Contribution of DSm Approach to the Belief Function Theory

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Abstract

In this study we engage in belief processing which enables managing of multiple and overlapping elements of a frame of discernment. Our focus is directed on DSm approach, which was originally introduced as a generalization of the Dempster-Shafer theory. Paradoxically, later it was presented also as a special case of the Dempster-Shafer approach applied on such frame of discernment. In this paper we discuss what is new in DSm approach, what are the benefits of DSm approach, and what is its real contribution to belief function theory.

Keywords: Belief functions, Dempster-Shafer theory, DSm theory, Constraints, Overlapping elements, Exclusive elements, Non-separable elements.

1 Introduction

Belief functions are one of the widely used formalisms for uncertainty representation and processing that enable representation of incomplete and uncertain knowledge, belief updating, and combination of evidence. They were originally introduced as a principal notion of the Dempster-Shafer (D-S) Theory or the Mathematical Theory of Evidence [13].

For a combination of beliefs, Dempster's rule of combination is used in D-S theory. Since the Dempster-Shafer theory publication, a series of modifications of Dempster's rule was suggested and alternative approaches were created. The classical ones are Dubois-Prade's rule [10], Yager's rule of belief combination [18] and Smets' Transferable Belief Model (TBM) [16, 17].

A new approach is the Dezert-Smarandache Theory (DSmT) with its DSm rule of combination. There are two main differences: 1) mutual exclusivity of elements of a frame of discernment is not assumed in general; mathematically, this means that belief functions are not defined on the power set of the frame but on a so-called hyper-power set, i.e., on the Dedekind lattice defined by the frame; 2) a new combination mechanism which overcomes the problems with conflict among the combined beliefs and which also enables a dynamic fusion of beliefs.

The classical Shafer's frame of discernment with exclusive elements is considered the special case of a so-called hybrid DSm model.

On the other hand, as it was presented in [8], we can also consider the DSm approach a special case of the Dempster-Shafer theory working on a frame of discernment with overlapping elements. The same is true for both the basic DSm free model and any hybrid DSm models. To prove this, an outline of the Dempster-Shafer theory on frames with over-

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lapping elements was formalized [8].

Subsequently, the following questions arise: What is really new in the DSm approach? Do we need DSmT or is it better to translate everything to Dempster-Shafer working with overlapping elements? What is a contribution of the DSm approach? The discussion of these questions is presented in this study.

Useful preliminaries are reminded in Section 2. The third section briefly introduces DSmT. Two models of a case of D-S theory equivalent to DSmT are outlined in Section 4. Contributions of DSmT to belief function theory are outlined in Section 5: DSm hyper-power set, DSm hybrid models, dynamic fusion, new combination and conditioning rules. Section 6 presents notes on applications and computational complexity of DSmT.

2 Preliminaries

Basic notions of the classic theory of evidence [13] are defined on an exhaustive finite *frame of discernment* $\Theta = \{\theta_1, ..., \theta_n\}$, whose elements are mutually exclusive.

A *basic belief assignment (bba)* is a mapping $m : \mathcal{P}(\Theta) \longrightarrow [0, 1]$, such that $\sum_{A \subseteq \Theta} m(A) = 1$, $m(\emptyset) = 0$ (the values of bba are called *basic belief masses (bbm)*.[1] A *belief function (BF)* is a mapping $Bel : \mathcal{P}(\Theta) \longrightarrow [0, 1]$, $Bel(A) = \sum_{\emptyset \neq X \subseteq A} m(X)$. We will further use conjunctive combination rule, Dempster's combination and conditioning rules [13] and Dubois-Prade rule [10].

3 A Brief Introduction to DSm Theory

The theory is a new approach to BF processing which appeared in 2002 [9], and is in permanent dynamic evolution (see DSm books [14, 15]; a new volume is announced to appear in 2008).

[1] $m(\emptyset) = 0$ is often assumed in accordance with Shafer's definition [13]. A classical counter example is Smets' Transferable Belief Model (TBM) which admits $m(\emptyset) \geq 0$.
More generally, bbas and BFs can be defined in the same way on Borel fields (σ-fields) [12].

