Possibilistic Influence Diagrams Using Information Fusion

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Abstract

This paper proposes a new approach for decision making under uncertainty based on influence diagrams and possibility theory. The so-called *possibilistic influence diagrams* extend standard influence diagrams in order to avoid difficulties attached to the specification of both probability distributions relative to chance nodes and utilities relative to value nodes. In fact, generally, it is easier for experts to quantify dependencies between chance nodes via possibility distributions and to provide a set of numerical utilities and a possibility distribution relative to each consequence and each utility.

Keywords: decision theory, influence diagrams, possibility theory.

1 Introduction

Graphical decision models provide efficient decision tools. In fact, it allow a compact and a simple representation of decision problems under uncertainty. Influence diagrams (IDs) (Howard and Matheson, 1981) are a popular framework representing a decision maker's belief and preferences about a sequence of decisions to be made under uncertainty [8]. An ID is composed by a graphical component which is a directed acyclic graph (DAG) and a numerical component quantifying this DAG.

In this paper, we are interested in these decision models initially proposed by [8]. The evaluation of IDs generates optimal decisions while maximizing the decision maker's expected utilities. Within proposed evaluation algorithms, we can distinguish direct methods [12] which operate directly on IDs or indirect methods [3, 10, 11, 15] which transform them into a secondary structure and then evaluate these structures.

The quantification of IDs can be done by experts, in such a case they express their uncertainty relative to variables by probability distributions and their preferences through utilities. Nevertheless, in most real problems it is not obvious to provide exact probability distributions and it is easier to express uncertainty qualitatively by ranking different states of the world.

Moreover, decision makers may encounter several difficulties when expressing their utilities and it may be more flexible to allow them providing a set of utilities and a possibility distributions relative to each utility and each consequence.

In such situations, standard IDs cannot be applied, thus, our idea is to extend them using a non-classical theory of uncertainty for specifying their numerical component. Namely, we have opted to use possibility theory, initially proposed by Zadeh [14] and developed by Dubois and Prade [4] since it offers a natural and simple framework to handle imperfect data.

In previous works, we have developed qual-

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itative possibilistic IDs [7] i.e. those where dependencies between chance nodes are expressed via qualitative possibility distribution and value nodes are quantified using ordinal utility. In addition, we have proposed qualitative possibilistic IDs based on qualitative binary possibilistic utility [6] when the decision maker should provide a preferential relation between different consequences, an ordinal scale and he should classify himself as either pessimistic or optimistic or neutral.

In this paper, we will develop another variant of possibilistic IDs where decision makers can provide a set of utilities and a possibility distribution relative to each utility and each consequence.

The success of indirect methods in the standard framework, has motivated us to propose an indirect method to evaluate these models. More precisely, the proposed evaluation method is based on the transformation of possibilistic IDs into possibilistic networks [1] and on making inference in this secondary structure using the appropriate propagation algorithms.

This paper is organized as follows: Section 2 provides the necessary background on possibility theory. Section 3 presents possibilistic IDs. Section 4 proposes an indirect evaluation method to generate optimal decisions. Finally, section 5 proposed a conclusion and future work.

2 Background of possibility theory

Possibility theory was initially proposed by Zadeh [14] and was developed by Dubois and Prade [4]. This section briefly recalls basic elements of possibility theory, for more details see [4].

The basic buildings block in the possibility theory is the notion of possibility distribution denoted by π, it is a mapping from the universe of discourse denoted by $\Omega = \{\omega_1 ... \omega_n\}$ to the unit interval $[0, 1]$.

This scale has two interpretations, a quantitative one when the handled values have a real sense and a qualitative one when the handled values reflect only an ordering between the different states of the world. In the first case, the *product operator* can be applied while in the second one, the *min operator* is used.

A possibility degree is the value from the interval $[0, 1]$ associated to each element ω of Ω. The possibility measure of any subset $\psi \subseteq \Omega$ is defined as follows:

$$\Pi(\psi) = max_{\omega \in \psi} \pi(\omega) \qquad (1)$$

A possibility distribution is said to be normalized, if $max_{\omega \in \psi} \pi(\omega) = 1$.

In the possibilistic framework, extreme forms of partial knowledge can be represented by *Complete knowledge* i.e. $\exists \omega_i \in \Omega, s.t$
$\pi(\omega_i) = 1$ *and* $\omega_j \neq \omega_i, \pi(\omega_j) = 0$ and the *total ignorance* i.e. $\forall \omega_i \in \Omega, \pi(\omega_i) = 1$.

