Uncertain Decision Making with Dempster-Shafer Theory

José M. Merigó
Department of Business Administration,
University of Barcelona,
Av. Diagonal 690,
08034 Barcelona, Spain
jmerigo@ub.edu

Montserrat Casanovas
Department of Business Administration,
University of Barcelona,
Av. Diagonal 690,
08034 Barcelona, Spain
mcasanovas@ub.edu

Abstract

We develop a new approach for decision making with Dempster-Shafer theory of evidence by using uncertain information represented in the form of interval numbers. We suggest the use of different types of uncertain aggregation operators in the problem. Then, we obtain new aggregation operators such as the belief structure - uncertain ordered weighted averaging (BS-UOWA) operator and the BS - uncertain hybrid averaging (BS-UHA) operator, among others. Some of their main properties are studied such as the distinction between descending and ascending orders and the analysis of different particular cases.

Keywords: Decision making, Dempster-Shafer theory, Interval numbers, Uncertain OWA operator.

1 Introduction

The Dempster-Shafer (D-S) theory [2,10] provides a unifying framework for representing uncertainty because it includes in the same formulation the case of risk and ignorance. Since its appearance, it has been used in a lot of situations [3,5-8,11,16,21,23].

Usually, when using the D-S theory in decision making, it is assumed that the available information are exact numbers [3,5-6,16,21,23]. However, this may not be the real situation found in the decision making problem. Sometimes, the available information is vague or imprecise and it is necessary to use another approach to assess it such as the use of interval numbers. In this paper, we will study the decision making problem with D-S belief structure using information given in the form of interval numbers.

In order to aggregate the uncertain information, we will use different types of uncertain aggregation operators. The reason for doing so is that we want to show that this problem can be modeled in different ways depending on the interests of the decision maker. Mainly, we will use the uncertain ordered weighted averaging (UOWA) operator [12] and the uncertain hybrid averaging (UHA) operator. Then, we will get as a result, new aggregation operators such as the belief structure - UOWA (BS-UOWA) operator and the BS-UHA operator. We will study some of the main properties of these aggregations and we will consider different families of uncertain aggregation operators in the problem such as the step-UOWA, the window-UOWA, the S-UOWA, the olympic-UOWA, the centered-UOWA, etc.

In order to do so, this paper is organized as follows. In Section 2 we briefly review some basic aspects such as interval numbers, the UOWA and the UHA operators. Section 3 briefly describes the main concepts of D-S theory. In Section 4, we present the new approach when using UOWA operators. In Section 5, we develop a similar analysis with

L. Magdalena, M. Ojeda-Aciego, J.L. Verdegay (eds): Proceedings of IPMU'08, pp. 425–432
Torremolinos (Málaga), June 22–27, 2008

UHA operators. Finally, in Section 6 we summarize the main conclusions of the paper.

2 Preliminaries

In this Section, we briefly describe the main concepts of the interval numbers, the UOWA and the UHA operator.

2.1 Interval Numbers

The interval numbers [9] are a very useful and simple technique for representing the uncertainty. It has been used in an astonishingly wide range of applications.

The interval numbers can be expressed in different forms. For example, if we assume a 4-tuple (a_1, a_2, a_3, a_4), that is to say, a quadruplet; we could consider that a_1 and a_4 represents the maximum and the minimum of the interval numbers and a_2 and a_3, the interval with the highest probability or possibility, depending on the use we want to give to the interval numbers. Note that $a_1 \leq a_2 \leq a_3 \leq a_4$. If $a_1 = a_2 = a_3 = a_4$, then, the interval number is an exact number and if $a_2 = a_3$, it is a 3-tuple known as triplet.

In the following, we are going to review some basic interval number operations as follows. Let A and B be two triplets, where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$. Then:

1) $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2) $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
3) $A \times k = (k \times a_1, k \times a_2, k \times a_3)$; for $k > 0$.

