Sensor Failure Detection within the TBM Framework: A Markov Chain Approach

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Abstract

This paper presents a sensor failure detection method based on the fusion of predicted and observed sensor data. The originality of our approach is the use of a Markov Chain to model the normal behavior of a sensor within the TBM framework. When fusion between predicted and observed data is done, three experts analyze conflict resulting from the fusion process and are able to detect an abnormal behavior of the sensor by looking for high increase of conflict. The testing results show that this method is efficient to detect sensor failure with a TBM approach.

Keywords:

Sensor Failure Detection, TBM, Markov Chain.

1 Introduction

In the sensors based applications framework, having a total confidence towards the sensor is a critical issue. A human or automatic system decision-making process, in accordance with the acquire sensors data, can generate important consequences. In this paper, we present a method to detect the sensors failures (the data sources, in a more general way) in order to not take into account data with errors in the final decision-making process and by pondering its influence at the final decision-making level.

In literature, one can distinguish three main methods for the detection of sensor dysfunction:

- The sensor is perfectly known and the provided values are limited [10],
- Several sensors produce redundant data on the same variable [12],
- A prediction-checking phase is used which enables detection of abnormal behaviour of one sensor only. The Kalman filter or particle filtering [11] is used to determine abnormal behaviour.

One can see that no work addresses the problem with a symbolic approach. This is one of our main constraints, due to our software architecture based on an ontology that handles a symbolic representation [4] to infer. Our work is based on the Transferable Belief Model framework (TBM) initiated by Philippe Smets [5]. Thus, a data source behaviour is represented with this theory represent with a set of masses on a discernment frame.

This article is organized as follows. First, we present how to model the data source behaviour with a stochastic process. In the second part, we detail our model-based method to detect a sensor failure thanks to the use of predicted values of the sensor. This method compares the actual values with the predicted ones to detect faults.

2 Data source modelling using a Markov Chain Model

The question is: How to detect an anomaly? First, it is necessary to know the normal data source behaviour in order to detect its abnormal behaviour.

We describe a data source as a system with a behaviour evolution through time series. Based on ontology [4], our architecture needs symbolic data to provide the best service. A data source has various known symbolic states that progress process through a sequence of states S with equal time intervals. Based on this presentation, we can define that a data source is a discrete-time stochastic process [2][7] and more particularly a Markov Chain Model (MCM) [1] which is a special type of discrete-time stochastic process. Among probabilistic processes, the MCM, employed in this paper, provides a powerful tool for analysing the

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system evolution through time series, and it has been applied in many fields of research.

MCM describes the evolution of a process through a sequence of states X with equal time intervals. A MCM moves from one state to the next controlled by the transition matrix T. The whole possible values are called the space of states where X_n is the state of the process at time n. If the conditional probability distribution of X_{n+1} on the last states is related to X_n alone, then:

$$P(X_{n+1} = x | X_0, X_1, X_2, \ldots, X_n) = P(X_{n+1} = x | X_n).$$

where X is an unspecified state of the process. Given the present state of the system Xn, matrix T provides the probability to go in one step from state X_n to state X_{n+1}, that is : $X_{n+1} = T \times X_n$.
This property indicates that it is possible to predict the future evolution of a system by determining the transition matrix T. The applicability of MCM to represent and predict has been discussed by previous paper [9]. Many work use Markov prediction. [13] uses Markov chain model to make link predictions that assist new users to navigate the Web site. Web users' navigation is modeling using Markov chains and prediction help users to find information more efficiently and accurately than simply following hyperlinks. In voice packet transmitting domain, Markov chain prediction is used for compensating when speech frames are missing [14]. It enables listener to hear something even when voice packets are lost, deleted, or excessively delayed in the network.

In this work, the normal behaviour of a data source will be related to an actual state and a previous state. Indeed, if the data source correctly follows the evolution of the MCM, *i.e.* a transition probability exists for the passage of a state to another, its behaviour will be defined as normal. However, when the data source goes through a state to another whereas the transition probability is null, then the behaviour is abnormal. Thanks to this property and to the predictivity concept of the data source behaviour, the dysfunction detection is possible.

