

Detection of defective sources with belief functions

Francois Delmotte
LAMIH, UVHC,
Le mont Houy
59313 valenciennes cedex 9
France
Francois.delmotte@univ-
valenciennes.fr

David Gacquer
LAMIH, UVHC,
Le mont Houy
59313 valenciennes cedex 9
France
david.gacquer@univ-
valenciennes.fr

Abstract

This paper studies the fusion of several sources with belief functions. Different operators have been defined but they have problems with conflicting data: rules are either very imprecise, or very sensitive. Discounting factors enable to weight the influences of sources, and solve some problems, but we have to estimate correctly these factors. We propose to estimate them from the conflicts between the sources and from past knowledge about the qualities of sources. With the assumption that conflicts come from defective sources, an algorithm is proposed to detect such sources and to lower conflicts

Keywords: belief functions, fusion.

1 Introduction

With belief functions we can represent uncertain variables, mix up several sources of information, and take decisions [1,17, 21]. This paper deals with the fusion problem [4, 5, 13, 20].

Several rules of fusion have been defined. For example a conjunction can be used, when we assume that all sources are reliable. Usually fusion rules cannot really cope with conflicting information, because results are very imprecise or undefined. Read [23] for a survey as well as a classification of many fusion rules. In this paper we propose to use the rule of Dempster which has firm theoretical roots. However, prior to the fusion step, we use the discounting approach to alter the sources when they are unreliable. This approach associates to each source a discounting factor to quantify the degree of confidence we have in it. This way we can precisely weight the

influence of a source, and this may solve some problems. However the meaning of such a discounting factor is a complex issue: on the one hand, when a fusion is carried out, a source can agree or disagree with the others. If sources agree, we will have a better confidence in what they claim, and they will get a better reliability. On the other hand, we may have at a given time some past knowledge about the quality of a source.

Usually in the literature these two notions are kept away and the discounting approach does not handle very well conflicting sources if it is based on wrong discounting factors (in case of defective sources for example). In this paper we claim that both static and dynamic reliabilities must be used in order to correctly estimate discounting factors. An algorithm combining these two reliabilities is presented in order to improve fusions.

The mechanism was previously introduced in paper [2] dealing with possibility theory. The difference between static and dynamic reliabilities was also introduced independently by Gaa et al [8] who speak of static and dynamic discounting factors. They also introduced a slightly different combination between static and dynamic reliabilities.

However their paper is mainly focused on the estimation of the static reliability by a training step, and they compare their results with other approaches. The dynamic component of the reliability is used in a single example without any further analysis.

In this paper we propose a mechanism detecting defective sources. Indeed it is assumed that conflicts come from defective sources: those having a high reliability factor, but also a high conflict with other reliable sources.

The second part presents the rule of Dempster and similarity relations between beliefs. We assume that readers are familiar with belief functions. The third part presents the discounting approach and the new algorithm. Then two examples are tackled. The first aims to compare previous rules of fusion. The second involves sensors with time-varying static reliabilities. This example highlights the detection part of our algorithm.

2 Belief functions

Given a frame of discernment X and two basic belief masses m_1 and m_2 , a belief m can be computed by the conjunctive rule \oplus^* [17]:

$$\forall A \subset X$$

$$m_{\oplus^*}^*(A) = \sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j) \quad (1)$$

The closed world assumption is taken. Therefore a step of normalization is introduced with the rule of combination of Dempster \oplus :

$$\forall A \subset X$$

$$m_{\oplus}(A) = \frac{\sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j)}{1 - \sum_{A_i \cap B_j = \emptyset} m_1(A_i) m_2(B_j)} \quad (2)$$

The amount $C(m) = m_{\oplus}^*(\emptyset) = \sum_{A_i \cap B_j = \emptyset} m_1(A_i) m_2(B_j)$

is introduced to assess in what extent sources are conflicting. When this amount is too high, it is possible to reject the hypothesis of the closed world and choose the open world assumption.