In the possibilistic approach, there are different combinations modes to assure the fusion of informations. The choice of the appropriate combination mode is related to the reliability of information's sources. The most known combination operators are the symmetric ones, namely the *conjunctive* and the *disjunctive* operators:

1. **The conjunctive fusion:** If all sources are reliable, then we can combine them using the intersection, the conjunctive operator $\otimes$ is defined as follows:

   $$\forall\ \omega \in \Omega, \quad \pi_{\wedge}(\omega) = \bigotimes_{i=1..n} \pi_i(\omega) \qquad (2)$$

   where π_i be the possibility distribution supplied by source i.

   $\otimes$ is a t-norms such that minimum or product or linear product according to the uncertainty scale's interpretation. Indeed, the min operator is supported by both quantitative and qualitative possibility distributions.

   However, the use of the product operator assumes that possibility degrees are numerical.

2. **The disjunctive fusion:** This mode of combination is applied when it is known for sure that at least one of the sources

is reliable but it is not known which one. The disjunctive operator $\bigoplus$ is defined as follows:

$$\forall\ \omega \in \Omega,\quad \pi_{\vee}(\omega) = \bigoplus_{i=1..n} \pi_i(\omega) \qquad (3)$$

$\bigoplus$ is a t-conorms such that maximum or probabilistic sum or Lukasievicz according to the uncertainty scale's interpretation. Indeed, all of these t-conorms can be applied in the quantitative setting. However, only the maximum operator can be applied in the qualitative setting.

The conditioning represents a special case of informations fusion. Indeed, it consists in revising our initial knowledge, represented by a possibility distribution π, which will be changed into another possibility distribution $\pi' = \pi(.|\psi)$ with $\psi \neq \emptyset$ and $\Pi(\psi) > 0$.

The two interpretations of the possibilistic scale induce two definitions of the conditioning:

- Min-based conditioning relative to the ordinal setting:

$$\pi(\omega|_m\psi) = \begin{cases} 1 & if\ \pi(\omega) = \Pi(\psi)\ \ and\ \ \omega \in \psi \\ \pi(\omega) & if\ \pi(\omega) < \Pi(\psi)\ \ and\ \ \omega \in \psi \\ 0 & otherwise \end{cases} \qquad (4)$$

- Product-based conditioning relative to the numerical setting:

$$\pi(\omega|_p\psi) = \begin{cases} \frac{\pi(\omega)}{\Pi(\omega)} & if \omega \in \psi \\ 0 & otherwise \end{cases} \qquad (5)$$

3 Possibilistic influence diagrams

Few works exist on possibilistic networks and existing ones concern reasoning under uncertainty without considering the decision aspect [1, 2].

Recently, Sabbadin et al. [5] have proposed possibilistic IDs using *optimistic and pessimistic utilities* [4] for the quantification of value nodes. Nevertheless, Giang et al. [6] noted that this utility framework is based on axioms relative to uncertainty attitude contrary to the VNM axiomatic system [9] based on risk attitude, which does not make a sense in the possibilistic framework since it represents uncertainty rather than risk. Moreover, to use pessimistic and optimistic utilities, the decision maker should classify himself as either pessimistic or optimistic which is not always obvious. To overcome these limitations, Giang et al. [6] propose a more generalized framework based on the axiomatic system of *possibilistic binary utility.*

In order to benefit from the simplicity of standard IDs and from the suitability of possibility theory for modeling qualitative uncertainty and utility, we have defined possibilistic influence diagrams [7].

Possibilistic IDs are a possibilistic adaptation of standard IDs, as the latter they have two components:

1. **A graphical component** defined by a directed acyclic graph (DAG), denoted by $G(N, A)$, where N is the set of chance, decision and value nodes and A is the set of arcs in the directed graph.

2. **A numerical component** evaluating different dependencies between chance nodes and utilities for value nodes.

    - For each chance node C_i, we should provide conditional possibility degree $\Pi(c_{ij} \mid pa(C_i))$ of each instance c_{ij} of C_i in the context of each instance of its parents. In order to satisfy the normalization constraint, these conditional distributions should satisfy, $\forall pa(C_i)$:

    $$max_{c_{ij}} \Pi(c_{ij} \mid pa(C_i)) = 1, \qquad (6)$$

    Note that for root chance nodes (i.e. $(Pa(C_i) = \emptyset)$, 6 corresponds to $max_{c_{ij}} \Pi(c_{ij}) = 1$.