3.1 Dedekind Lattice, Basic DSm Notions

Dezert-Smarandache Theory (DSmT) by Dezert and Smarandache [9, 14] allows mutually overlapping elements of a frame of discernment. Thus, a frame of discernment is a finite exhaustive set of elements $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$, but not necessarily exclusive in DSmT. As an example, we can introduce a three-element set of colours $\{Red, Green, Blue\}$ from the DSmT homepage[2]: DSmT allows that an object can have 2 or 3 colours at the same time: e.g. it can be both red and blue, or red and green and blue in the same time, it corresponds to a composition of the colours from the 3 basic ones, see Figure 1.

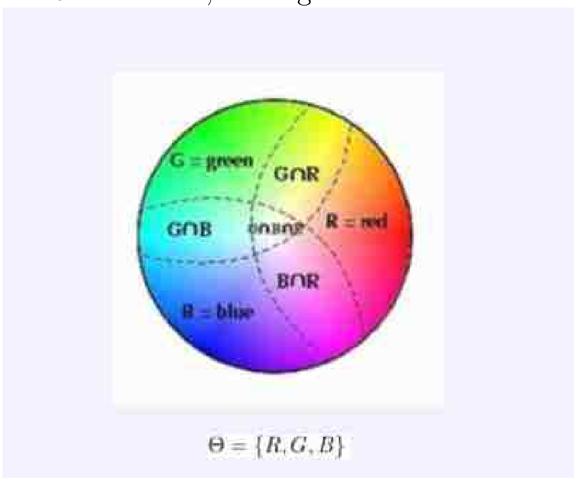

Figure 1: Three colour example of a hyper-power set (of the 3-element free DSm model).

DSmT uses basic belief assignments and belief functions defined analogically to the classic Dempster-Shafer theory, but they are defined on a so-called hyper-power set or Dedekind lattice (which does not satisfy the conditions of Borel field), instead of the classic power set of the frame of discernment.

The *Dedekind lattice*, more frequently called *hyper-power set* D^Θ in DSmT, is defined as the set of all composite propositions built from elements of Θ with union and intersection operators $\cup$ and $\cap$ such that $\emptyset, \theta_1, \theta_2, ..., \theta_n \in D^\Theta$, and if $A, B \in D^\Theta$ then also $A \cup B \in D^\Theta$ and $A \cap B \in D^\Theta$, no other elements belong to D^Θ ($\theta_i \cap \theta_j \neq \emptyset$ in general, $\theta_i \cap \theta_j = \emptyset$ iff $\theta_i = \emptyset$ or $\theta_j = \emptyset$).

[2] www.gallup.unm.edu/~smarandache/DSmT.htm

Thus the hyper-power set D^Θ of Θ is closed to $\cup$ and $\cap$ and $\theta_i \cap \theta_j \neq \emptyset$ in general, hence D^Θ is not a Borel field. Whereas the classic power set 2^Θ of Θ is closed to $\cup$, $\cap$ and complement, and $\theta_i \cap \theta_j = \emptyset$ for every $i \neq j$.

Examples of hyper-power sets. Let $\Theta = \{\theta_1, \theta_2\}$, we have $D^\Theta = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$, i.e., $|D^\Theta| = 5$. Let $\Theta = \{\theta_1, \theta_2, \theta_3\}$ now, we have $D^\Theta = \{\alpha_0, \alpha_1, ...\alpha_{18}\}$, where $\alpha_0 = \emptyset, \alpha_1 = \theta_1 \cap \theta_2 \cap \theta_3, \alpha_2 = \theta_1 \cap \theta_2, \alpha_3 = \theta_1 \cap \theta_3, ..., \alpha_{17} = \theta_2 \cup \theta_3, \alpha_{18} = \theta_1 \cup \theta_2 \cup \theta_3$, i.e., $|D^\Theta| = 19$ for $|\Theta| = 3$. Taking $\Theta = \{\theta_1, \theta_2, \theta_3\} = \{R, G, B\}$, this example corresponds to three colour example from Fig. 1.