The rule of Dempster cannot guarantee the continuity of the result. For example take two sources: $m_1(A) = 0.99$ and $m_1(B) = 0.01$, $m_2(C) = 0.99$ $m_2(B) = 0.01$. We obtain: $m_{\oplus}(B) = 1$. Then now consider $m_1(A) = 1$ and $m_2(C) = 1$. There is only a small variation in the data. However the result is totally different, and we cannot even get m_{\oplus} since $m_{\oplus}^*(\emptyset) = 1$.

Smets interprets high values of $m_{\oplus}^*(\emptyset)$ as a high conflict and request an expert system of some sort to solve this problem [23]. In this paper, we

propose such a system, based on the discounting approach.

It will use a distance between two belief functions. First let us introduce the operator $BetP$, to obtain the so-called pignistic probabilities [22]. Given a basic belief mass m , possibly not normalized, we obtain:

$$\forall x \in X$$

$$BetP[m](A) = \sum_{A \supset x} \frac{m(A)}{|A|(1-m(\emptyset))} \quad (3)$$

with $|B|$ the number of elements of B . Now the similarity between two measures m_i and m_j is defined by the following relation:

$$r_{ij} = \sum_{x \in X} \min(BetP[m_i](x), BetP[m_j](x)) \quad (4)$$

It is close to the similarity defined by Tessem [16]. We could also use

$$r_{ij} = 1 - 0.5 \sum_{x \in X} |BetP[m_i](x) - BetP[m_j](x)|,$$

or any other similarities previously defined [16]. In [16] a solution based on $1 - C(m)$ is favored. A distance can be computed from a similarity by:

$$d(m_i, m_j) = 1 - r_{ij} \quad (5)$$

3 Estimating discounting factors

3.1 Discounting factors

The discounting approach assumes that the quality of a source is represented by a scalar. Then this scalar is used to alter the source and to obtain a new reliable source. So this new source can be used by the rule of Combination (2) without worrying about the initial quality. The discounting $D_t[m]$ of m by the weight t is given by:

$$\forall A \subset X, A \neq X, D_t[m](A) = tm(A)$$

$$D_t[m](X) = tm(X) + 1 - t \quad (6)$$

Once all sources have been adapted by their discounting factor, they are fused by an intersection.

3.2 What meaning has a reliability?

There is a clear relation between the data supplied by a source, and its reliability. However the link is complex, and usually classification problems are used to estimate the reliability indices. In the previous papers the reliability of a source is assumed constant. Once

it is estimated, it is used without any further adjustments.

The approach proposed in this paper is based on the assumption that sensors can become defective a posteriori past the learning step. So conflicts between the sources are dynamically exploited in order to continuously adapt the reliabilities. We assume that the reliability of a source is time-varying. Then, by using conflicts between the sources, defective sources can be detected. To do so, the reliability indices used in this paper are assumed to involve two different parts: the classical reliability representing the quality of a source, called the static reliability t_s , and a new reliability t_d coming from the conflicts a source has with the other sources. Reliabilities t_d are computed each time a fusion is carried out.

Finally both reliabilities are combined into a single one $r = f(t_d, t_s)$, to be used as a discounting factor, for the true fusion. In this way, we can propose an algorithm that detects defective sources: those having a high static reliability, and at the same time severely conflicting with other sources.

In order to avoid a full conflict between the discounted sources to fuse, the following constraints have to be enforced: f is increasing from both arguments, $f(1,1) = 1$, $f(0,0) = 0$, $t_d > 0 \Rightarrow f(t_d, 0) > 0$ and $f(0, t_s) < 1$.

In [3, 7, 14], about classifications problems, training sets are used and discounting factors are optimized to minimize an error-based criteria according to the result of the fusion. This approach can be interpreted as using dynamic reliabilities. But static ones are avoided. In [18] another approach is used by fusing meta informations about the sources before fusing them. In [15], which deals with fault isolation, conflicts are solved by using static reliabilities. However such constant discounting factors are chosen without any justifications. In [8] these two definitions are introduced, as well as their combination to obtain a discounting factor. But in that paper we feel that dynamic reliabilities are not fully used. In particular if a source is supposed very reliable and has a high static reliability, but if it contradicts the other sources,

then it is possible that it has broken down. Such case is not handled in [8]. With the assumption that the only cause of conflict between reliable sources is a failure, we will be able to detect such failures and to improve the fusion step.

