Risk Prediction in a Space-Time Markov Model applied to Propagating Contamination

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Abstract

Our concern is crisis situations involving propagating threats such as toxic clouds, pandemics, contamination, flows or oil slicks. Emergency planning may be then complex, and decision-making support (DS) tools based on risk criteria can be helpful. The evaluation of quantitative risk needs for a mathematical model taking into account either the cost to pay if a given point is contaminated (damage, casualties...), either the associated probability.
The present work proposes original mathematical expressions for a recursive implementation of the mid-term predicted risk at each point of the threatened area.
An application of the proposed risk prediction model is shown on a dangerous cloud, with numerical results.
Keywords: Risk assessment, Markov models, propagating phenomena, NRBC (bio radio chemical nuclear) threat, crisis management, disaster

1 Introduction

This paper addresses the need for risk assessment in DS tools for crisis management in large disasters involving a propagating danger. The purpose of Quantitative Risk Assessment (QRA) is explained in [2] [12]. The Risk Assessment task aims at answering three possible questions: identifying the possible dangerous events, their probability and their cost.
The approach described below, addresses both the 2nd and the 3rd questions by providing a cost-probability risk (e.g. the expected number of future casualties), using the probability that the propagating phenomenon, which can be for example a growing cloud, reaches one point before a given delay. At each time $t_0 > 0$, the cloud's geography is known exactly, for example thanks to aerial or satellite imaging.
Markov models are well known to be representative of many phenomena and they are used in many domains [16]. Their interest in comparison to dynamical system models [15] is that they are probabilistic, so they allow to take into account the uncertainty of the systems. Based on Markov models, consisting of comprehensive representations of possible chains of events, we are able to compute, from the observation of the cloud at t_0, a cost-probability risk prediction for a future time $t_0 + t$. In the sequel, observation will be replaced by a simulation based on a space-time Markov modeling of the cloud's propagation.
The proposed work provides a mid- and long-term risk prediction model for each point of the threatened area. It constructs maps of the predicted cost-probability risk, based on the geography (location of population and assets) and on a space-time Markov chain modeling of the propagating phenomenon. From a simulation of the current state of the propagation at time t_0, the risk is calculated at each point of the studied area for each future time t recursively as a function of the risk map at $t - 1$. The calculated risk can be integrated on the overall area in order to show its evolution as a function of future time; this curve

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can help the planning of the intervention. The calculated risk can also be shown in a map that can be used directly by planning tools.
These phenomena have been modeled precisely as deterministic linear dynamic systems. The Markov chain approach can be associated (by fitting in average), and this allows taking into account the probabilistic aspect of the risk.

2 Existing 2D Markov models and works

2.1 Markov chains

Here will be defined the Markov chains [13] [6], which are one-dimension models for stochastic processes. Let $\Omega = \{a_1, a_2...a_N\}$ be a finite set of the N possible values that can be taken by a random variable x. One supposes that a probability distribution Pr can be defined on Ω^n, which is the set of all possible n-length sequences in Ω for a given integer n.

Definition 2.1. The probability Pr on is a *Markov chain* as soon as it satisfies:

$$Pr(x_1, x_2...x_n) = Pr(x_1) \prod_{t=2}^{n} Pr(x_t|x_{t-1})$$

The values taken by the random variable x at each time t are called *states*. A Markov chain is characterized by the triple (Ω, Q, P_0) where $P_0 = Pr(x_1)$ is the so-called initial probability and Q is the $N \times N$ transition matrix, defined by $Q = (q_{ij})_{1 \leq i,j \leq n}$ where $q_{ij} = Pr(x_{t+1} = a_i|x_t = a_j)$.

Definition 2.2. A *Hidden Markov Model* (HMM) is a 5-uple $(\Omega_x, \Omega_y, Q, K, P_0)$ where (Ω_x, Q, P_0) is a Markov chain, and the observation is a random variable y taking values in Ω_y and such taht the Markov kernel of y given x is K.

A Markov kernel is the matrix of conditional probabilities of the occurrence of one of the two random variables given the other. In a HMM, the internal states x of the Markov chain are not known, except through the knowledge of y. To estimate x_t from an observed sequence y_t, algorithmes have been proposed such as the Baum-Welch algorithm, the Viterbi algorithm [14].