Note that other operations could be studied [9] but in this paper we will focus on these ones.

2.2 Uncertain OWA Operator

The uncertain OWA (UOWA) operator [12] is an aggregation operator that uses uncertain information represented by interval numbers. The reason for using the UOWA is because sometimes it is necessary to use interval numbers to correctly assess uncertain decision problems because the expected results are not clear. It can be defined as follows.

Definition 1. *Let Ω be the set of interval numbers. An UOWA operator of dimension n is a mapping UOWA: $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then*:

$$UOWA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j b_j \quad (1)$$

where b_j is the jth largest of the $\tilde{a}_i$ and the $\tilde{a}_i$ are interval numbers.

An interesting issue to consider in the UOWA operator is how to develop the reordering of the arguments because now, they are interval numbers. For simplicity, we recommend to use the average of the interval number in order to establish a criterion for comparing interval numbers. Note that a more complete approach is the use of a weighted average in the comparison.

Note that it is possible to study a wide range of properties such as the distinction between descending (DUOWA) and ascending (AUOWA) orders.

2.3 Uncertain Hybrid Averaging Operator

The uncertain hybrid averaging (UHA) operator is an extension of the HA operator [13] that uses uncertain information represented in the form of interval numbers. It uses in the same formulation the uncertain weighted average (UWA) and the UOWA operator. Then, with this operator we can represent the subjective probability and the attitudinal character of a decision maker in the same problem. The main advantage is that it can represent uncertain situations that can not be assessed with exact numbers or singletons, but it is possible to use interval numbers. Then, the decision maker gets a more complete view of the decision problem.

Definition 2. *Let Ω be the set of interval numbers. An UHA operator of dimension n is a mapping UHA: $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then*:

$$UHA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j b_j \quad (2)$$

where b_j is the jth largest of the $\hat{a}_i$ ($\hat{a}_i = n\omega_i\tilde{a}_i$, $i = 1,2,\ldots,n$), $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the $\tilde{a}_i$, with $\omega_i \in [0, 1]$ and the sum of the weights is 1, and the $\tilde{a}_i$ are interval numbers.

Note that it is possible to distinguish between the descending UHA (DUHA) and the ascending UHA (AUHA) operator.

The UHA operator is commutative, monotonic and idempotent. It is not bounded by the maximum and the minimum because we may find some situations where the aggregation gives higher and lower results than the maximum and the minimum, respectively.

Different families of UHA operators are found by using a different manifestation of the weighting vector such as the UA, the UWA, the UOWA, the step-UHA, the olympic-UHA, the median-UHA, the window-UHA, the S-UHA, the centered-UHA, etc. For more information, see [4].

3 The Dempster-Shafer Theory

The D-S theory provides a unifying framework for representing uncertainty as it can include the situations of risk and ignorance as special cases. Note that the case of certainty is also included as it can be seen as a particular case of risk and ignorance.

Definition 3. *A D-S belief structure defined on a space X consists of a collection of n nonnull subsets of X, B_j for $j = 1,\ldots,n$, called focal elements and a mapping m, called the basic probability assignment, defined as, m*: $2^X \to [0, 1]$ *such that*:

(1) $m(B_j) \in [0, 1]$.

(2) $\sum_{j=1}^{n} m(B_j) = 1$. (3)

(3) $m(A) = 0, \forall A \neq B_j$.

As said before, the cases of risk and ignorance are included as special cases of belief structure in the D-S framework. For the case of risk, a belief structure is called Bayesian belief structure if it consists of *n* focal elements such that $B_j = \{x_j\}$, where each focal element is a singleton. Then, we can see that we are in a situation of decision making under risk environment as $m(B_j) = P_j = \text{Prob}\ \{x_j\}$.

The case of ignorance is found when the belief structure consists in only one focal element *B*, where *m(B)* essentially is the decision making under ignorance environment as this focal element comprises all the states of nature. Thus, $m(B) = 1$. Other special cases of belief structures are studied in [10].