    - For each value node V_i, there are several ways to represent decision maker's preferences on the set of consequences, namely using cardinal utility, ordinal utility, possibilistic utility or as well as a compound utility.

Note that likewise standard IDs, decision nodes in possibilistic IDs are not quantified.

Different combinations between the quantification of chance and utility nodes offer several kinds of possibilistic IDs which can be regrouped into three principal classes:

- *Product-based possibilistic IDs* where both dependencies between chance nodes and value nodes are quantified in a genuine numerical setting.
- *Min-based possibilistic IDs or qualitative possibilistic ID* where both dependencies between chance nodes and value nodes are quantified in a qualitative setting used for encoding an ordering between different states of the world.
- *Mixed possibilistic IDs* where dependencies between chance nodes and value nodes are not quantified in the same setting.

Product-based and min-based possibilistic IDs represent homogeneous possibilistic IDs and mixed possibilistic IDs are the heterogeneous ones.

In a previous study, we have developed qualitative possibilistic IDs which represent one case of homogeneous ones [7].

In this work, we propose another case of these possibilistic IDs when dependencies between chance nodes are expressed by a quantitative possibility distributions.

For each value node, the decision maker should provide a set of numerical utilities, denoted by UT, which can occured.

However, the decision maker is unable to determine the exact value of utility of each consequence. Namely, they can't affect each utility UT_i from the set UT to the appropriate consequence.

Indeed, the decision maker should define a possibility distributions relative to each consequence x and to each utility UT_i.

These possibility distributions should satisfy, $\forall x \in X$:

$$max_{UT_i \in UT} \Pi(U(x) = UT_i) = 1 \quad (7)$$

Example 1 *Let us state a simple decision problem represented by a possibilistic ID as represented in figure 1. It contains 3 chance nodes (A, B, C), 1 decision node (D) and 1 value node (V).*

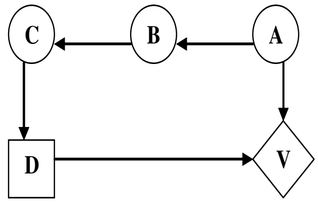

Figure 1: An example of influence diagram

The possibility distributions for the chance nodes A, B and C are presented in table 1.

For the utilities, DMs affirm that the possible values of utilities are {4, 7, 8, 10}.

For the sake of simplicity we will denote $(U(A, D) = 4)$ by U_1, $(U(A, D) = 7)$ by U_2, $(U(A, D) = 8)$ by U_3 and $(U(A, D) = 10)$ by U_4.

Table 1: A priori and conditional possibility distributions for chance nodes

A	$\Pi(A)$	A	B	$\Pi(B\|A)$	B	C	$\Pi(C\|B)$
T	1	T	T	0.9	T	T	1
F	0.6	F	T	0.2	F	T	0.3
		T	F	1	T	F	0.2
		F	F	1	F	F	1

The possibility distribution relative to each consequence and utility is represented by table 2:

Table 2: The possibility distribution $\Pi(U(A, D) = UT_i)$

A	D	$\Pi(U_1)$	$\Pi(U_2)$	$\Pi(U_3)$	$\Pi(U_4)$
T	d_1	0.2	0.1	0.3	1
F	d_1	1	0.1	0.2	0.1
T	d_2	0.6	0.1	0.3	1
F	d_2	0.5	0.1	0.3	1

4 Evaluation of possibilistic influence diagrams

Given a possibilistic ID, we should evaluate it in order to generate optimal decisions. As we have mentioned in the introduction, there are two approaches to evaluate standard IDs, namely, direct and indirect ones.

The evaluation of possibilistic IDs, proposed in [5], is based on an indirect evaluation method which transforms them into decision trees. Such evaluation method was not successful in the probabilistic framework since, contrary to those based on Bayesian networks, it does not use independencies encoded by IDs to save some computations since decision trees are not able to represent independencies [15]. This argument remains available in the possibilistic framework, as it only concerns the graphical component which is the same in the two frameworks.

In addition, direct evaluation methods [12] require heavy computations since they are based on arc reversal and node deletion, contrary to indirect ones which are based on the transformation of IDs into Bayesian networks. This explains the great development of indirect methods in the probabilistic case [3, 10, 11, 15].

The success of indirect evaluation methods for standard IDs, motivates us to develop an indirect evaluation method for possibilistic IDs. Our choice is reinforced by the fact that a possibilistic counterpart of Bayesian networks has been developed as well as their propagation algorithms [1].