An application of such HMMs has been proposed to detect oil slicks in SAR images [10]. A one-dimensional signal is first obtained from the images by a Peano scan, and then a 1D HMM identification is performed.

2.2 Markov random fields

A *Markov random field* (MRF) is a 2D Markov model. The Markov condition is applied in the space domain, i.e. to the pixel's neighbors, instead of the time domain [7]:

Definition 2.3. A *Markov random filed* is a bidimensional stochastic process $X(i, j)$ satisfying:

$$\begin{aligned} &Pr\big(X(i,j)|X(k \neq i, l \neq j)\big) \\ &= Pr\big(X(i,j)|X(i+\delta_x, j+\delta_y\big) \end{aligned}$$

with $\delta_x, \delta_y \in \{-1; 0; +1\}$.

The MRF model is purely spatial, it still does not take the time into account.

2.3 Other approach

A spatio-temporal model has been proposed for a pandemics [4]. It uses a Gibbs model for the spatial properties, which involves an energy between neighbor pixels. The corresponding mathematical expression contains an exponential law, which is complex to solve; the authors limited then their study to the one-dimension case.

2.4 Novelty of the proposed approach

Various decision-support tools for disaster management are developed on the principle of the fine modelling of a physical phenomenon associated to a GIS [8], [5]. Few have been proposed in that domain to provide to the decision-maker, in addition to the model, a quantitative risk measurement [9], [15], [16].
The proposed space-time Markov model used here for the propagating phenomenon is apparented to a MRF. But the originality of this work is in the associated computation of a risk criterion.

3 Principle of the proposed approach

The propagating phenomenon will be denoted as a cloud for more clarity, but the idea will be

exactly the same for any propagating dangerous phenomenon.
The model used to describe the actual state of the cloud ate each time, and to forecast its possible future state, will be first described. Then, general criteria for risk measurement will be reminded. In the following, the algorithm for the measurement of the risk criterion in the model will be given.

3.1 The propagation model

The time t is supposed to be sampled at a regular period that is chosen to be adapted to the propagation speed. The model output is a two-dimensional map representing the state of each location, represented by a pixel in an image.
The space-time function $c(M,t)$ expresses the fact that the pixel located at point M is contaminated at time t. The idea is first to express $c(M,t)$ as a function of its neighbors in space and in time:

$$c(M,t) = f\big(\{c(V_i, t-1)/1 \leq I \leq 8\}\big)$$

where the points V_i are the M neighbors. At time $t = 0$ the cloud covers one single pixel, and between each temporal period $]t, t+1]$ it propagates from one pixel to its neighbors following the markovian rule described in table 1.

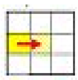

Figure 1: Influence of a next neighbour

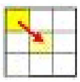

Figure 2: Influence of a diagonal neighbour

Table 1: Probability of contamination from one pixel to its neighbor

Contaminated at $t-1$	**Next neighbor**	**Diagonal neighbor**
Scheme	See fig. 1	See fig. 2
Probability to be contaminated at t	p	$\frac{p}{\sqrt{2}}$

If there are several contaminated neighbors, their influence on the current pixel are statistically independent. Thus the probability $Pr(c)$ for the current pixel to be contaminated at t is:

$$Pr(c) = 1 - (1-p)^N.(1 - \frac{p}{\sqrt{2}})^D \qquad (1)$$

where N is the number of contaminated next neighbors, and D is the number of contaminated diagonal neighbors. This is a space-time Markov chain, since the probability for one pixel to be contaminated at time t depends only on state (contaminated or not) in its neighborhood: the neighbor pixels (for the spatial point of view), and the previous date (for the temporal point of view). One can notice that p depends on the ratio of the cloud velocity upon the chosen time period. The latter must be short enough. Furthermore, the propagation may be anisotropic if there is wind; in that case one must introduce four values for p (depending on the relative position of the neighbor pixel). Such a model can be simulated with random draws. An example is given in figure 5.