To avoid a misunderstanding of Figure 1, we have to further note, that $G \cap B \cap R \subseteq G \cap R \subseteq G$, i.e., 'green' is represented by four fields in Figure 1, $G \cap R$ by two fields, and similarly for all other single colours and intersections.

We have to note, that this model is useful when processing pictures with continuous palette of colours. On the other side it is not adequate for the case when only 8 or 16 distinct colours is used, because we cannot address the single basic colours without simultaneous consideration of all its intersections with the other colours in the DSm free model. E.g., any belief mass assigned to red R is always plausible also to all its intersections $R \cap G \subset R$, $R \cap B \subset R$, and $R \cap G \cap B \subset R$. There is no complement, no negation in DSmT.

3.2 DSm Models

If we assume a Dedekind lattice (hyper-power set) according to the above definition without any other assumptions, i.e., all elements of an exhaustive frame of discernment can mutually overlap, we refer to the *free DSm model* $\mathcal{M}^f(\Theta)$, i.e. the DSm model free of constraints. In general, it is possible to add exclusivity or non-existential constraints into DSm models; in such cases we then speak about *hybrid DSm models.*

An *exclusivity constraint* $\theta_1 \cap \theta_2 \overset{\mathcal{M}_1}{\equiv} \emptyset$ says that elements θ_1 and θ_2 are mutually exclusive in model $\mathcal{M}_1$, whereas both of them can overlap with θ_3. If we assume exclusivity constraints $\theta_1 \cap \theta_2 \overset{\mathcal{M}_2}{\equiv} \emptyset$, $\theta_1 \cap \theta_3 \overset{\mathcal{M}_2}{\equiv} \emptyset$, $\theta_2 \cap \theta_3 \overset{\mathcal{M}_2}{\equiv} \emptyset$, another exclusivity constraint directly follows: $\theta_1 \cap \theta_2 \cap \theta_3 \overset{\mathcal{M}_2}{\equiv} \emptyset$. In this case all the elements of the 3-element frame of discernment $\Theta = \{\theta_1, \theta_2, \theta_3\}$ are mutually exclusive as in the classic Dempster-Shafer theory, and we call such hybrid DSm model as *Shafer's model* $\mathcal{M}^0(\Theta)$.

A *non-existential constraint* $\theta_3 \overset{\mathcal{M}_3}{\equiv} \emptyset$ brings a piece of additional information about a frame of discernment saying that θ_3 is impossible; it forces all the belief masses of $X \subseteq \theta_3$ to be equal to zero for any basic belief assignment in model $\mathcal{M}_3$. It represents a sure meta-information with respect to generalized belief combination which is used in a dynamic fusion.

3.3 The DSm Rules of Combination

There is a series of combination rules in DSmT. Originally in [9, 14], only the DSm classic and the hybrid DSm rules were considered, i.e. the generalized conjunctive rule and a slightly extended generalization of the Dubois-Prade rule, (for detail see [5]). For a possibility of better comparison of the DSmT with the classic Dempster-Shafer theory also Dempster's and Yager's rules were generalized to the DSm hyper-power sets [5]. In addition, a series of new rules was later defined [15].

4 An equivalent case in D-S theory

We briefly sketch k-tuple [8] and Hájek's propositional formula [11] models in this section. Further we have to mention also Cholvy's translation [2] of DSmT to D-S theory which is based on logical interpretations of the language Θ.

4.1 k-tuple model

Let us suppose that not just one element of Θ is effected as in the classic D-S case, but that at least one (i.e., one or several elements) can be effected simultaneously (a colour of an object can be composed from two or three basic ones, there can be not a single murderer, but a couple or group of them, etc., ...). Thus we suppose k-tuples $(t_1, ..., t_k)$ of elements of Θ, where $1 \leq k \leq n$. Bbas and BFs are defined on the power set of all these k-tuples. When assuming also 0-tuple or empty tuple $(\,)$, we obtain "double" power set $\mathcal{P}(\mathcal{P}(\Theta))$ of Θ. Nevertheless, we can simply ignore it as $m((\,))$ should always be equal to 0 (see [8] for detail). To obtain the free DSm model $\mathcal{M}^f$ as a special case of $\mathcal{P}(\mathcal{P}(\Theta))$ the *non-separability constraint* was defined in [8].