4 UOWA Operator in Decision Making with D-S Theory

In this Section, we describe the process to follow when using the D-S theory in decision making with uncertain information represented in the form of interval numbers. For doing so, we will use the UOWA operator for aggregating the information because it provides a parameterized family of uncertain aggregation operators that includes the uncertain maximum, the uncertain minimum and the UA, among others. We will also analyze different families of UOWA operators to be used in the aggregation.

4.1 Decision Making Approach

A new method for uncertain decision making with D-S theory is possible by using uncertain aggregation operators in the problem. Note that we will consider the UOWA and the UHA operators but it is also possible to consider other cases such as the use of different types of uncertain generalized means and uncertain quasi-arithmetic means. The motivation for using interval numbers appears because sometimes, the available information is not clear and it is necessary to assess it with another approach such as the use of interval numbers. Although the information is uncertain and it is difficult to take decisions with it, at least we can represent the best and worst possible scenarios and the most possible ones. The decision process can be summarized as follows.

Assume we have a decision problem in which we have a collection of alternatives $\{A_1, \ldots, A_q\}$

with states of nature $\{S_1, \ldots, S_n\}$. $\tilde{a}_{ih}$ is the uncertain payoff, given in the form of interval numbers, to the decision maker if he selects alternative A_i and the state of nature is S_h. The knowledge of the state of nature is captured in terms of a belief structure m with focal elements $B_1, \ldots, B_r$ and associated with each of these focal elements is a weight $m(B_k)$. The objective of the problem is to select the alternative which gives the best result to the decision maker. In order to do so, we should follow the following steps:

Step 1: Calculate the uncertain payoff matrix.

Step 2: Calculate the belief function m about the states of nature.

Step 3: Calculate the attitudinal character of the decision maker $\alpha(W)$ [14].

Step 4: Calculate the collection of weights, w, to be used in the UOWA aggregation for each different cardinality of focal elements. Note that it is possible to use different methods depending on the interests of the decision maker [1,4-6,12,15-22,24-25].

Step 5: Determine the uncertain payoff collection, M_{ik}, if we select alternative A_i and the focal element B_k occurs, for all the values of i and k. Hence $M_{ik} = \{a_{ih} \mid S_h \in B_k\}$.

Step 6: Calculate the uncertain aggregated payoff, V_{ik} = UOWA(M_{ik}), using Eq. (1), for all the values of i and k.

Step 7: For each alternative, calculate the generalized expected value, C_i, where:

$$C_i = \sum_{r=1}^{r} V_{ik} m(B_k) \tag{4}$$

Step 8: Select the alternative with the largest C_i as the optimal.

4.2 The BS-UOWA Operator

Analyzing the aggregation in *Steps* 6 and 7 of the previous subsection, it is possible to formulate in one equation the whole aggregation process. We will call this process the belief structure - UOWA (BS-UOWA) aggregation. It can be defined as follows.

Definition 4. *A BS-UOWA operator is defined by*

$$C_i = \sum_{k=1}^{r} \sum_{j_k=1}^{q_k} m(B_k) w_{j_k} b_{j_k} \tag{5}$$

where w_{j_k} is the weighting vector of the kth focal element such that $\sum_{j=1}^{n} w_{j_k} = 1$ and $w_{j_k} \in [0,1]$, b_{j_k} is the j_kth largest of the $\tilde{a}_{i_k}$ and the $\tilde{a}_{i_k}$ are interval numbers, and $m(B_k)$ is the basic probability assignment.

Note that q_k refers to the cardinality of each focal element and r is the total number of focal elements.

The BS-UOWA operator is monotonic, commutative, bounded and idempotent. We can prove these properties with the following theorems.