3.2 Fitting with a dynamical system model

Relationships between HHMs and Linear Dynamical Systems have been studied [11]. Fine models exists for natural physical phenomena. For example, one is proposed [9] [17] for oil slicks; it involves differential equations. Physical exact models do not take into account any random components. In fact phenomena are deterministic in average, which appears in large scale analysis. They are affected by random aspects, which are due to many factors, and assimilated as *noise* at a signal processing point of view.
The Markov modelling takes the random aspect into account. To be validated, it must have an average equal to the deterministic model. In the proposed approach, the fitting between the two models will be obtained by adjusting the p parameter, which is related to the propagation space.

3.3 The principle of risk measurement

The notion of risk [2] relies on the idea of *cost*, which designates any prejudice, damage, or loss. Depending on the application domains, this cost can be of various natures: financial loss, human losses (number of casualties), unavailability of some resources or supply means... In fact the cost

is often a combination of several of these aspects. But in this work the resulting cost is supposed to be a single random value.
The so-called QRA methodology (Quantitative Risk Assessment) [12] is composed of phases which aim at answering to the following questions:

- Qualitative analysis: What is likely to happen ? Here, the answer is: a given point, which is now outside the cloud, may be contaminated at a future time and thus people may die.
- How likely is it to happen ? The answer is a probability.
- If it happens, what can be the consequences (cost) ? The cost here the cloud's level of severity.

The present work is concerned by these two last aspects, which are summarized in the lines 2 and 3 of the table 2, by proposing a criterion and methods for the evaluation.

What may happen ?	Qualitative analysis
How likely is it to happen ?	Probability
What are the consequences ?	Cost

Table 2: The three aspects of risk

The notion of risk can be summarized as an uncertainty about a future possible cost to be paid.
The cost c is then considered as a random variable whose probability density is $p(c)$ (see an example on figure 3). One will denote as $Pr(.)$ the associated probability measure. Numerous risk criteria have been proposed from that random varialbe, in particular in the financial domain [1]. They can be grouped in three categories that will be desribed below: the average, the variability of the cost, and the worst case. All the expressions are particular cases of partial centered momentums, which are well-known under more general forms in the financial domain (Stone and Fichburn risk measures [1]).

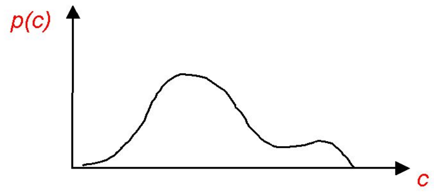

Figure 3: Probability density of cost

3.3.1 The average risk

The idea is to answer to the question: *In average, what will it cost ?* [3]. In that category, there is the cost average 2, that will be called *EC risk* (Expected Cost Risk) and denoted as $< c >$:

$$< c >= \int_{-\infty}^{+\infty} c\, p(c) dc \qquad (2)$$

3.3.2 The cost variability

Here the risk is defined by the fact that the future cost is rather impredictable. This uncertainty on cost can be shown for example in the variance $\sigma^2(c)$ 3:

$$\sigma^2(c) = \int_{-\infty}^{+\infty} (c- < c >)^2 p(c) dc \qquad (3)$$

3.3.3 The worst case risk

Now one considers the probability that could occur the most dreaded case (for example a breakdown in an important engine, a fire, a death...). It can correspond to one single situation or a set of dreaded situation called *risks*. Their list is then established during the qualitative risk analysis phase. They correspond to the max cost cases, or more precisely to the occurrence of a cost higher than a certain level c_0 (see figure 4).

This notion of risk assessment implies a fine estimation of the probabilies $Pr(c_{max})$ or $Pr(c \geq c_0)$, which depends on the knowledge of the application domain: statistics, phenomena modeling. What we can write is 4. In the financial domain, this is the *probability of ruin* [1]:

$$Pr(c \geq c_0) = \int_{c_0}^{+\infty} p(c) dc \qquad (4)$$

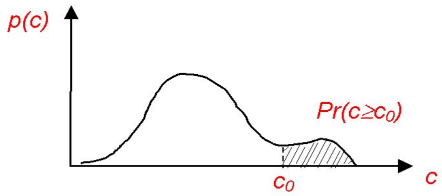

Figure 4: Probability for the cost c to be greater than a threshold c_0

The worst case probabilities can be weightened by their corresponding cost C_{max} and called *cost-probability* (CP) risk:

$$C_{max}Pr(C_{max})$$

One can also calculate the CP risk by an integral:

$$R = \int_{c_0}^{+\infty} cp(c)dc$$

3.4 The proposed risk evaluation algorithm

The retained criterion for the overall risk measurement is the expected cost, since one wants to know what can be the cost (how many casualties can be made) at a given place and at a given future time. The first step is to calculate the probability for the place to be contaminated at that time, which is a worst case risk criterion.