3.3 Presentation of the algorithm

The first step is to assess dynamic reliabilities. They will be based on similarities.

Dynamic reliabilities $t_{i,D}$

They are computed for each source i . $t_{i,D}$ is given by:

$$t_{i,D} = \frac{\sum_{j=1, j \neq i}^n t_{j,S} r_{ij}}{\sum_{j=1, j \neq i}^n t_{j,S}} \quad (7)$$

For example a low static reliability source can have conflicts with other sources, but it will not lower too much their dynamic reliability. Of course other relations can be used.

Reliabilities r_i for each source i .

We propose a linear sum, depending on $l_0 \in (0,1)$. Here $l_0 = 0.5$. It can be optimized, but this is beyond the scope of this paper.

$$r_i = l_0 t_{i,D} + (1-l_0) t_{i,S} \quad (8)$$

Similarity between the discounted sources

This step is used to detect defective sources. The similarity is:

$$A_i = \sum_{x \in X} \min \left(\text{BetP} \left[D_{i_1} [m_1] \right] (x), \text{BetP} \left[D_{i_2} [m_2] \right] (x) \right) \quad (9)$$

Of course if more than two sources are fused, the *min* must be computed with all sources. We can remark that:

$$A_i = 1 + \sum_{x \in X} \min_j \left(\left(\sum_{\substack{A \subset X \\ A \neq X \\ A \supset x}} t_j \frac{m_j(A)}{|A|} \right) + \frac{t_j m_j(X) - t_j}{|X|} \right) \quad (10)$$

Adaptation Step

Discounting factors are given by:

$$t_i = \lambda r_i = \lambda (l_0 t_{i,D} + (1-l_0) t_{i,S}) \quad (11)$$

with λ ranges from 1 to 0. In case of conflict, and thus in case of low A_i , λ is reduced to 0. This will increase A_i , since all sources will tend to the ignorance. It is obvious that the corresponding index, noted A_λ , is given by:

$$A_\lambda = 1 + \lambda(A_i - 1) \quad (12)$$

When A_λ has to reach a minimal level of agreement LoA , the value of λ is given by:

$$\lambda = \min\left(1, \frac{LoA - 1}{A_i - 1}\right) \quad (13)$$

Note that such an approach can be designed with the similarity based on $1 - C(m)$ [15] but the expression for λ is, for n sources:

$$\lambda = \min\left(1, \sqrt[n]{\frac{1 - LoA}{C(m)}}\right) \quad (14)$$

Detection of defective sources

We call here defective those sources having a low discounting factor. A signal Def is created for each source. If $Def(i) = 0$ the i^{th} source is said defective, if $Def(i) = 1$ it is said working. With t_i given by (11), we propose the next relation to assess Def :

$$Def(i) = \begin{cases} 0 & \text{if } t_i < LoW \\ 1 & \text{otherwise} \end{cases} \quad (15)$$

with LoW a scalar to choose. The relation between LoA and LoW is studied in section 5, as well as an optimization step. Indeed it is clear that the detection is sensitive to these two parameters. For example with (15), if a reliability is decreased, the corresponding source is likely to be identified as defective with high values of LoW .

Static reliabilities are unavailable

In this case, we take $l_0 = 1$. By assuming constant $t_{i,S}$, we get:

$$t_{i,D} = \frac{1}{n-1} \sum_{j=1, j \neq i}^n r_{ij} \quad t_i = \lambda t_{i,D} \quad (16)$$

Only (15) can be used. Of course, such discounting factors can be used as static reliabilities for future fusions.

