the initial performance profile and $P^I = C_\mu(\vec{P}^I)$ the associated overall performance. The problem to be solved is to identify the most efficient strategy to improve the overall performance, *i.e.* the least costly improvement of the elementary performances to achieve an expected overall performance $P^* > P^I$. What is the minimal investment w.r.t. each criterion to reach P^*? Let us note $\vec{\delta}^* = (\delta_1^*, \delta_2^*, ..., \delta_n^*)$ the solution to this problem. $\vec{\delta}^*$ is thus associated to the most efficient strategy w.r.t. a given C_μ model and a predefined set of cost functions associated to each criterion of the PMS. A cost function $c_i(P_i^I, \delta_i)$ is associated to each criterion *i*: $c_i(P_i^I, \delta_i)$ represents the cost for an improvement of δ_i from P_i^I. For sake of simplicity, $c_i(P_i^I, \delta_i)$ is supposed to be a linear function w.r.t. δ_i: $c_i(P_i^I, \delta_i) = cu_i.\delta_i$ with cu_i a unit cost. The problem (**P₁**) of the most efficient improvement can then be formulated as follows:

Objective function

$$\min c(\vec{P}, \vec{\delta}) = \sum_{i=1}^{n} c_i(P_i^I, \delta_i) \qquad \textbf{(4)}$$

Constraints

$$C_\mu(\vec{P} + \vec{\delta}) = P^* \quad (\textit{Behavioral constraint})$$
$$\forall i, \quad 0 \le \delta_i \le 1 - P_i^I \quad (\textit{Bound constraints})$$

The piecewise linearity of C_μ enables to tackle ($\mathbf{P_1}$) as a linear programming problem. Indeed, C_μ behaves as a *WAM* on each simplex $H_{(.)} = \left\{ \vec{P} \in [0,1]^n / 0 \le P_{(1)} \le \cdots \le P_{(n)} \le 1 \right\}$. This remark enables to break down the initial problem into $n!$ linear programming sub problems. Nevertheless, this solving can be considered only for low *n* values [5].

Another idea consists in considering the problem as a whole and introducing linear programming considerations [8][9]. To that end, let us first notice that guaranteeing a potential solution belongs to a given $H_{(.)}$ implies adding $(n-1)$ constraints in the problem definition: $\forall i, P^I_{(i)} + \delta_{(i)} \le P^I_{(i+1)} + \delta_{(i+1)}$. Next, noticing that realizable solutions related to a linear programming problem belong to a convex hull, the associated vertices $\vec{x}$ have a particular profile due to the 3 types of inequalities involved in the problem modeling:

(a) $\forall i,\ \ 0 \le \delta_{(i)}$

(b) $\forall i,\ \ \delta_{(i)} \le 1 - P_{(i)}$ **(5)**

(c) $\forall i, P_{(i)} + \delta_{(i)} \le P_{(i+1)} + \delta_{(i+1)}$.

A vertex $\vec{x}$ is thus defined by *n* equations: $(n-1)$ of the preceding constraints brought to equality conjointly with $C_\mu(\vec{P}^I + \vec{\delta}) = \sum_{i=1}^{n} \Delta\mu_{(i)}.(\vec{P}^I + \vec{\delta})_{(i)} = P*$ where the $\Delta\mu_{(i)}$'s are the coefficients of the linear expression of C_μ in simplex $H_{(.)}$ defined by inequalities of type (c). The set of constraints is generated for any simplex $H_{(.)}$ and all the vertices are computed. The minimal distance between $\vec{P}^I$ and a vertex gives the solution to the global problem.

Now, let us remark that, after some rearrangement, a vertex $\vec{x}$ is a vector with 3 distinct blocks of coordinates:

- (a) unchanged coordinates w.r.t. the initial vector $\vec{P}$ ($\delta_{(i)} = 0 \Rightarrow x_{(i)} = P_{(i)}$),
- (b) coordinates equal to 1 ($\delta_{(i)} = 1 - P_{(i)} \Rightarrow x_{(i)} = 1$),
- (c) a subset of coordinates at the same value β ($P_{(i)} + \delta_{(i)} = P_{(j)} + \delta_{(j)} \Rightarrow x_{(i)} = x_{(j)}$).

Linear programming results involve that $\vec{P}^I + \vec{\delta}^*$ can only take remarkable values as coordinates. Indeed, after some rearrangement, it means that $\vec{P} + \vec{\delta}^*$ can always be rewritten under the following form denoted **F** [7]: $[1,...,1,\beta,...,\beta,P_{(i)},...,P_{(j)}]^T$ **(6)**

This is a relevant piece of information for decision-making that generalizes the obvious result that is obtained with a *WAM*, e.g. $[1,...,1,\beta,P^I_{(i)},...,P^I_{(j)}]^T$. More details are provided in [9].