More precisely, we will develop a possibilistic counterpart of Cooper's method [3] for the particular case of influence diagram with a unique value node, since it represents the basis of existing indirect methods.

Thus, the principle of our evaluation algorithm is to transform decision and value nodes into chance nodes in order to obtain a possibilistic network, and then to use this secondary structure to compute maximal expected utilities via a propagation process. These two major phases are detailed in what follows.

4.1 Transformation phase

This phase consists in transforming decision and value nodes into chance nodes.

4.1.1 Decision nodes transformation

Each decision node D_i in the possibilistic ID is transformed into a chance node which should be quantified. In the probabilistic case, this quantification is ensured by an equi-probable distribution. Nevertheless, this is not really appropriate, since equi-probability represents randomness rather than total ignorance. This problem can be overcome in the possibilistic framework where our ignorance about the new chance node can be suitably represented via a uniform possibility distribution. More formally:

$$\Pi(d_{ij} \mid_p pa(D_i)) = 1, \quad \forall d_{ij}, pa(D_i) \qquad (8)$$

Example 2 *The ID presented in figure 1 has one decision node D. The possibility distribution of the new chance node D obtained by 8 is presented in table 3:*

Table 3: The possibility distribution $\Pi(D \mid C)$

C	D	$\Pi(D \mid C)$
T	d_1	1
F	d_2	1
T	d_2	1
F	d_1	1

4.1.2 Value node transformation

The first step is to transform numerical utilities (UT_i) into a possibility distribution by rescaling the set of numerical utilities into the unit interval [0,1] as follows, $\forall pa(V) \in Pa(V)$:

$$\Pi(v = T \mid_p pa(V)) = \frac{UT(pa(V)) - UT_{min}}{UT_{max} - UT_{min}} \qquad (9)$$

$$\Pi(v = F \mid_p pa(V)) = \frac{UT(pa(V)) - UT_{max}}{UT_{min} - UT_{max}} \qquad (10)$$

where UT_{max} and UT_{min} are the maximal and the minimal utility levels in UT.

Indeed, we have two informations about each consequence x, since it is characterized by two possibility degrees i.e. the new possibility distribution issued by applying (9 and 10) the possibility distribution $\Pi(U_i)$ given by DMs.

Our idea is to merge these two informations using the product operator because it concerns a conjunctive fusion in quantitative setting presented in section 2.

Then, for each consequence we have several possibility levels relative to each value of utility. Namely, the number of possibility levels is equal to the number of utility in the set UT.

To combine these several possibility levels, the max operator will be used as it's matter of a disjunctive fusion in a quantitative setting as presented in section 2.

Note that the resulting possibility distribution relative to the new chance node V may be sub-normalized. In order to satisfy the normalization constraint, the obtained possibility distribution should be transformed as follows: $\forall pa(V) \in Pa(V)$, $\forall v \in \{T, F\}$

If $max(\Pi(v|_p pa(V)), \Pi(\neg v|_p pa(V))) = \Pi(v|_p pa(V))$

$\Rightarrow \Pi(v|_p pa(V)) = 1.$

Otherwise: $\Rightarrow \Pi(v|_p pa(V)) = \frac{\Pi(v|_p pa(V))}{\Pi(\neg v|_p pa(V))}$

Example 3 *Let us present the transformation of the possibilistic ID presented in example 1. The obtained possibilistic network is presented in figure 2.*

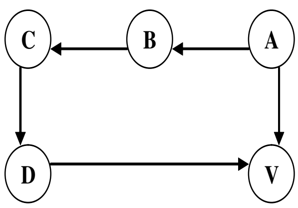

Figure 2: The obtained possibilistic network

At the beginning, the set of numerical utilities is transformed into a possibility distribution using 9 and 10 as presented in table 4.

As we have said before, each consequence has two informations: $\Pi(V \mid A, D)$ *and* $\Pi(U_i)$ $\forall i \in \{1, 2, 3, 4\}$.

Table 4: The transformation of the utilities into a possibility distribution

UT(A,D)	V	$\Pi(V \mid A, D)$
4	T	0
7	T	1/2
8	T	2/3
10	T	1
4	F	1
7	F	1/2
8	F	1/3
10	F	0

Let us denote $\Pi(V \mid A, D) \wedge \Pi(U_i)$ *by* Π_{V_i}, $\forall i \in \{1, 2, 3, 4\}$.

Namely, Π_{V_i} *represents the possibility distribution issued by the conjunctive fusion of these informations using the product operator as presented in table* 5.