4 Comparison with other rules

4.1 Other rules of fusion

Whatever the rules are, the problem is to reallocate the mass given to \emptyset . The operator Yag of Yager proposes to shift this mass to X :

$$\begin{aligned} Yag(m_1, m_2)(x) &= \\ & \sum_{x=A_i \cap B_j \neq \emptyset} m_1(A_i) m_2(B_j) \\ Yag(m_1, m_2)(X) &= m_1(X) m_2(X) \\ & + \sum_{A_i \cap B_j = \emptyset} m_1(A_i) m_2(B_j) \end{aligned} \quad (17)$$

The operator Dub of Dubois and Prade proposes to give this mass to the union of elements which, by their intersection, give this empty set:

$$\begin{aligned} Dub(m_1, m_2)(x) &= \sum_{x=A_i \cap B_j \neq \emptyset} m_1(A_i) m_2(B_j) \\ & + \sum_{\substack{x=A_i \cup B_j \\ \emptyset=A_i \cap B_j}} m_1(A_i) m_2(B_j) \end{aligned} \quad (18)$$

The consensus operator [10] equally considers the sources, with the exception that a relative weight γ is introduced in the fusion when sources are conflicting, according to a particular definition called "dogmatism". But this relative weight cannot be used when the data are not absolutely "dogmatic". We present only the main relations. For any basic belief mass m we define three functions:

$$\begin{aligned} b(x) &= \sum_{y \subseteq x} m(y), \quad d(x) = \sum_{y \cap x = \emptyset} m(y), \\ u(x) &= \sum_{y \cap x \neq \emptyset, y \not\subseteq x} m(y) \end{aligned} \quad (19)$$

We do not have $\sum_{x \in X} b(x) = 1$. We only have

$$\forall x \in X, b(x) + d(x) + u(x) = 1.$$

$k(x) = u_1(x) + u_2(x) - u_1(x)u_2(x)$ is called the dogmatism between m_1 and m_2 . k depends on x , because the consensus operator is different for all x in X . So if X has p elements, we have to compute $3p$ functions, and perform $3p$ fusions. Indeed the fusion rule Con is defined by:

If $k(x) \neq 0$:

$$\begin{aligned} b_{Con(b_1^s, b_2^s)}(x) &= (b_1(x)u_2(x) + b_2(x)u_1(x)) / k(x) \\ d_{Con(b_1^s, b_2^s)}(x) &= (d_1(x)u_2(x) + d_2(x)u_1(x)) / k(x) \\ u_{Con(b_1^s, b_2^s)}(x) &= (u_2(x)u_1(x)) / k(x) \end{aligned} \quad (20)$$

If $k(x)=0$, a relative weight $\gamma(x)$ is introduced. It may vary according to the elements x . Then the *Con* operator is defined by:

$$\begin{aligned} b_{Con(b_1^x, b_2^x)}(x) &= (b_1(x)\gamma(x) + b_2(x)) / (\gamma(x) + 1) \\ d_{Con(b_1^x, b_2^x)}(x) &= (d_1(x)\gamma(x) + d_2(x)) / (\gamma(x) + 1) \\ u_{Con(b_1^x, b_2^x)}(x) &= 0 \end{aligned} \quad (21)$$

In [9] a simple way to obtain $Con(m_1, m_2)$ is not presented

Finally we will compare our result with the approach of Lefevre et al, [9, 11, 12]. $m^*(\emptyset)$ is cut in several pieces that are given to all the subsets of the reference set X . The process is quite complex, but the authors show that their fusion rule is a special case of the discounting approach, with a special choice of the discounting factors t_i . So a training step is proposed to learn the t_i .

It is obvious that if we have a database, static reliabilities can be directly assessed, and these reliabilities will have a physical meaning, as in [14, 15]. This is why we claim that the approach of Lefevre et al could be a way to learn the static reliabilities.

So to summarize, this approach is used like the discounting approach, with a special choice of the discounting factors t_i .