If we are not interested in $\mathcal{M}^f$ as a model (as a definition domain), but we are interested in BFs defined on it; we need that any focal element X contains with any r-tuple $r = (v_1, ..., v_r)$ also all s-tuples, $r \leq s \leq n$; $w = (w_1, ..., w_s)$ which includes v ($v_i \in w$ for all $i = 1, ...r$), v is subtuple of w, i.e. w is supertuple of v. Hence, we can simply take all BFs defined on double power set of Θ, such that $m(X) = 0$ for all X such that X does not contain all super tuples of its elements. This set of BFs is closed under intersection and union of focal elements, thus it is also closed under the conjunctive rule of combination as there are no conflicts there. Hence we have just the set of DSm BFs, but defined on $\mathcal{P}(\mathcal{P}(\Theta))$ this time, without need for any additive non-separability constraints.

To obtain a set of DSm BFs defined on hybrid DSm models, we have to adopt *exclusivity* and *non-existential* constraints.

4.2 Hájek's model

Bbas and BFs are defined on sets of formulas constructed from literals of propositional variables corresponding to elements of Θ. There are atomic formulas consisting of literals of all elements of Θ and conjunction &; e.g. $r\&g\&b$, $r\&\neg g\&b$, $r\&\neg g\&\neg b$, etc. Bbas and BFs are classically defined on the power set $\mathcal{P}(At)$ of all atomic formulas At. We can consider sets of atomic formulas as disjunctions; e.g., $\{r\&g\&b, r\&g\&\neg b\} \sim r\&g\&b \vee r\&g\&\neg b = r\&g$.

Thus we have a "double" power set of Θ again, this time the full one as we need also $\neg r\&\neg g\&\neg b$, because of formulas construction. Nevertheless we can define $m(\&_{i=1,...,n}\,\theta_i) = 0$. When defining $m(X) = 0$ for all X such that $\exists\theta_j$ in X which has only negated instances $\neg\theta_j$ in X, we obtain (after replacing & and $\vee$ with $\cap$ and $\cup$) the same set of bbas and BFs as they are defined on the free DSm model without using any additional type of constraint again.

4.3 'Differences' of DSmT

There are some features in DSmT which seem quite different from D-S approach, or completely inconsistent with D-S. As an illustrative example, we have to mention the fact that plausibility is always 1 when working with the free DSm model. It holds true that $Pl(\theta_i) = 1$ for any θ_i from any frame of discernment Θ.

This seems totally incompatible with D-S approach, but it is not. It is caused by the fact, that DSmT does not work with real singletons in fact. The real singletons are $\{(\theta_i)\}$ in k-tuple model or $\theta_i \,\&\, (\&_{j\neq i}\neg\theta_j)$ in Hájek's model. These singletons are not addressable in DSmT which works with sets $\{(t_1, ..., t_k)\,|\,\theta_i \in \{t_1, ..., t_k\}, k \leq n\}$ or $\{\&_{i=1,...,n} l_j\,|\, l_i = \theta_i$, and $l_j = \theta_j$ or $l_j = \neg\theta_j$ for $j \neq i\}$. The only real singleton the DSmT works with is $\bigcap_{i=1,...,n}\theta_i$. Its plausibility 1 reflects the fact, that is, indeed, the sum of plausibilities of all singletons which should be greater or equal to 1, and it holds true.

5 New features in DSmT

5.1 DSm hyper-power set

Hyper-power set D^Θ is simpler than "double power set" with non-separability constraints, and moreover, no additional definition constraints for BFs have to be supposed.

The same structure also represents different types of conflicts arising within combination of classic BFs. This structure was used in minC combination already in the first version of [4] in 2000, later formalized in [6]. In minC combination it represents internal working structure of belief masses assigned to conflicts when two or more classic belief functions are combined.