3.4.1 Mid-term risk of contamination

One wants to forecast the risk for a given place to be contaminated at a future time. The purpose is to evaluate in particular the threat for the population in the surrounding areas which have not yet been contaminated by the cloud.
Given the cloud situation at t_0, the mid-term risk can be mathematically expressed thanks to the Markov chain property given at 1. For a given pixel M, let's introduce $b(M,t)$ which is the probability that M is not contaminated (nc) at t given that it was not contaminated at $t-1$:

$$b(M,t) = \prod_{i=1}^{8} \big(Pr(nc(V_i, t-1)) \\ +Pr(nc|c(V_i, t-1).Pr(c(V_i, t-1))\big)$$

where $v_i, i = 1...8$ are the M neighbors. If we introduce $a(M,t)$ which is the probability that the pixel is not contaminated at t, we get:

$$a(M,t) = a(M,t-1).b(M,t)$$

and

$$b(M,t) = \prod_{V\in\mathcal{N}(M)}(p.a(V,t-1)+1-p) \\ \times \prod_{V\in\mathcal{D}(M)}\big(\tfrac{p}{\sqrt{2}}a(V,t-1)+1-\tfrac{p}{\sqrt{2}}\big)$$

where $\mathcal{N}(M)$ and $\mathcal{D}(M)$ are the sets of M's next and the diagonal neighbors respectively.
So, we showed in this section how to compute the probability $b(M,t)$ that the place M remains uncontaminated. We obtein:

$$Pr(c(M,t)) = 1 - b(M,t)$$

3.4.2 Including the local vulnerability

Let's introduce a severity parameter $G \in [0;1]$. G is the probability of local damage, e.g. the probability that one person die during one time period at one contaminated location. That means if at t there are $N_u(t)$ unharmed people there, the expected number of (new) casualties is $GN_u(M,t)$, so when the cloud grows, the expected number of remaining unharmed people is updated by

$$N_u(M, t_0+1) = (1-G)N_u(M,t_0) \text{ if } c(M,t)$$

So, at a given pixel M, the number of people that are expected to die at $t_0 + t$ is

$$r(M,t) = G < N_u(M,t-1) > Pr(c(M,t-1))$$

where $< N_u(M,t-1) >$ is the expected number of unharmed people at $t_0 + t - 1$:

$$< N_u(M,t-1) >= N_u(M,t_0) - R(M,t-1)$$

$N_u(M,t_0)$ is the initial number of unharmed people (at t_0) and $R(M,t)$ is the global EC risk (expected total number of casualties):

$$R(M,t) = R(M,t-1) + r(M,t)$$

.

The expected number of casualties can easily be computed at each pixel, so we obtain a map as shown at figure 9.

4 Results

The proposed algorithm has been implemented on a simulation of a dangerous cloud.
An explosion takes place in an urban area, at time $t = 0$ and all the local agencies fire brigade, ambulance and police receive the alert. The Police warn the Fire Brigade about the risks of a dirty bomb. The explosion is believed to have involved a chemical release and a consequent dangerous cloud starts to grow, threatening the population. The figure 5 shows an example simulated with the space-time Markov model described at 1.

Then, the Fire Brigade being the agency equipped with suitable Protective equipment is responsible for the initial incident assessment and the Crisis Response Coordination Commander acts as a decision maker to organize the threatened population evacuation. Such an operation is not a trivial matter, since people will not abandon their houses if they do not have any serious reason to do so. Furthermore, it raises traffic and logistic problems that should be managed. Thus, it is necessary to have a precise risk evaluation at one's disposal as a criterion to motivate such a decision.
One important data for the problem is the map of the population density, as proposed in figure 6.

The computation of the risk with the proposed algorithm provides the results showed at figures 7, 8, 10 and 9.

One can notice that this figure 9 is coherent with the fact that the number of casualties increases with the cloud's surface, thus the threatened are essentially located near the cloud's periphery (see figure 7), so they grow linearly.
Numerical results of the risk $R(t)$ are shown in our example at figure 8, and its integration over the whole area (total number of people) is shown at figure 10.