4.2 The comparison

The reference set is $X = \{A, B, C\}$. The two sources are:

$$\begin{aligned} m_1(\{A\}) &= a, \quad m_1(\{B\}) = b \\ m_2(\{C\}) &= a, \quad m_2(\{B\}) = b \end{aligned}$$

with $a + b = 1$

Since there are only two sources and no static reliability, discounting factors will be equal for both sources: $t = b$. The two sources, after the discounting step, become:

$$\begin{aligned} D[m_1](\{A\}) &= at \quad D[m_1](\{B\}) = bt \\ D[m_1](X) &= 1 - t \\ D[m_2](\{B\}) &= bt \quad D[m_2](\{C\}) = at \\ D[m_2](X) &= 1 - t \end{aligned} \quad (22)$$

The intersection between the discounted sources is given by table 1.

Table 1. Intersection between the two discounted sources. ($D[m_2]$ in columns)

	$X, 1-t$	C, at	B, bt
$X, 1-t$	$X, (1-t)^2$	$C, at(1-t)$	$B, bt(1-t)$
B, bt	$B, bt(1-t)$	\emptyset, abt^2	B, b^2t^2
A, at	$A, at(1-t)$	\emptyset, a^2t^2	\emptyset, abt^2

(9) gives $A_i = a + (1-a)^2$ so when $LoA \leq 0.75$ no adaptation with (11)-(13) is required.

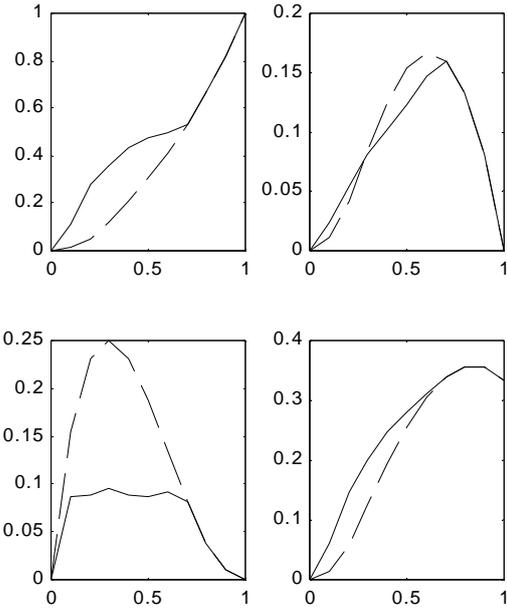


Figure 1. Fusion with our approach, $a \in [0, 1]$. From left to right: $m(X)$, $m(A)$, $m^*(\emptyset)$, $BetP(A)$. $LoA = 0.9$ for '- -', $LoA = 0.5$ for '-'

When a tends to 1, we tend to the absolute ignorance, with $m(X) = 1$. Clearly masses are continuous functions of the varying parameter a . When $a = 1$, $t = 0$, and we are in a total ignorance. For $LoA = 0.9$, $m(X) = 1$ is bigger and $m^*(\emptyset)$ before normalization is lower than for $LoA = 0.5$.

This result can be compared with other rules. With the approach of Yager, masses are:

$$m(X) = 1 - b^2, \quad m(B) = b^2$$

With the approach of Dubois, masses are:

$$m(B) = b^2, \quad m(B, C) = m(B, A) = ab,$$

$$m(A, C) = a^2$$

For the consensus operator functions b, d, u have to be defined for the three elements and the two sources:

$$b_1(A) = a, \quad d_1(A) = 1 - a$$

$$b_1(B) = 1 - a, d_1(B) = a$$

$$b_1(C) = 0, d_1(C) = 1$$

Replacing A by C gives the functions for m_2 . Then the 2 sources can be fused. Here we are in what is called in [15] an absolute dogmatism for the 2 sources, since $u_i(x) = 0, i = 1, 2, \forall x \in X$. Therefore the relative weight $\gamma(x)$ of the sources is introduced. Since sources m_1 and m_2 are considered equally, $\gamma = 1$ and constant. The final result is:

$$b(B) = 1 - a = m(B), d(B) = a$$

$$b(A) = a/2 = m(A), d(A) = (2 - a)/2$$

with the same results for C. Here the basic belief mass m can be introduced since $\sum_{x \in X} b(x) = 1$, but usually this is not the case.