Even before, the structure was used by Ph. Besnard et al. [1] in 1996[3]. General mass functions and general belief functions are defined in [1] in the same way as the generalized bbms and BFs in DSmT (and also in the same way as in D-S theory).

5.2 DSm hybrid models — the most important contribution of DSmT

Constraints enable less size definition domain adequate for specific applications. Presenting a DSm hyper power set, we have recalled 3-element R-G-B example in Section 3. This free DSm model fits with pictures displayed on a screen or with photographs.

Black-R-G-B example: Let us suppose now a picture printed using Black, Red, Green, and Blue toners, where any pixel is either black or coloured, in such a way that its colour is composed from three basic colours R, G, B as it is above.

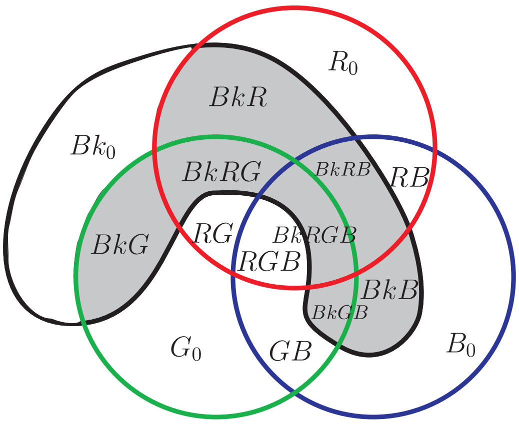

Figure 2: A general version of Black-R-G-B example of hybrid DSm model, $\Theta_4 = \{Bk, R, G, B\}$.

Hence we have either classic two-element frame of discernment $\Theta_2 = \{Bk, Cr\}$, when distinguishing only black pixels from coloured ones, or 4-element frame of discernment $\Theta_4 = \{Bk, R, G, B\}$ when distinguishing also all three overlapping colours of toners, as in Figure 2. Red $= R$ is union of 8 fields of the Figure: $R_0 \cup BkR \cup RG \cup RB \cup BkRG \cup BkRB \cup RGB \cup BkRGB$; $Bk \cap R$ is union of 4 fields: $BkR \cup BkRG \cup BkRB \cup BkRGB$; $Bk \cap R \cap G \cap B$ is the single field $BkRGB$, etc. But, in this case, we have a constraint that black pixel cannot be coloured and vice versa. I.e., we have constraints that $Bk \cap R \equiv \emptyset$, $Bk \cap G \equiv \emptyset$, $Bk \cap B \equiv \emptyset$ (or simply that $Bk \cap \{R, G, B\} = Bk \cap (R \cup G \cup B) \equiv \emptyset$), from the first equivalence we obtain also $Bk \cap R \cap G \equiv \emptyset$, $Bk \cap R \cap B \equiv \emptyset$, $Bk \cap R \cap G \cap B \equiv \emptyset$, and from the second or the third one also $Bk \cap G \cap B \equiv \emptyset$. Thus Black $= Bk \equiv Bk_0$.

When removing fields corresponding to the empty set (coloured grey in Figure 2) and 'smoothing' shape of the non-empty remainder, we can obtain a tuned version of the figure (Figure 3). Shapes of the fields, direct / undirect neighbouring of the fields or continuity/discontinuity of the graphical presentation do not play any role in either of the figures.

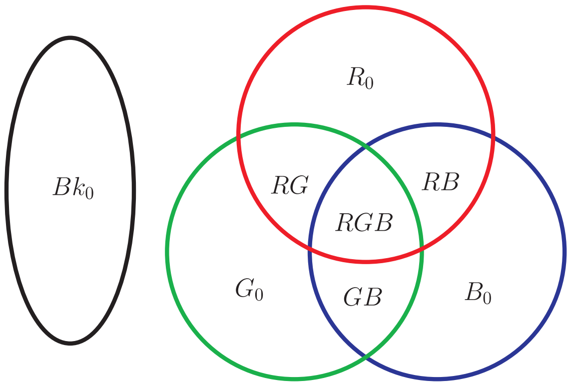

Figure 3: A tuned version of Black-R-G-B example.