Let us still note that problem (**P$_1$**) can be easily extended in (**P'$_1$**) when more severe bound constraints related to the application are to be introduced. Constraints $\forall i,\ 0 \le \delta_i \le 1 - P^I_i$ are replaced by $\forall i,\ 0 \le \delta^l_i \le \delta_i \le 1 - \delta^r_i - P^I_i$ where δ^l_i and δ^r_i are threshold parameters issued from the application (e.g. improvement w.r.t. criterion *i* cannot exceed 30% but must be over 10%).

2.3 The improvement control problematic

From this viewpoint, an efficient improvement to reach the overall performance level P^* merely depends on an initial performances profile $\vec{P}^I$, a set of *n* linear cost functions and an aggregation operator C_μ. It means that efficiency only depends on a static viewpoint of improvement: the optimization profile (**P$_1$**) merely determines the setpoint $\vec{P}* = [P^*_1..P^*_n] / C_\mu(P^*_1,..,P^*_n) = P*$ to be reached. No dynamical aspects of improvement are considered in this modeling. The problem to be solved now is to determine a step by step evolution in time for $\vec{P}$ from $\vec{P}^I$ to $\vec{P}*$. $\vec{P}*$ is the setpoint of this control problem, $\vec{P}$ the controlled variable.

The basic idea is to define some remarkable points $\vec{P}^k$ of the trajectory from $\vec{P}^I$ to $\vec{P}*$ to plan a step by step expected evolution of $\vec{P}^k$. Let us note that our viewpoint is dedicated to the managers' team because our decision-making support system only relies on the PMS perception of the company's health. It does not consider further operational or physical constraints related to the implementation of the improvement. Indeed, the aggregation model only captures expectations, preferences or wills of the company's managers. Interactions thus express expected negative or positive synergies between criteria, but they are not to be confused with statistical correlations between parameters of the physical and operational world [3]. Providing the series of intermediate points $\vec{P}^k$ from $\vec{P}^I$ to $\vec{P}*$ enables to define the guidelines that the managers *would like* the company improvements follow. This is a purely managerial viewpoint. The contribution of a criterion to the global improvement has then a key role in the following of this paper.

3 Improvement control logics

3.1 A statistical viewpoint: the worth index

This first viewpoint is inspired of the work of Labreuche in [7]. In that paper, the author proposes an index of importance to determine the criteria on which a candidate should improve first in order to improve as much as possible his overall score. His results are transposed here in the case of industrial performance improvement. Let us note that no quantitative objective $P*$ is provided in Labreuche's issue, the qualitative aim is only to do one's best.

Let us first briefly recall the notion of worth index like it is developed in [7]. This work was initiated in

[11] when no initial profile $\vec{P}^I$ is specified. Labreuche's aim is to come up with advices on the criteria on which the company performance should be improved first from a statistical viewpoint. This identification depends on the aggregation model H as well as on the partial performances $\vec{P}^I = (P_1^I, P_2^I, ..., P_n^I)$. To give some advices on the criteria that should be improved, Labreuche proposes to introduce an index denoted by $\omega_A^C(H)(\vec{P}^I)$ for the aggregation function H and the initial profile $\vec{P}^I$. For any $A \subset C$, $\omega_A^C(H)(\vec{P}^I)$ will be the worth for the profile $\vec{P}^I$ to be improved in criteria among A, subject to the evaluation function H. He constructs $\omega_A^C(H)(\vec{P}^I)$ that will be large if improving $\vec{P}^I$ w.r.t. criteria A yields a large improvement in the overall evaluation $H(\vec{P})$. The recommended set of criteria to be improved first is the coalition $A^* \subset C$ that maximizes $\omega_A^C(H)(\vec{P}^I)$. More accurately, improving the criteria in A^* maximizes the odds the overall performance reaches the highest level as possible. Labreuche proposes an axiomatic construction of worth index $\omega_A^C(H)(\vec{P}^I)$. In the following the only case $H = C_\mu$ is considered.

Let us consider two profiles $\vec{P}^I, \vec{P}'$ and $A \subset C$. The following notations are introduced: $\left[P'_A, P^I_{C\setminus A}\right]$ is the compound profile whose partial performances are such that $P_i' > P_i^I$ if $i \in A$ else P_i^I. For any subset of criteria $A \subset C$, P_A is the restriction of $\vec{P}$ on A.