Theorem 1 (Commutativity). *Assume f is the BS-UOWA operator, then*

$$f(\tilde{a}_{1_1}, \ldots, \tilde{a}_{q_1}, \ldots, \tilde{a}_{q_r}) = f(\tilde{a}^*_{1_1}, \ldots, \tilde{a}^*_{q_1}, \ldots, \tilde{a}^*_{q_r}) \tag{6}$$

where $(\tilde{a}^*_{1_1}, \ldots, \tilde{a}^*_{q_1}, \ldots, \tilde{a}^*_{q_r})$ is any permutation of $(\tilde{a}_{1_1}, \ldots, \tilde{a}_{q_1}, \ldots, \tilde{a}_{q_r})$ for each focal element k.

Theorem 2 (Monotonicity). *Assume f is the BS-UOWA operator, if $\tilde{a}_{i_k} \geq \tilde{a}^*_{i_k}$ then,*

$$f(\tilde{a}_{1_1}, \ldots, \tilde{a}_{q_1}, \ldots, \tilde{a}_{q_r}) \geq f(\tilde{a}^*_{1_1}, \ldots, \tilde{a}^*_{q_1}, \ldots, \tilde{a}^*_{q_r}) \tag{7}$$

Theorem 3 (Boundedness). *Assume f is the BS-UOWA operator, then*

$$\min\{a_i\} \leq f(\tilde{a}_{1_1}, \ldots, \tilde{a}_{q_1}, \ldots, \tilde{a}_{q_r}) \leq \max\{a_i\} \tag{8}$$

Theorem 4 (Idempotency). *Assume f is the BS-UOWA operator, if $\tilde{a}_{i_k} = \tilde{a}$ for all $i \in N$, then*

$$f(\tilde{a}_{1_1}, ..., \tilde{a}_{q_1}, ..., \tilde{a}_{q_r}) = \tilde{a} \quad (9)$$

From a generalized perspective of the reordering step, it is possible to distinguish between descending and ascending orders by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DUOWA and w^*_{n-j+1} the jth weight of the AUOWA operator. Then, we obtain the BS-DUOWA and the BS-AUOWA operators.

4.3 Families of BS-UOWA Operators

By using a different manifestation in the weighting vector of the UOWA operator, we are able to develop different families of UOWA and BS-UOWA operators. As it can be seen in Definition 4, each focal element uses a different weighting vector in the aggregation step with the UOWA operator. Therefore, the analysis needs to be done individually.

For example, it is possible to obtain the uncertain maximum, the uncertain minimum, the UA and the UWA. The uncertain maximum is found if $w_1 = 1$ and $w_j = 0$, for all $j \neq 1$. The uncertain minimum is obtained if $w_n = 1$ and $w_j = 0$, for all $j \neq n$. The UA is found when $w_j = 1/n$, for all $\tilde{a}_i$ and the UWA is obtained if the ordered position of b_j is the same than the position of $\tilde{a}_i$.

Other families of UOWA operators could be used in the BS-UOWA operator such as the step-UOWA, the S-UOWA, the olympic-UOWA, the window-UOWA and the centered UOWA operator, among others. Note that recently, it is appearing a wide range of papers dealing with the problem of determining OWA weights. In this subsection we simply give a general overview commenting some basic cases that are applicable in the UOWA operator.

The step-UOWA operator is found when $w_k = 1$ and $w_j = 0$, for all $j \neq k$. Note that the median-UOWA can be seen as a particular case of this situation when the number of arguments is odd.

A further interesting family is the S-UOWA operator. In this case, we can distinguish between three types: the "orlike", the "andlike", and the "generalized" S-UOWA operator. The generalized S-UOWA operator is obtained when $w_1 = (1/n)(1 - (\alpha + \beta)) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta)) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta))$ for all $j = 2$ to $n - 1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Note that if $\alpha = 0$, we get the andlike S-UOWA and if $\beta = 0$, the orlike S-UOWA. Also note that if $\alpha + \beta = 1$, we get the uncertain Hurwicz criteria.