We have to once more remind Besnard's approach: DSm hybrid models are closely related to Besnard's *evidential structure*, where the set of couples of contradictive elements of Θ is defined on $(\Theta, \wedge, \vee)$. Nevertheless, the interpretations of both very similar structures are different in these approaches. DSm hybrid models serve as a reduction of definition domain of generalized bbas and BFs, whereas the evidential structure serves to distinguish among different types of contradiction; here the noncontradictive BFs are defined as a new

[3] Besnard et al. do not use intersection and union of sets $\cap$, $\cup$, but a general distributive lattice $(\Theta, \wedge, \vee)$ with general lattice operations $\wedge$, $\vee$. Nevertheless, the structure $(\Theta, \wedge, \vee)$, which is called *propositional space* in [1] is the same as the DSm hyper-power set $D^\Theta = (\Theta, \cap, \cup)$.

notion of *consistent belief functions* [1].

5.3 Dynamic fusion of belief functions

DSmT enables also a dynamic fusion, this means that independently of a hybrid DSm model $\mathcal{M}$ in use, input BFs can be defined on the free DSm model $\mathcal{M}^f$ or in any hybrid DSm model with less constraints than $\mathcal{M}$ has[4], and the results of combination are in the given hybrid DSm model $\mathcal{M}$. This means that DSm combination rules include some kind of conditioning by used model $\mathcal{M}$.

This new feature of joining of combination and conditioning by model would be possible to be replaced by step-wise application of the static combination and of the corresponding conditioning by the given DSm model $\mathcal{M}$. Nevertheless, despite such a potential simplification, this is the contribution of DSmT as it arises from various hybrid DSm models.

5.4 New combination and conditioning rules

A series of combination rules has been defined in DSmT since its appearance. The original classic and hybrid DSm combination rules [9, 14] were recognized to be the conjunctive and Dubois-Prade rules of combination, which are defined on the free or hybrid DSm models respectively, see [5].

Besides these, there is a series of new combination rules defined in DSm book 2 [15]: Proportional Conflict Redistribution (PCR) rules PCR1, ..., PCR5 (Smarandache & Dezert) and generalized PCR rules (Martin & Osswald), the Conflict free rule (Dambreville) and qualitative operators q-DSmC, q-DSmH and q-PCR (Smarandache & Dezert). These rules should be analysed from the point of view of the actual relation of DSm and D-S approaches to evaluate which ones bring a real enrichment of the belief function theory.

There is also a long series of 31 Belief Conditioning Rules (BCR) in DSmT (Smarandache & Dezert) [15]. It is a series of complicated combinatorically defined formulas without any deep analysis and comparison with Dempster's conditioning. A real contribution should be performed by two of them BCR12 and BCR17 recommended by their authors. Moreover, BCR12 is claimed to be a generalization of Dempster's conditioning, but unfortunately it has been observed, that it does not hold true in full generality [7]. Thus BCR rules should be carefully analysed and compared with the classic Dempster's conditioning rule.

[4]In the case of inputs from more constrained DSm models there is no problem (no conflict with model $\mathcal{M}$) and the static combination is sufficient.

5.5 Other DSmT contributions

There is a lot of other theoretic aspects studied in DSmT (see Parts I of both the volumes of DSm book). Some of them are brand new, some of them are related to problems which have already been solved in D-S theory.

5.6 Summary of DSmT contribution

There are many other results and techniques in DSmT which should be compared with those used in classic belief function processing in a new light shed to DSmT in [8]. Nevertheless, we can agree with Prof. B. Solaiman that DSmT gives several good tools for engineering applications, see Preamble of DSm book 2 [15].