A possible formula for the worth index is:

$$\omega_A^c(C_\mu)(\vec{P}^I) = \frac{1}{\prod_{i\in A}(1-P_i^I)} \int_{P_A'\in[P_A^I,1_A]} [C_\mu(P_A', P_{C\setminus A}^I) - C_\mu(\vec{P}^I)].dP_A' \qquad \textbf{(7)}$$

$\omega_A^c(C_\mu)(\vec{P}^I)$ is thus the mean value of gain related to the aggregated performance calculated over all the expected values the improvement can take $P_A' \in [P_A^I, 1_A]$ in the upper hypercube (Figure 1). Let us note that the upper bound of the integral is 1_A (no quantitative objective P^* is specified).

When cost functions w.r.t. partial improvements are introduced, a natural extension is:

$$\omega_A^c(C_\mu)(\vec{P}^I) = \frac{1}{\prod_{i\in A}(1-P_i^I)} \int_{P_A'\in[P_A^I,1_A]} \frac{[C_\mu(P_A', P_{C\setminus A}^I) - C_\mu(\vec{P}^I)].dP_A'}{c(P_A^I, P_A' - P_A^I)} \qquad \textbf{(8)}$$

The benefit $C_\mu(P_A', P_{C\setminus A}^I) - C_\mu(\vec{P}^I)$ is replaced by the benefit to cost ratio $\dfrac{C_\mu(P_A', P_{C\setminus A}^I) - C_\mu(\vec{P}^I)}{c(P_A^I, P_A' - P_A^I)}$.

In this framework, coalition $A^* / \max_A \omega_A^c(C_\mu)(\vec{P}^I)$ results from a statistical interpretation: indeed, $\omega_{A^*}^C(H)(\vec{P}^I)$ provides the criteria that maximize the expectancy of $C_\mu(P_A', P_{C\setminus A}^I) - C_\mu(\vec{P}^I) / c(P_A^I, P_A' - P_A^I)$ appearing as a variate. No accurate value can be provided for the overall improvement with $\omega_{A^*}^C(H)(\vec{P}^I)$, it only warranties that criteria in A^* maximize the odds to reach a high overall performance.

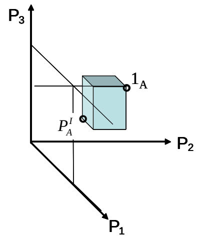

Figure 1: The upper hypercube P_A^I — 1_A
lower bound P_A^I - upper bound 1_A

This statistics based advice is natural for the MCDM or theory games community, but somewhat disturbing in the industrial systems engineering community. We are going to see how Labreuche's model can be slightly modified and integrated in a step by step improvement procedure when a quantitative goal P^* is assigned.

Thus, we authorize an upper bound P_C^{Sup} not necessarily equal to 1_C (as a consequence the upper bound in **(8)** will be P_A^{Sup} and not 1_A). We will note it:

$$\omega_A^c(C_\mu)(\vec{P}^I, P_C^{Sup}) \text{ and } \omega_A^c(C_\mu)(\vec{P}^I, 1_C) = \omega_A^c(C_\mu)(\vec{P}^I) \qquad \textbf{(9)}$$

Now let us back to the control problem of $\vec{P}$ from $\vec{P}^I$ to $\vec{P}^*$. $\omega_A^c(C_\mu)(\vec{P}^I, P_C^{Sup})$ can be used for strategic control purposes. Let us take $P_C^{Sup} = \vec{P}^*$ where $\vec{P}^*$ is solution of **(P$_1$)**. The aim is to plan a step by step expected evolution in time for $\vec{P}^k$ from $\vec{P}^I$ to $\vec{P}^*$.

The algorithm (**A$_1$**) is then the following:

Given $\vec{P}^I, C_\mu, \forall i, cu_i$
$k = 0, \vec{P}^0 := \vec{P}^I$
While $C_\mu(\vec{P}^k) < C_\mu(\vec{P}^*)$
Compute
$A^* / \omega_{A^*}^c(C_\mu)(\vec{P}^k, \vec{P}^*) = \max_{A\subset C} \omega_A^c(C_\mu)(\vec{P}^k, \vec{P}^*)$
Improve partial performances w.r.t. criteria in A^* till time $k+1$
Check improved performances be kept in upper hypercube P_A^k —— P_A^*
Note $\vec{P}^{k+1}$ the new attained performance profile at $k+1$
Evaluate overall performance $C_\mu(\vec{P}^{k+1})$
$k = k + 1$
End while
The series of points $\vec{P}^k$ defines the required trajectory