The olympic-UOWA operator is found if $w_1 = w_n = 0$, and for all others $w_j = 1/(n - 2)$. Note that the window-UOWA operator can be seen as a generalization of this case.

The centered-UOWA operator is found if the aggregation is symmetric, strongly decaying and inclusive. It is symmetric if $w_j = w_{j+n-1}$. It is strongly decaying when $i < j \leq (n + 1)/2$, then $w_i < w_j$ and when $i > j \geq (n + 1)/2$ then $w_i < w_j$. It is inclusive if $w_j > 0$. Note that it is possible to consider different particular situations of this operator by softening the second condition with $w_i \leq w_j$ instead of $w_i < w_j$ and by removing the third condition.

Finally, if we assume that all the focal elements use the same weighting vector, then, we can refer to these families as the BS-uncertain maximum, the BS-uncertain minimum, the BS-UA, the BS-UWA, the BS-step-UOWA, the BS-S-UOWA, the BS-olympic-UOWA, the BS-window-UOWA, the BS-centered-UOWA, etc.

5 UHA Operator in Decision Making with D-S Theory

In some situations, the decision maker could prefer to use another type of uncertain aggregation operator such as the UHA operator. The main advantage of this operator is that it uses the characteristics of the UWA and the UOWA in the same aggregation. Then, if we introduce this operator in decision making with D-S theory, we are able to develop a unifying framework that includes in the same formulation probabilities, UWAs and UOWAs.

In order to use this type of aggregation in D-S framework we should consider that now in *Step* 3, when calculating the collection of weights to be used in the aggregation, we are using two weighting vectors because we are mixing in the same problem the UWA and the UOWA.

In *Step* 5, when calculating the uncertain aggregated payoff, we should use the UHA operator instead of the UOWA operator by using Eq. (2).

In this case, it is also possible to formulate in one equation the whole aggregation process. We will call it the BS-UHA operator.

Definition 5. *A BS-UHA operator is defined by*

$$C_i = \sum_{k=1}^{r} \sum_{j_k=1}^{q_k} m(B_k) w_{j_k} b_{j_k} \tag{10}$$

where w_{j_k} *is the weighting vector of the kth focal element such that* $\sum_{j=1}^{n} w_{j_k} = 1$ *and* $w_{j_k} \in [0,1]$, b_{j_k} *is the* j_k*th largest of the* $\hat{a}_i$ *(*$\hat{a}_i = n\omega_i\tilde{a}_i$*,* $i = 1,2,\ldots,n$*),* $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ *is the weighting vector of the* $\tilde{a}_i$*, with* $\omega_i \in [0, 1]$ *and the sum of the weights is 1, and the* $\tilde{a}_{i_k}$ *are interval numbers, and* $m(B_k)$ *is the basic probability assignment.*

As we can see, the focal weights are aggregating the results obtained by using the UHA operator. Note that if $\omega_i = 1/n$ for all i, then, Eq. (10) is transformed in Eq. (5).

In this case, we could also study different properties and particular cases of the BS-UHA operator, in a similar way as it has been explained for the BS-UOWA operator such as the distinction between descending (BS-DUHA) and ascending (BS-AUHA) orders.

When aggregating the collection of uncertain payoffs of each focal element, it is also possible to consider a wide range of families of UHA operators such as the uncertain hybrid maximum, the uncertain hybrid minimum, the uncertain Hurwicz hybrid criteria, the UA, the UWA, the UOWA operator, the olympic-UHA, the S-UHA, the centered-UHA, the median-UHA, etc.

6 Numerical example

In order to illustrate the new approach, we are going to develop a numerical example. We will consider a decision making problem about selection of strategies. We will develop the analysis considering different types of uncertain aggregation operators such as the UA, the UWA and the UOWA.

Assume a company that operates in Europe and North America is analyzing the general policy for the next year and it considers 3 possible strategies to follow: A_1, A_2, A_3.