6 Applications and Complexity

6.1 Applications of DSmT

A variety of applications of DSmT from various application areas was published in Parts II of both volumes of DSm book [14, 15]. Majority of real applications deals only with 2 or 3-element frames of discernment or the size of frame is not specified. This is not a case of Martine & Osswald application of generalized PCR rules to sonar imagery and radar targets, see Chap. 11 in [15]. The authors use 7-element frame $\Theta = \{A, B, C, D, E, F, G\}$, where they note A = rock, B = cobble, C = sand, D = sild, E = ripple, F = shadow, and G = other to differentiate a type of sediment. Used hybrid model(s) is (are) not specified.

Unfortunately, there is no specific hybrid DSm model explicitly presented in Parts II of both volumes of DSm book [14, 15], only the free DSm model, Shafer's model, and 'any' hybrid DSm models are referred there.

There are neither any specific constraints presented there. With the exception of academic Tweety problem which is solved on 4-element frame of discernment $\Theta = \{b, p, f, \overline{f}\}$ with the constraint $f \cap \overline{f} \equiv \emptyset$, see Chap. 12 in [15].

6.2 Computational complexity

Computational complexity of DSmT is dependent on the size of the domain which exponentially grows with the number of elements in Θ, it is already problem of Dempster-Shafer theory which works on $\mathcal{P}(\Theta)$ and, naturally, it is significantly worse for the D-S theory on double power set $\mathcal{P}(\mathcal{P}(\Theta))$, which cardinality is 2^{2^n} in general, see Table 1, as it is used in Hájek's model or $|\mathcal{P}(\mathcal{P}(\Theta)\setminus\{\emptyset\})\setminus\{\emptyset\}| = 2^{2^n-1} - 1$ as in k-tuple model.

Table 1: Cardinalities of domains.

$\|\Theta\|=n$	$\|\mathcal{P}(\Theta)\|$	$\|D^\Theta\|$	$\|\mathcal{P}(\mathcal{P}(\Theta))\|$
2	4	5	$2^4=16$
3	8	19	$2^8=256$
4	16	167	$2^{16}=65536$
5	32	7580	$2^{32}=4294967296$

A question arises, whether DSmT is applicable for more than 3-element frames in practice. The enormous cardinality holds true in general, when general belief functions are processed. When working with a special class of BFs the cardinalities decrease and DSmT can applied.

We have to mention here a Djiknavorian & Grenier's reduction of DSmH rule (Chap. 15 in [15]), where a reduction algorithm, a MatLab tool, and dependence of execution parameters on $|\Theta|$ are presented. The n-ary combination was tested with regards to $|\Theta| = n$, number of sources s, number of focal elements f, execution time t and memory size m, and graphical results were presented there.

Time t versus n (or s) is exponential for $s=5$ (resp. $n=5$), $n=3$–9 (resp. $s=3$–9) for BFs with $f=6$ focal elements. Time t versus f is almost linear for $n=3$, $s=5$ for BFs with $f=3$–9 focal elements. Memory size m versus n is 'acceptable' for $s=5$, $n=3$–9, $f=6$; memory m versus s is linear for $n=5$, $s=3$–9, $f=6$.

Thus BFs with only 6 focal elements are processable upto 9 elements even for n-ary combination of 5 sources, for detail see Chap. 15 in [15]. Unfortunately, this nicely presented testing was performed only for theoretical academic data, not for data from any real application.

6.3 Application summary

From the above results we can see that DSmT is, in spite of its very high general theoretical computational complexity, applicable to adequate real applications.

A new challenge for presentation of DSmT is presentation of several real hybrid DSm models to completely remove any doubt whether DSm hybrid models are useful for real applications or a mathematical game only. Another challenge is to present some complexity testing analogous to Djiknavorian & Grenier's one for some real application data.

7 Conclusion

We have outlined two alternative representations of DSmT within the general framework of the Dempster-Shafer approach and pointed out the important differences and new contributions of DSmT to belief processing.

DSm approach offers many different contributions which enrich the belief function theory. But it seems rather a model within general framework of Dempster-Shafer or evidence theory than a new different standalone theory itself.

DSm approach can be considered as a General DSm Model which includes a series of specializations of its definition domain, the free DSm model and various hybrid DSm models. Nevertheless, we always have to keep in mind that DSmT performs a special model of the theory of belief functions in full generality.

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