3 Sensor failure detection by fusion of predicted/observed data

3.1 General method

The implemented method is based on filtering methods that make a prediction from observations. Based on this philosophy, we defined a method for sensor failure detection composed of four principal parts:

- Observation
- Prediction
- Fusion
- Checking

The observation part is the set of masses at the t moment provided by the data source to diagnose.

The prediction part provides an estimation of the state of the data source at the t+1 moment.

The fusion part merges the observed and the predicted values.

The checking part checks the result from the fusion part and detects the disagreement between the observed and the predicted values. This checking part is carried out on the conflict m_{fusion} *(Ø)* resulting from the fusion part.

The general diagram is the following one:

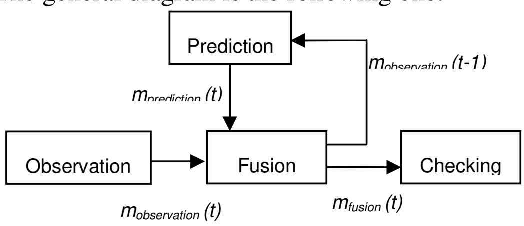

Figure 1 : General diagram for sensor failure detection

The observation provides a set of masses $m_{observation}$ on the whole of hypothesis and disjunctions of hypothesis of the discernment frame. Thus, it is possible to build a predicted set of masses $m_{prediction}$ calculated with an evolution model. In our case, this evolution model is a MCM [8] [3]. From this MCM, it is possible to carry out a Markovian prediction.

3.2 The Markovian prediction

In the Markovian prediction framework [6], a probabilistic vector of observation $X_{observation}$ is in input of the prediction part, corresponding to the current state of the data source. The estimation of the Markovian vector of prediction $X_{prediction}$ based on the transition matrix *T*, is obtained by:

$$X_{prediction} = X_{observation} * T \qquad (1)$$

For example, the matrix T is defined modelling 3 states. From the X0 state at t, the process can remain in X0 or go to X1 at the next moment t+1. In the same manner, from the X2 state at t, the process can remain in X2 or go to X1 at the next moment. When the process is in the state X1 at t, the process can remain in X1, or go in X0 or X2 at the next moment. Thus, the previous description is represented into a transition matrix T and its corresponding directed graph :

T=

	X0	X1	X2
X0	1/2	1/2	0
X1	1/3	1/3	1/3
X2	0	1/2	1/2

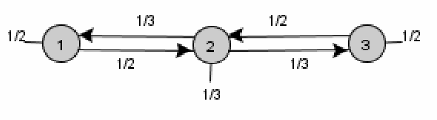

Figure 2 : Transition matrix T and its corresponding directed graph

When a transition from a state to another is possible, the probability of transition is strictly positive, if not when a transition is impossible, the probability of transition is null.

With this representation, an observation vector $X_{Observation} = [1\ 0\ 0]$, i.e. the process is in the X0 state at the moment t=0, provides the prediction vector $X_{Prediction} = X_{Observation} * T = [0.5\ 0.5\ 0]$. Hence, this prediction shows that when the process is in the X0 state with a high probability, it will be in the X0 or X1 state at the next moment with the same probability.

As we can see with this example, the Markovian prediction needs a set of probabilities in input and provides a set of probabilities in output, *i.e.* a predicted set of probabilities. However in the input of our system, the data source provides an observed set of masses.

It means that it is necessary to convert this observed set of masses in order to allow the estimation of the Markovian prediction. The pignistic probability calculus proposed by Smets[5] is a solution to this problem. It is possible to calculate the Markovian prediction on the set of pignistic probabilities. The predicted set of probabilities must be converted too into a predicted set of masses in order to be able to carry out a fusion between predicted and observed set of masses.