A curve like in figure 10 can be helpful to the decision makers in emergency planning, since it shows the number of potential casualties as a function of the time they will spend to inter-

Figure 5: Cloud evolution as a function of time

Figure 6: Population density in the studied area

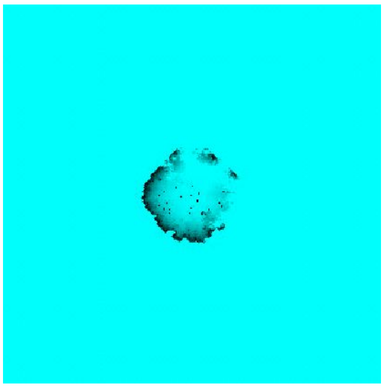

Figure 7: Map of the short-term ($t_0 + 1$) threatened people for $t_0 = 50$

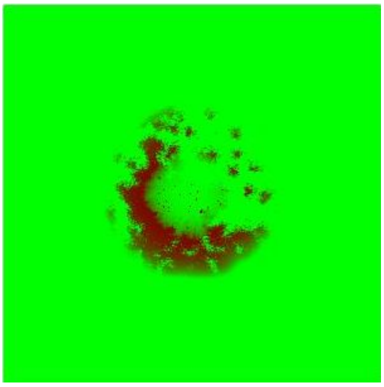

Figure 8: Map of the long-term threatened people (expected casualties risk at future time $t = 30$ given the known situation at $t_0 = 50$)

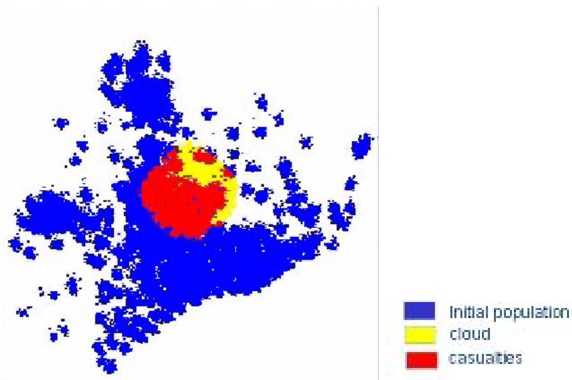

Figure 9: Map of the population, the cloud and the casualties for $t_0 = 50$

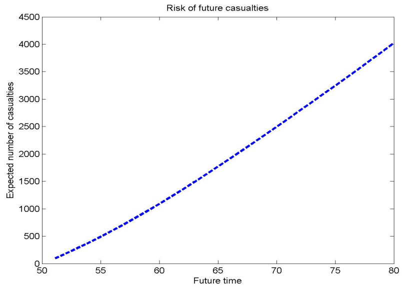

Figure 10: Long-term risk of future casualties for $t_0 = 50$ and future time $t = 30$

vene. This information may be crucial in large-scale disasters when another area is affected by another cause of damage at the same time, but the rescuing means are limited; particularly if the curve like figure 10 is quite nonlinear, it helps the decision-maker to know when he can wait and when the emergency will be greater.

5 Conclusion

One of today's worrying threats of crisis are propagating contaminations (like toxic gas, rays, infections...) that can be initiated accidentally or by an attack. Disaster and emergency managers (the Crisis Response Coordination Commander) and all the stakeholders often need to make decisions under uncertainty and complex situations. They need a solid understanding of the situation supported by relevant risk assessment tools including 2D visualization.

The aim of QRA (quantitative risk assessment) within a crisis response operation, is to determine the probability that a given hazard will evolve, including also its consequences. For this purpose the cost-probability is the criterion used in this paper to express the risk. A mathematical model for the phenomenon is then necessary. However such phenomena have often well-known deterministic behaviour in average, they are subject to probabilistic perturbations or unknown parameters; this is why Markov models can be suited to them.

The proposed approach is then a new algorithm for risk measurement in a space-time Markov modeling of the contamination propagation, providing a mathematical expression of the cost-probability short-term, mid-term and long-term risk that can be computed recursively to visualize at each time a risk map of the threatened area. An illustration is proposed on a simulated example.

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