Note that this step by step procedure is possible because $\vec{P}^*$ remains a solution of (**P$_1$**) with any $\vec{P}^k$ as initial profile in the upper hypercube $\vec{P}^I - \vec{P}^*$ (L_1 norm is used in (**P$_1$**) and no regression is authorized w.r.t. any criterion). (**A$_1$**) warranties that from any $\vec{P}^k$ to $\vec{P}^{k+1}$, the improved criteria correspond to the ones that maximize the odds to reach a high level of performance at time $k+1$. This is an optimum in the sense of statistics: on an average, the criteria of A^* warranty the maximal expectancy for overall improvement at $k+1$. It does not mean that each intermediary $\vec{P}^k$ corresponds to an efficient improvement! Constraining the improvement to be efficient at each intermediary $\vec{P}^k$ corresponds to another viewpoint that is now explained.

3.2 The local efficiency viewpoint

Let us consider again the control problem of $\vec{P}$ from $\vec{P}^I$ to $\vec{P}^*$. We are no more interested in maximizing the odds to reach a high performance at each step *k*, as the worth index method suggests it, the strategic decisive factor is now warranting efficiency at any intermediary point $\vec{P}^k$.

The basic idea is to "locally" reuse (**P$_1$**) (more exactly (**P'$_1$**)) to define such a series of points $\vec{P}^k$. Indeed, intermediate points $\vec{P}^k$ are needed when P^* appears as an ambitious setpoint that will require a long time before reached. Managers have to define short terms and progressive improvements for which it is easier to plan an adequate implementation. These intermediate points can be envisaged as locally efficient improvement. The guidelines consist in providing short terms objectives that will be reached in an efficient manner and proceed thus, step by step, until $\vec{P}^*$. This is **A$_2$** algorithm:

> Given $\vec{P}^I, C_\mu, \forall i, cu_i$
>
> Provide a real series of overall objectives P^k such that $C_\mu(\vec{P}^I) \le P^k \le P^*$ (for example, if $\vec{P}^*$ is reached in p steps, we may choose $P^k = C_\mu(\vec{P}^I) + k.\dfrac{P^* - C_\mu(\vec{P}^I)}{p}, k = 1,..,p$. This is a mere suggestion; the idea is merely to provide short terms and progressive improvements).
>
> $k = 0, \vec{P}^0 := \vec{P}^I$
>
> While $C_\mu(\vec{P}^k) < C_\mu(\vec{P}^*)$
>
> Solve (**P'$_1$**) with $\vec{P}^k$ as initial point and P^{k+1} as setpoint and bound constraints: $\forall i, \delta_i^l = 0, \delta_i^r = 1 - P_i^*$; it provides $\vec{P}^{k+1}$
>
> $k = k+1$
>
> End while
>
> The series of points $\vec{P}^k$ defines the required trajectory

This strategy warranties a local efficient improvement as soon as $\vec{P}^k$ is reached. The implementation is both locally and globally efficient when $\vec{P}^*$ is finally reached. Local efficiency is another way envisaging the trajectory from $\vec{P}^I$ to $\vec{P}^*$, which obeys a different decisive factor as the one in the worth index method. They correspond to clearly different attitudes w.r.t. the potential risk to fail.

3.3 Semantic comparison

From a semantic viewpoint, this second alternative is close to a control theory modeling, whereas the first one was rather games theory oriented with a statistical semantic. Both can be easily justifiable: the first one ensures that improved criteria correspond to the maximal expectancy for the expected overall gain at any time. The second one relies on a decisional criterion based upon local and global efficiency. Both have advantages—they are justifiable at any time—but suffer some drawbacks. Indeed, the worth index logics will provide the most statistical profitable criteria but nothing can be said about the resulting expected gain. For example, the maximal worth index can be associated to a criterion where the improvement margin is extremely reduced (the company's partial performance w.r.t. this criterion is already excellent and even if perfection (i.e., 1) is reached, the overall performance will not be significantly improved. Furthermore, at each $\vec{P}^k$ the worth index relies on statistical considerations, thus the performances evaluation at time $k+1$ can be discouraging in practice! On the other hand, (**P$_1$**)-based logics warranties efficiency at each $\vec{P}^k$ if and only if $\vec{P}^k$ is precisely reached. Moreover, it necessitates managers to be able to define the series of real values P^k.

4 Criteria contribution and Monitoring

Now let us consider a more quantitative and theoretical comparison of both control logics. The basic notion beyond these two strategies is the notion of criterion contribution to the improvement of the overall performance.