For the operator of Lefevre, it is assumed that $t = 0.5$, the mean value of our own t .

The next figure compares our approach ($LoA = 0.5$) with the four others.

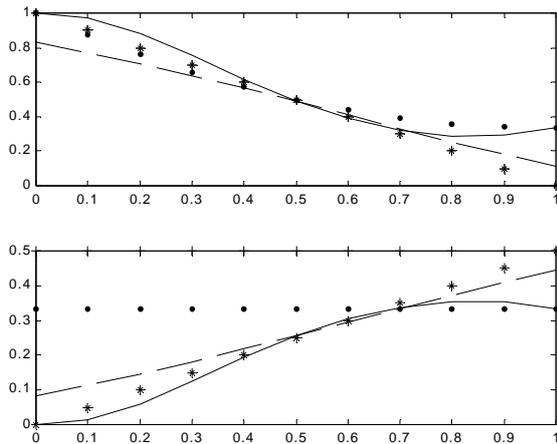


Figure 2. From the top to the bottom: $BetP(B)$ and $BetP(A)$, for five fusion rules: ‘-’ our approach, ‘- -’ Lefevre’s, ‘*’ consensus operator, ‘●’ Yager’s, ‘+’ Dubois’.

If a decision about the correct solution has to be taken, differences are:

- for the approach of Yager, B is always the solution, except when $a = 1$ when we are in a state of full ignorance, with A, B, C sharing the same $BetP = 1/3$.
- for the approach of Dubois, B is the solution when $a \leq 0.68$, and then we hesitate between A and C ,

- we get the same results with the consensus operator (there is a difference on a for the third decimal),
- we get the same results with the method of Lefevre, based on an static reliability $t = 0.5$, but the transition occurs for $a = 0.71$,
- we get the same result with our method, except than when $a = 1$ we are in a full ignorance. In this case A, B, C share the same $BetP = 1/3$.

This figure hides that with the approach of Yager, Lefevre and our’s, $m(X)$ has a high level when a tends to 1, reflecting a high level of ignorance. The consensus operator and the one of Dubois are also different.

We lack of place to discuss the detection step, but if it is used, the two sources may be declared as defective depending on a, LoA and LoW . This issue is tackled in the next section.

5 Example with defective sensors

5.1 Problem

Three sources are fused. They have explicit time-varying static reliabilities. The frame of discernment is $X = \{x_1, \dots, x_n\}$ with $n = 100$. The variable varies randomly between two measures. We assume that the x_i are ordered.

Sensors are prone to breakdowns, and by means of our algorithm, we will study 6 probabilities:

- Status_OK, corresponding to a standard report (no alarm and no defective sensor),
- False_Alarm, corresponding to alarms while the three sensors are running correctly,
- Detection_OK, when they are alarms and one (or more) sensor is defective,
- Robust_Fusion, when there is no alarm, one and only one sensor is defective, and errors are acceptable,
- Non_Detection, when there is no alarm, one (or more) sensor is defective, and errors are unacceptable,
- Incorrect_Robust_Fusion, when there is no alarm, there is only one working sensor, and errors are acceptable.

The problem is to minimize $P(\text{Error}) = P(\text{False_Alarm}) + P(\text{Non_Detection}) +$

$P(\text{Incorrect_Robust_Fusion})$ while maximizing the other probabilities.

5.2 Data

Sensor's reports are modeled by simple support functions, whatever their status (working or not). The report of the source S_i is:

Working state: $m_i(A) = a_i, m_i(X) = 1 - a_i$

Defective state: $m_i(B) = a_i, m_i(X) = 1 - a_i$

With A and B sets of cardinal c_i (at most) centered around an x_0 . For B x_0 is chosen with a uniform probability on X .

Let be the correct state of the universe x . x_0 for A is shifted from x with an offset equals at most to d_i . This offset is based on a uniform law between $[-d_i, d_i]$. Figure 3 shows an example where the set A is truncated and does have a cardinal lower than c_i because the computed set with a cardinal c_i does not belong to X .