In order to evaluate these strategies, the group of experts of the company considers that the key factor is the economic situation for the next year. Then, depending on the situation, the expected benefits for the company will be different. The experts have considered 5 possible situations for the next year: S_1 = Negative growth rate, S_2 = Growth rate near 0, S_3 = Low growth rate, S_4 = Medium growth rate, S_5 = High growth rate. The expected results are shown in Table 1. Note that the results are 3-tuple interval numbers or triplets.

After having analyzed the information in detail, the experts have found some probabilistic information about which state of nature will occur. This information is represented by the following belief structure about the states of nature.

Belief structure

$B_1 = \{S_1, S_2, S_3\} = 0.3$

$B_2 = \{S_2, S_3, S_4\} = 0.3$

$B_3 = \{S_3, S_4, S_5\} = 0.4$

The experts have established the following weighting vectors for both the UWA and the UOWA.

Weighting vector

$W = (0.3, 0.3, 0.4)$

$W = (0.1, 0.2, 0.2, 0.2, 0.3)$

Now, it is possible to calculate the aggregated payoffs that are shown in Table 2.

Table 1: Payoff matrix

	S_1	S_2	S_3	S_4	S_5
A_1	(20,30,40)	(50,60,70)	(30,40,50)	(60,70,80)	(60,70,80)
A_2	(60,70,80)	(40,50,60)	(30,40,50)	(70,80,90)	(30,40,50)
A_3	(50,60,70)	(50,60,70)	(40,50,60)	(30,40,50)	(40,50,60)

Table 2: Aggregated results

	UA	UWA	UOWA	UHA
V_{11}	(33.3,43.3,53.3)	(33,43,53)	(32,42,52)	(28,36,44)
V_{12}	(46.6,56.6,66.6)	(48,58,68)	(45,55,65)	(45,55,65)
V_{13}	(50,60,70)	(51,61,71)	(48,58,68)	(57,68.5,80)
V_{21}	(43.3,53.3,63.3)	(42,52,62)	(42,52,62)	(33,41,49)
V_{22}	(46.6,56.6,66.6)	(49,59,69)	(45,55,65)	(45,55,65)
V_{23}	(43.3,53.3,63.3)	(42,52,62)	(42,52,62)	(46.5,58,69.5)
V_{31}	(46.6,56.6,66.6)	(46,56,66)	(46,56,66)	(37,45,53)
V_{32}	(40,50,60)	(39,49,59)	(39,49,59)	(39,49,59)
V_{33}	(36.6,46.6,56.6)	(37,47,57)	(36,46,56)	(46.5,58,69.5)

Table 3: Uncertain generalized expected value

	UA	UWA	UOWA	UHA
A_1	(44,54,64)	(40.7,50.7,60.7)	(42.3,52.3,62.3)	(44.7,54.7,64.7)
A_2	(44.3,54.3,64.3)	(44.1,54.1,64.1)	(42.9,52.9,62.9)	(42,52,62)
A_3	(40.6,50.6,60.6)	(40.3,50.3,60.3)	(39.9,49.9,59.9)	(41.4,51.4,61.4)

Once we have the aggregated results, we have to calculate the uncertain generalized expected value. The results are shown in Table 3.

As we can see, depending on the uncertain aggregation operator used, the results may lead to different decisions. In this example, the optimal choice is A_2 if we use the UA, the UWA and the UOWA, and A_1 if we use the UHA.

7 Conclusions

We have studied the decision making problem with D-S theory of evidence when the available information is given in the form of interval numbers. We have seen that this method is simple and complete because it considers all the different situations that could happen in the problem. We have suggested the use of different types of uncertain aggregation operators in the D-S framework. As a result, we have obtained new aggregation operators that consider the D-S framework such as the BS-UOWA and the BS-UHA operators. We have analyzed some of the main properties of these aggregations such as the use of different families of UOWA and UHA operators in the decision problem. We have also developed an example where we have seen the applicability of the new approach.