Let us thus consider the following problem: what is the contribution of criterion i in the efficient improvement from $\vec{P}^I$ to $\vec{P}^*$? This contribution cannot be a priori computed with initial data of (**P$_1$**): $\vec{P}^I, C_\mu, \forall i, cu_i$ and P^*. Indeed, criteria contributions depend on the dynamics of the improvement from $\vec{P}^I$ to $\vec{P}^*$. Let us consider the following illustration (Figure 2).

Three possible paths $t_{j=1,2\ et\ 3}$ from $\vec{P}^I$ to $\vec{P}^*$ are represented in the figure 2 example. The global cost C^* is provided by (**P$_1$**): it is the same for all paths t_j from $\vec{P}^I$ to $\vec{P}^*$, whereas the criteria contributions to $C_\mu(\vec{P}^*) - C_\mu(\vec{P}^I)$ depend on the path. The contributions of criterion 1, $C_1^{t_j}$, $j = 1, 2$ *and* 3 are: $C_1^{t_1} = \Delta\mu_1.(P_1^* - P_1^I)$, $C_1^{t_2} = \Delta\mu_1^{'}.(P_1^* - P_1^I)$ and $C_1^{t_3} = \Delta\mu_1.dp + \Delta\mu_1^{'}.(P_1^* - P_1^I - dp)$.

Thus, the a priori contribution of a criterion to $C_\mu(\vec{P}^*) - C_\mu(\vec{P}^I)$ is not a precise quantity. In the following, a method is proposed to compute the lower and upper endpoints of the interval of all possible values for the contribution of any criterion *i*. The aim is to provide the minimal and maximal expected profitability. A criterion necessarily contributes at least up to $C_i^N = \min_t C_i^t$, but it is possible the contribution reaches $C_i^\Pi = \max_t C_i^t$. $[C_i^N, C_i^\Pi]$ characterizes the imprecision of the a priori contribution of criterion *i* to $C_\mu(\vec{P}^*) - C_\mu(\vec{P}^I)$.

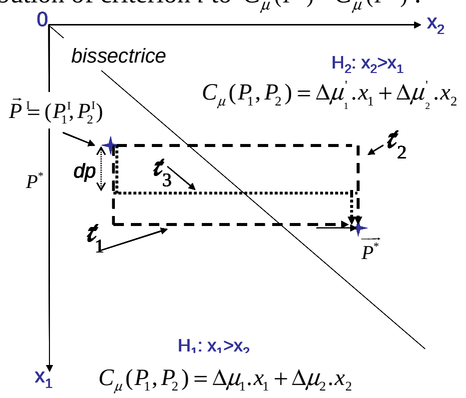

Figure 2. Contributions and trajectories

We give now the principle of the $[C_i^N, C_i^\Pi]$ computation. This is a three steps procedure.

Step 1: Non oriented complete graph Γ is first built; it links all the $n!$ simplexes $H_\sigma = \{\vec{P} \in [0,1]^n / 0 \le P_{\sigma(1)} \le .. \le P_{\sigma(n)} \le 1\}$ together.

Let be H_{σ_I} the simplex $\vec{P}^I$ belongs to and H_{σ_F} the one of $\vec{P}^*$. H_{σ_I} is the source of Γ and H_{σ_F} its sink.

- For each H_σ, it is checked that there exists at least one point $\vec{P}(\sigma)$ such that: $\forall i \in \{1;..;n\}$, $P_i^I \le P_i(\sigma) \le P_i^*$;
- If there does not exist such a point, then H_σ is deleted as all the arcs whose H_σ is an endpoint;
- Finally, when each node H_σ has been checked, a filtered graph Γ_F is obtained.

Step 2: For each node H_σ in Γ_F, compute the range of allowed values for $P_i(\sigma)$ for each criterion *i*: $[B_i^{Inf}(\sigma); B_i^{Sup}(\sigma)]$. The computation is given by the following expressions:

$$B_i^{Inf}(\sigma) = \max_{j \le \sigma^{-1}(i)} P_{\sigma(j)}^I \; ; \; B_i^{Sup}(\sigma) = \min_{j \ge \sigma^{-1}(i)} P_{\sigma(j)}^* \qquad \textbf{(10)}$$