Table 5: The conjunctive fusion

V	A	D	Π_{V_1}	Π_{V_2}	Π_{V_3}	Π_{V_4}
T	T	d_1	0	0.05	0.2	1
T	F	d_1	0	0.05	0.13	0.1
T	T	d_2	0	0.05	0.2	1
T	F	d_2	0	0.05	0.2	1
F	T	d_1	0.2	0.05	0.1	0
F	F	d_1	1	0.05	0.066	0
F	T	d_2	0.6	0.05	0.1	0
F	F	d_2	0.5	0.05	0.1	0

For each consequence we have four choices (as presented in table 5*), the max operator will be used for the disjunctive fusion as presented in table* 6.

Table 6: The possibility distribution of the value node V

V	A	D	$\Pi(V \mid A, D)$
T	T	d_1	1
T	F	d_1	0.13
T	T	d_2	1
T	F	d_2	1
F	T	d_1	0.2
F	F	d_1	1
F	T	d_2	0.6
F	F	d_2	0.5

4.2 Propagation phase

The possibilistic network issued from the transformation phase can be used to generate optimal decisions by computing the Maximal Expected Utility (MEU) relative to each decision node. This computation is ensured by selecting and applying the appropriate propagation algorithm according to the DAG structure.

The computation of the MEU starts by the last decision node D_m to the first one D_1. For the node D_i, we should integrate already computed optimal decisions i.e. those relative to $D_1, .., D_{i-1}$. More formally, for each decision D_i, let:

$$P(D_i, E) = (\Pi(v = T \mid Pa(V))\Pi(Pa'(V) \mid d_{ij}, E)).$$

where $Pa'(V)$ denotes the set of chance nodes in $Pa(V)$ and E denotes the set of evidence.

Note that $\Pi(pa'(V) \mid d_{ij}, E))$ is computed via the product-based propagation algorithm in quantitative possibilistic networks. Indeed, two product-based propagation algorithms have been defined according to the nature of the DAG in the possibilistic causal network [1]. Namely, the possibilistic adaptation of the centralized version of Pearl's algorithm is used when the DAG is singly connected, and the possibilistic adaptation of junction trees propagation are appropriate for multiply connected DAGs.

Once $P(D_i, E)$ is computed for each decision D_i we can compute the MEU as follows:

$$MEU(D_i, E) = max_{d_{ij}} \Sigma_{pa'(V)} P(D_i, E) \quad (11)$$

Example 4 *Let us continue with the same example. Suppose that we receive a certain information saying that the variable C takes the value T.*

Since the obtained possibilistic network presented in figure 2 is a multiply connected DAG, the possibilistic adaptation of junction trees propagation is used to compute $\Pi(A \mid D, C = T)$ as presented in table 7.

Finally, these values are used to apply equation (11) to compute the MEU which is equal to 0.81. Then, we can conclude that the optimal decision D^* is d_1.

Table 7: The computation of $\Pi(A \mid D, C = T))$

A	D	C	$\Pi(A \mid D, C = T))$
T	d_1	T	1
F	d_1	T	0.2
T	d_2	T	1
F	d_2	T	0.19

5 Conclusion

This paper proposes a new approach for decision making under uncertainty using IDs in the possibilistic framework.

Indeed, dependencies between chance nodes are quantified using possibility distributions. Then, decision makers should define for each value node a set of possible numerical utilities which are characterized by a possibility distribution relative to each consequence and each utility.

To evaluate these possibilistic IDs, we have proposed an indirect evaluation method based on information fusion in the possibilistic setting and on Cooper's evaluation method.

The proposed approach, has been implemented in a *Possibilistic Influence Diagram Toolbox (PIDT)* which can be seen as a decision support system.

As a future work, we can first distinguish a direct improvement of our proposal concerning possibilistic IDs with several value nodes to deal with multi objective decision problems when uncertainty is modeled in a possibilistic setting.

An interesting line of research will be to extend our work to mixed influence diagrams in order to treat the case where experts and decision makers are heterogeneous i.e. they express their uncertainty in both qualitative and quantitative setting.

Another line of research will be to use our possibilistic decision models in a real decision problem. We are in particular interested by the semantic web domain.

Indeed, an interesting work about the construction of a retrieval model based on IDs has been developed [13]. This model aims to ensure the efficiency and the personalization of the research process in the web.

In fact, the presented model is based on standard influence diagrams in order to provide to the user a suitable and a pertinent information according to his requirement.

Our idea is to use possibilistic IDs to construct this model, since the user is not able to define his utility by numerical values.

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