In future research, we expect to continue developing new extensions of the decision making problem with D-S theory by introducing other aggregation operators and other types of information, and applying it in different decision making problems such as financial decision making or human resource selection.

References

[1] T. Calvo, G. Mayor, R. Mesiar (2002). *Aggregation Operators: New Trends and Applications*, Physica-Verlag, New York.

[2] A.P. Dempster (1967). Upper and lower probabilities induced by a multi-valued mapping, *Annals of Mathematical Statistics* 38, 325-339.

[3] K.J. Engemann, H.E. Miller, R.R. Yager (1996). Decision making with belief structures: an application in risk management, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* 4, 1-26.

[4] J.M. Merigó (2007). *New extensions to the OWA operators and their application in business decision making*, Unpublished thesis, Department of Business Administration, University of Barcelona.

[5] J.M. Merigó, M. Casanovas (2006). Ordered weighted geometric operators in decision making with Dempster-Shafer belief structure, In *Proceedings of the 13th SIGEF Congress*, Hammamet, Tunisia, pp. 709-727.

[6] J.M. Merigó, M. Casanovas (2007). Decision making with Dempster-Shafer theory using induced aggregation operators, In *Proceedings of the 4th International Summer School on AGOP*, Ghent, Belgium, pp. 95-100.

[7] J.M. Merigó, M. Casanovas (2007). Using fuzzy OWA operators in decision making with Dempster-Shafer belief structure, In *Proceedings of the 16th AEDEM International Conference*, Krakow, Poland, pp. 475-486.

[8] J.M. Merigó, M. Casanovas, L. Martínez (2007). Linguistic decision making using Dempster-Shafer theory of evidence, In *Proceedings of the 14th SIGEF Congress*, Poiana-Brasov, Romania, pp. 658-671.

[9] R.E. Moore (1966). *Interval Analysis*, Prentice Hall, Englewood Cliffs, NJ.

[10] G.A. Shafer (1976). *Mathematical Theory of Evidence*, Princeton University Press, Princeton, NJ.

[11] R.P. Srivastava, T. Mock (2002). *Belief Functions in Business Decisions*, Physica-Verlag, Heidelberg.

[12] Z.S. Xu, Q.L. Da (2002). The Uncertain OWA Operator, *International Journal of Intelligent Systems* 17, 569-575.

[13] Z.S. Xu, Q.L. Da (2003). An Overview of Operators for Aggregating the Information, *International Journal of Intelligent Systems* 18, 953-969.

[14] R.R. Yager (1988). On Ordered Weighted Averaging Aggregation Operators in Multi-Criteria Decision Making, *IEEE Transactions on Systems, Man and Cybernetics B* 18, 183-190.

[15] R.R. Yager (1992). On generalized measures of realization in uncertain environments, *Theory and Decision* 33, 41-69.

[16] R.R. Yager (1992). Decision Making Under Dempster-Shafer Uncertainties, *International Journal of General Systems* 20, 233-245.

[17] R.R. Yager (1993). Families of OWA operators, *Fuzzy Sets and Systems* 59, 125-148.

[18] R.R. Yager (1994). On weighted median aggregation, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* 2, 101-113.

[19] R.R. Yager (1996). Quantifier Guided Aggregation Using OWA Operators, *International Journal of Intelligent Systems* 11, 49-73.

[20] R.R. Yager (2004). Uncertainty modeling and decision support, *Reliability Engineering and System Safety* 85, 341-354.

[21] R.R. Yager (2007). Centered OWA operators, *Soft Computing* 11, 631-639.

[22] R.R. Yager, M. Fedrizzi, J. Kacprzyk (1994). *Advances in the Dempster-Shafer theory of evidence*, John Wiley & Sons, New York.

[23] R.R. Yager, J. Kacprzyk (1997). *The Ordered Weighted Averaging Operators: Theory and Applications*, Kluwer Academic Publishers, Norwell.