Step 3: Γ_F defines a set of paths $Path_k, k = 1..m$ with H_{σ_I} as source and H_{σ_F} as sink without cycles. For each node H_σ in $Path_k$, we know $[B_i^{Inf}(\sigma); B_i^{Sup}(\sigma)]$ and $\Delta\mu_i(\sigma)$, the linear coefficient of C_μ for criterion *i* in simplex H_σ. Let be $\varepsilon_i^{Path_k}$ the set of series of disjoint intervals $I_i(\sigma)$ such that $I_i(\sigma) \subseteq [B_i^{Inf}(\sigma); B_i^{Sup}(\sigma)]$ and $\bigcup_{\sigma / H_\sigma \in Nodes(Path_k)} I_i(\sigma) = [P_i^I; P_i^*]$. It can be then computed, with $L(I)$ the length of interval I:

$$\min_{\varepsilon_i^{Path_k}} \sum_{\sigma / H_\sigma \in Nodes(Path_k)} \Delta\mu_i(\sigma).L(I_i(\sigma)) \text{ and}$$

$$\max_{\varepsilon_i^{Path_k}} \sum_{\sigma / H_\sigma \in Nodes(Path_k)} \Delta\mu_i(\sigma).L(I_i(\sigma))$$

Finally:

$$C_i^N = \min_{Path_k} \min_{\varepsilon_i^{Path_k}} \sum_{\sigma / H_\sigma \in Nodes(Path_k)} \Delta\mu_i(\sigma).L(I_i(\sigma))$$

$$C_i^\Pi = \max_{Path_k} \max_{\varepsilon_i^{Path_k}} \sum_{\sigma / H_\sigma \in Nodes(Path_k)} \Delta\mu_i(\sigma).L(I_i(\sigma))$$

Note that C_i^N and C_i^Π can be indexed by time *k* for monitoring purposes. Indeed, $[C_i^N(k), C_i^\Pi(k)]$ can be computed at each point $\vec{P}^k$ of the trajectory from $\vec{P}^I$ to $\vec{P}^*$. We have then the following relation: $[C_i^N(k), C_i^\Pi(k)] \supseteq [C_i^N(k+1), C_i^\Pi(k+1)]$. It means that the imprecision related to the contribution of a criterion to the overall improvement $C_\mu(\vec{P}^*) - C_\mu(\vec{P}^I)$ naturally decreases as $\vec{P}^k$ becomes closer to $\vec{P}^*$. When $\vec{P}^*$ is reached at $k = k^*$, $C_i^N(k^*) = C_i^\Pi(k^*)$, i.e. $L([C_i^N(k^*); C_i^\Pi(k^*)]) = 0$: there is of course no more imprecision related to the a posteriori contribution. The manager has thus at his disposal an estimation in time of the range of authorized values for the contribution of each criterion. These computations enable to compare quantitatively any two control strategies from $\vec{P}^I$ to $\vec{P}^*$ in terms of criteria profitability. It is illustrated in the following section for the local efficiency and the worth index control strategies.

5 Case study

The case study concerns a SME producing kitchens and it is detailed in [12]. The overall objective of the company is to continuously increase its productivity. Let us suppose that a partial break-down of the strategic objective related to the productivity rate into 4 basic criteria is processed: *Stocks, Equipment availability, Operators' skill, Quality*. The overall performance is defined as the aggregation of these 4 associated indicators with a C_μ. All the numerical pieces of information exposed hereafter have been obtained by asking the management staff according to the Macbeth methodology [12]. Table 1 provides the current performances $\vec{P}^I = (P_1^I, \ldots, P_n^I)$, the relative importance of each criterion in the SME activity and the estimated cu_i costs to improve the performances. In this example, we use a 2-additive C_μ. In this case, we have:

$\Delta\mu_{(i)} = \nu_{(i)} + \frac{1}{2}\sum_{j>i} I_{(i)(j)} - \frac{1}{2}\sum_{j<i} I_{(i)(j)}$ where the $\nu_{(i)}$ are the Shapley indexes and the $I_{(i)(j)}$ the interaction coefficients between criteria *i* and *j* (Table 2).

Table 1. Weights, costs and initial performances

	Indicators	ν_i	cu_i	$\vec{P}^I$
C_1	Stocks	0.30	1 000k€	0.80
C_2	Equipment availability	0.25	3 000k€	0.25
C_3	Quality	0.30	2 000k€	0.75
C_4	Operators' skill	0.15	3 000k€	0.50

Table 2. Interactions coefficients

Interactions between	value
Stocks - Equipment availability	0.30
Stocks - Quality	0.25
Operators' skill - Quality	0.30

The current aggregated performance is $C_\mu(\vec{P}^I) = C_\mu(0.8, 0.25, 0.75, 0.5) = 0.483$. The expected overall performance is $P^* = 0.9$. Solving (**P₁**) with $\vec{P}^I$ as initial point and $P^* = 0.9$ provides $\vec{P}^* = (1, 1, 1, 0.636)$ which corresponds to a global improvement cost $C^* = 3358k€$. That is the static viewpoint of this efficient improvement. Now $P^* = 0.9$ appears as an ambitious setpoint so that the manager makes up his mind to reach it in three years. He has then to define the way he is going to achieve this goal. He decides to set intermediate targets each year. The two strategies described in this paper are envisaged (see respectively algorithms $\mathbf{A_1}$ and $\mathbf{A_2}$).