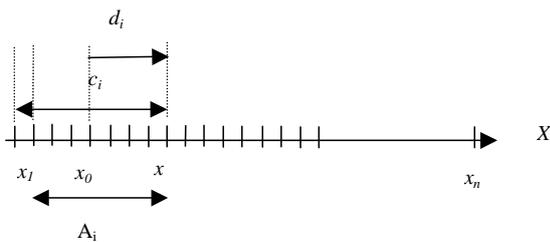


Figure 3. Report from a source.

We take $c_1 = 9, c_2 = 9, c_3 = 11, d_1 = 4, d_2 = 5, d_3 = 6$. Since the offset d_3 is larger than $(c_3 - 1)/2$, the focal element of sensor 3 (and 2) may have an empty intersection with x . This justifies the interest of a fusion between the sensors. We also have $a_1 = 0.7, a_2 = 0.7, a_3 = 0.9$.

The probability a sensor is working on the interval $[0, T]$ is defined as the reliability in the theory of reliability [19]. With t the elapsed time since the beginning, it is given for the three sensors S_1, S_2 and S_3 by exponential laws with constant parameters:

$$t_{i,S} = r_i = e^{-\mu_i T} \quad (23)$$

with $\mu_1 = 25 \cdot 10^{-6}$, $\mu_2 = 10^{-6}$, $\mu_3 = 8 \cdot 10^{-7}$, meaning that Mean Time To Failure are respectively equal to 11h, 28h and 35h (take $1/\mu_i$).

When a fusion is carried out, an estimated state \hat{x} is computed as the expected utility based on

the $BetP$ function, with $x_i = i$. The level of acceptable errors is set to 10%.

5.3 Results

In this example, we add a discrete filter to smooth results about the detection. The input of the filter will be Def_i and its output Def_i' . The i^{th} source is said defective if $Def_i' < 0.5$.

We choose a first order filter, of equation:

$$Def_i'(k) = \alpha Def_i'(k-1) + (1-\alpha) Def_i(k) \quad (24)$$

with k an integer, $\Delta t = 2$ minutes, and with initial state $Def_i'(0) = 1$. α is a parameter to choose. In this example $\alpha = 0.9$, meaning that a fully working source is stated defective after 6 successive failure.

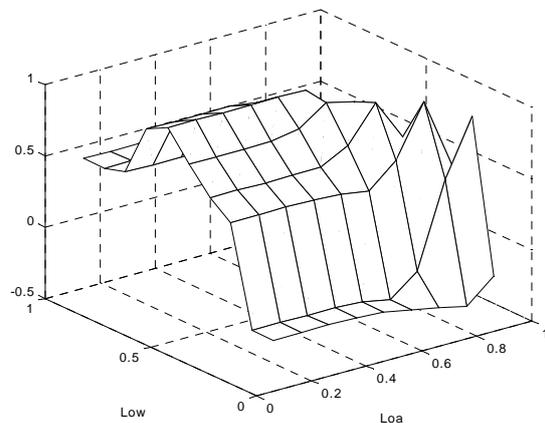


Figure 8. $P(\text{Status_Ok}) + P(\text{DetectionOk}) + P(\text{Robust_Fusion}) - P(\text{False_Alarm}) - P(\text{Non_Detection}) - P(\text{Incorrect_Robust_Fusion})$.

The optimization of LoA and LoW was based on a search with 10 values for LoA and 9 for LoW . Another optimization step based on numerical algorithm could improve the results. The best combination is given by $LoW = 0.25$ and $LoA = 0.8$ with $P(\text{False_Alarm}) + P(\text{Non_Detection}) + P(\text{Incorrect_Robust_Fusion}) = 1\%$.

6 Conclusion

In this paper a method to compute dynamically discounting factors from past information and conflicts has been proposed, as well as a detection of defective sources, based on the analysis of the adapted discounting factors. Two examples show the interest of such an approach.

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