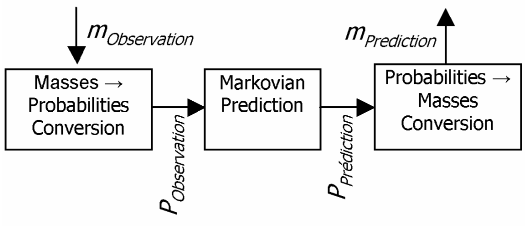

Figure 3 : Conversion sets to enable Markovian prediction

As the set of probabilities is complete ($\sum P_{Prediction}(Xi) = 1$), it is then possible to directly convert this set of probabilities into a set of masses. The probabilities on the singleton hypothesis are directly placed on the sets of masses on the corresponding singleton hypothesis :

$$m_{prediction}(Xi) = P_{prediction}(Xi) \quad (2)$$

3.3 Observed and predicted set of masses fusion

To carry out fusion between the predicted set of masses $m_{prediction}$ and the observed set of masses $m_{observation}$, we use the Smets operator ∩ to isolate the conflict resulting from the fusion between $m_{prediction}$ and $m_{observation}$:

$$m_{fusion} = m_{prediction} \cap m_{observation} \quad (3)$$

and

$$m_{fusion}(\varnothing) = \sum_{A \cap B = \varnothing} m_{prediction}(A).m_{observation}(B) \quad (4)$$

The set of masses m_{fusion} is the result of the fusion between the predicted and the observed set of masses at a moment t. $m_{observation}$ is the set of masses coming from the observation and $m_{prediction}$ is the set of masses coming from the prediction based on the MCM. This fusion is based on the Smets operator ∩ that enable to isolate the conflict between the predicted and the observed set of masses, i.e. the difference between the predicted and the observed state.

To detect a failure, our method is based on the conflict analysis between the predicted and observed set of masses. To detect a failure, the conflict $m_{fusion}(Ø)$ has to be analyse. If conflict appears, it can be due to two hypotheses. The first one is a false prediction (due to a bad MCM) and the second one is conflict due to the sensor failure. We assume the second hypothesis and a right MCM of the sensor. To illustrate this theory, one sensor with the frame of discernment (FOD) Θ = {X0, X1,X2} and following the behaviour of Figure 2, provides :

At t=0 :

Observed set of masses t=0	Predicted set of masses t=1
$m_{Obervation}$ (X0) = 0.7	$m_{Prédiction}$(X0) = 0.4745
$m_{Obervation}$ (X1) = 0	$m_{Prédiction}$(X1) = 0.4745
$m_{Obervation}$ (X2) = 0	$m_{Prédiction}$(X2) = 0.0495
$m_{Obervation}$ (X0 ∪ X1) = 0.3	$m_{Prédiction}$(X0 ∪ X1) = 0
$m_{Obervation}$ (X0 ∪ X2) = 0	$m_{Prédiction}$(X0 ∪ X2) = 0
$m_{Obervation}$ (X1 ∪ X2) = 0	$m_{Prédiction}$(X1 ∪ X2) = 0
$m_{Obervation}$ (X0 ∪ X1 ∪ X2) = 0	$m_{Prédiction}$(X0 ∪ X1 ∪ X2) = 0

The prediction process begins at t=1. At t=0, the predicted set of masses is equal to the observed set of masses.

At t=1 :

Observed set of masses t=1	Merged set of masses t=1 ($m_{Obervation}$(Xi) (t=1) ∩ $m_{Prédiction}$(Xi) (t=1))
$m_{Obervation}$ (X0) = 0.9	m_{fusion} (X0) = 0.4745
$m_{Obervation}$ (X1) = 0	m_{fusion} (X1) = 0.04745
$m_{Obervation}$ (X2) = 0	m_{fusion} (X2) = 0
$m_{Obervation}$ (X0 ∪ X1) = 0.1	m_{fusion} (X0 ∪ X1) = 0
$m_{Obervation}$ (X0 ∪ X2) = 0	m_{fusion} (X0 ∪ X2) = 0
$m_{Obervation}$ (X1 ∪ X2) = 0	m_{fusion} (X1 ∪ X2) = 0
$m_{Obervation}$ (X0 ∪ X1 ∪ X2) = 0	m_{fusion} (X0 ∪ X1 ∪ X2) = 0
	m_{fusion}(Ø) = 0.47655