- <u>Local efficiency logics</u>. The manager chooses more pragmatic and short-term overall performances for each end-year, i.e., P^1 and P^2. Optimization problem ($\mathbf{P_1}$) is first solved with $\vec{P}^I$ as initial profile and P^1 as expected overall performance: it provides $\vec{P}^1$ at the end of first year. Then ($\mathbf{P_1}$) is solved with $\vec{P}^1$ and P^2: the result is $\vec{P}^2$ at the end of second year. Finally, it remains the improvement from $\vec{P}^2$ to $\vec{P}^*$ last year. He justifies his strategy as a step by step efficient improvement implementation. The computations are summarized in Table 3 (left part).

- <u>Worth index logics</u>. The worth index is computed at the beginning of each year. It provides the criteria to be first improved. At the end-year, the performance vector is observed. Worth index is recalculated with this new vector until $\vec{P}^*$ is reached. Results are summarized in Table 3 (right part). Thus, $A^* = \{c_2\}$ first year (maximal worth index is got for $\omega_{A^*}^{C}(C_\mu)(\vec{P}^I, \vec{P}^*) = \omega_{\{C_2\}}^{C}(C_\mu)(\vec{P}^I, \vec{P}^*) = 0{,}01303$) and $\vec{P}^3 = (0.8, 0.65, 0.75, 0.5)$ is reached, then $A^* = \{c_1, c_2, c_3\}$ second year (maximal worth index is got for $\omega_{A^*}^{C}(C_\mu)(\vec{P}^I, \vec{P}^*) = \omega_{\{C_1,C_2,C_3\}}^{C}(C_\mu)(\vec{P}^I, \vec{P}^*) = 0{,}01147$) and $\vec{P}^4 = (1, 0.9, 0.8, 0.5)$ is achieved. Last year, it remains improvement from $\vec{P}^4$ to $\vec{P}^*$. The manager justifies his strategy as the step by step most statistically profitable implementation.

Table 3. Left Part: Trajectory with local efficiency; Right Part: Trajectory with the worth index

	1st year				1st year			
		2nd year				2nd year		
			3rd year				3rd year	
$\vec{P}$	$\vec{P}^I$	$\vec{P}^1$	$\vec{P}^2$	$\vec{P}^*$	$\vec{P}^I$	$\vec{P}^3$	$\vec{P}^4$	$\vec{P}^*$
C_1	0.8	0.8	0.914	1	0.8	0.8	1	1
C_2	0.25	0.759	0.914	1	0.25	0.65	0.9	1
C_3	0.75	0.759	0.914	1	0.75	0.75	0.8	1
C_4	0.5	0.5	0.5	0.636	0.5	0.5	0.5	0.636
$C_\mu(\vec{P})$	0.483	0.69	0.8	0.9	0.483	0.643	0.768	0.9

Finally, for a quantitative comparison of both control strategies, interval $[C_i^N(k), C_i^\Pi(k)]$ ($k = 3$ years) that provides the lower and upper bounds of the contribution of criterion *i* are computed with $\vec{P}^I$ as initial point. Results are reported in Table 4. Column « Improvement » gives the coordinates of vector $\vec{\delta}^*$ from $\vec{P}^I$ to $\vec{P}^*$. Column « costs » reports the cost corresponding to each criterion for the optimal improvement. Columns $C_i^N(t)$ and $C_i^\Pi(t)$ give the minimal and maximal expected contributions for an efficient improvement from $\vec{P}^I$ to $\vec{P}^*$. Column 6 (resp. 7) reports the corresponding minimal (resp. maximal) expected profitability for each criterion. Columns 8 and 9 respectively give the a posteriori contribution and profitability of each criterion to $C_\mu(\vec{P}^*) - C_\mu(\vec{P}^I)$ when a locally efficient strategy is applied, whereas columns 10 and 11 provide the same data for the worth index logic. Note that both strategies provide opposite effects on this example: the most profitable criteria for one are the less ones for the other one. Indeed, for the local efficiency control strategy, the most profitable criterion is *Stocks* and the less profitable one is *Quality*; in revenge, the most profitable criterion is *Quality* and the less profitable one is *Stocks* for the worth index control strategy.

Conclusion

Optimization problem (**P$_1$**) is a basic useful tool to determine what an efficient improvement is when the overall performance of the company is modeled as the aggregation of elementary performances. However, they only provide the setpoint $\vec{P}*$, the expected performance profile for this improvement. That is the static viewpoint of what an efficient improvement should be. The way reaching $\vec{P}*$ necessitates other decisive factors that will set the discrete dynamics of the improvement. Criteria contributions play a key role in this choice. Worth index and local efficiency logics provide two different dynamics to implement the efficient improvement that leads to $\vec{P}*$. An algorithm has been provided to compute the expected contributions of criteria from any initial point to $\vec{P}*$. It enables to compare in time control strategies in terms of criteria profitability.

Acknowledgments

We thank our colleagues from the Listic (Université de Savoie), Lamia Berrah, Vincent Clivillé and Gilles Mauris because they initiated us to industrial engineering and encouraged this work. We also thank Christophe Labreuche from Thalès Research Group for his relevant remarks and advices.

References

[1] L. Berrah, G. Mauris, and F. Vernadat, Information aggregation in industrial performance measurement: rationales, issues and definitions. *International Journal of Production Research*, 2004, vol. 42:(20) p. 4271-4293.

[2] U.S. Bititci, Modelling of performance measurement systems in manufacturing enterprises. *International Journal of Production Economics*, 1995, vol. 42:(2) p. 137-147.

[3] R. Felix. Relationships between goals in multiple attribute decision making Source, Fuzzy Sets and Systems 1994, vol. 67:(1), 47 - 52

[4] P. Kueng and A. J. W. Krahn, Building a process performance measurement system: Some early experiences. *Journal of Scientific & Industrial Research*, 1999, vol. 58:(3-4) p. 149-159.

[5] L. Berrah, G. Mauris, and J. Montmain, Diagnosis and improvement indexes for a multi-criteria industrial performance synthesized by a Choquet integral aggregation. *The International Journal of Management Science, OMEGA*, 2008, vol. 36 :(3) p. 340-351.

[6] A. Neely, The performance measurement revolution: why now and what next? *International Journal of Operations and Production Management*, 1999, vol. 19 p. 205-228.

[7] C. Labreuche, Determination of the criteria to be improved first in order to improve as much as possible the overall evaluation. *IPMU'04*, 2004, Perugia, Italy.

[8] S. Sahraoui, J. Montmain, L. Berrah and G. Mauris, User-friendly optimal improvement of an overall industrial performance based on a fuzzy Choquet integral aggregation. *IEEE International Conference on Fuzzy Systems*, 2007, London, UK.

[9] S. Sahraoui, J. Montmain, L. Berrah and G. Mauris. Decision-aiding functionalities for industrial performance improvement. *4th IFAC Conference on Management and Control of Production and Logistics*, 2007, Sibiu, Romania.

[10] L. Berrah, *et al.*, Global vision and performance indicators for an industrial improvement approach. *Computers in Industry*, 2000, vol.43:(3) p. 211-225.

[11] M. Grabisch and C. Labreuche, How to improve acts: an alternative representation of the importance of criteria in MCDM. *Int. J. of Uncertainty, Fuzziness, and Knowledge-Based Systems.* 2001, Vol.9:(2) p. 145-157.

[12] V. Clivillé, L. Berrah, et al. Quantitative expression and aggregation of performance measurements based on the Macbeth multi-criteria method. *International Journal of Production Economics*. 2007, vol.105:(1), p. 171-189.

Table 4. Expected and observed contributions and profitabilities

Criterion	Improv^t	Cost k€	$C_i^N(t)$	$C_i^\Pi(t)$	Minimal Expected Profitability (%)	Maximal Expected Profitability (%)	Observed Contribution Strategy 1 local efficiency	Observed profitability (%) Strategy 1	Observed Contribution Strategy 2 worth index	Observed profitability (%) Strategy 2
C_1	0.2	200	0,0099	0,11	4.95^{E}-3	5.5^{E}-2	0,11	5.5^{E}-2	0.01	5^{E}-3
C_2	0.75	2250	0,24	0,3	1.067^{E}-2	1.33^{E}-2	0,24	1.067^{E}-2	0.3	1.33^{E}-2
C_3	0.25	500	0,02875	0,06875	5.75^{E}-3	1.375^{E}-2	0,02875	5.75^{E}-3	0.06875	1.375^{E}-2
C_4	0.136	408	0,0374	0,0374	9.17^{E}-3	9.17^{E}-3	0,0374	9.17^{E}-3	0.0374	9.17^{E}-3
Somme		3358					0,41615		0,41615	