Predicted set of masses t=2
$m_{Prédiction}$(X0) = 0.492
$m_{Prédiction}$(X1) = 0.492
$m_{Prédiction}$(X2) = 0.02
$m_{Prédiction}$(X0 ∪ X1) = 0
$m_{Prédiction}$(X0 ∪ X2) = 0
$m_{Prédiction}$(X1 ∪ X2) = 0
$m_{Prédiction}$(X0 ∪ X1 ∪ X2) = 0

At t=2 :

Observed set of masses t=2	Merged set of masses t=2
$m_{Obervation}$ (X0) = 0	m_{fusion} (X0) = 0
$m_{Obervation}$ (X1) = 0	m_{fusion} (X1) = 0.123
$m_{Obervation}$ (X2) = 0.75	m_{fusion} (X2) = 0.065
$m_{Obervation}$ (X0 ∪ X1) = 0	m_{fusion} (X0 ∪ X1) = 0
$m_{Obervation}$ (X0 ∪ X2) = 0	m_{fusion} (X0 ∪ X2) = 0
$m_{Obervation}$ (X1 ∪ X2) = 0.25	m_{fusion} (X1 ∪ X2) = 0
$m_{Obervation}$ (X0 ∪ X1 ∪ X2) = 0	m_{fusion} (X0 ∪ X1 ∪ X2) = 0
	m_{fusion}(Ø) = 0.861

This example presents three sensor data acquisition. At t=0, the predicted set of masses is unknown, that is why the fusion of predicted and observed set of masses is not achieved. However, it is possible to predict the set of masses for t=1. We can observe that this predicted set of masses follows the right behaviour of the sensor, i.e. when the sensor is in the state X0, it can remain in X0 or go to X1 at t+1 (defined by an equi-masses on $m_{Prediction}$(X0) and $m_{Prediction}$(X1)).

At t=1, it is possible to merge the observed and predicted set of masses. We can observe that the conflict m_{fusion}(Ø) = 0.47655 is relatively high, due to the prediction that can not choose between X0 or X1.

At t=2, the conflict between the predicted and observed set of masses is very high (m_{fusion}(Ø) = 0.861), due to an observation that is trending to X2 and the prediction that is trending to X0 or X1. This conflict is justified by the Markovian model of the sensor. At t=1, the sensor trends to be in the state X0 and at t=2, it trends to be in X2, which is an impossible behaviour of the sensor (transition probability between X0 and X2 is equal to 0; *P(X2|X0) = 0*)

3.4 Failure detection

Here the problem is to detect an abnormal behaviour of the sensor by analysing the conflict between the observed and predicted set of masses. As we see in the previous example, when the sensor follows an abnormal behaviour, the conflict increases quickly. However, even if the sensor follows its normal behaviour, conflict is appearing too. This redundant conflict is due to the imprecise Markovian prediction that can not choose between probable states but just provides a tendency. This redundant conflict is continuous during the functioning of the sensor and is not significant to express an abnormal behaviour of the sensor.

On the contrary, with multiple examples, we observed a quick increase of the conflict followed of a quick decrease when an abnormal behaviour occurs. To detect a failure, we define three experts based on the temporal conflict signal :

1. Expert 1 discusses about the direction between two consecutive points.
2. Expert 2 discusses about the difference value between two consecutive points.
3. Expert 3 discusses about the variation value between N consecutive points.

The main goal of those three experts is to merge them to detect a high increase of the conflict for a short duration that characterise an abnormal behaviour.

4 Experimental results

We implemented our method with simulated data. This simulation uses a sensor in input of the system and provides a set of masses $m_{observation}$ on the discernment frame Θ : $\Theta = \{X0, X1, X2\}$. To describe the behaviour of the sensor, we use the Markov Chain Model of Figure 2. The Figure 4 presents the experimental data set $m_{observation}$ ($m(X0)(t)$, $m(X1)(t)$, $m(X2)(t)$, $m(X0\cup X1)(t)$, $m(X0\cup X2)(t)$, $m(X1\cup X2)(t)$. $m(X0\cup X1\cup X2)(t)$ doesn't appears because it never occurs).

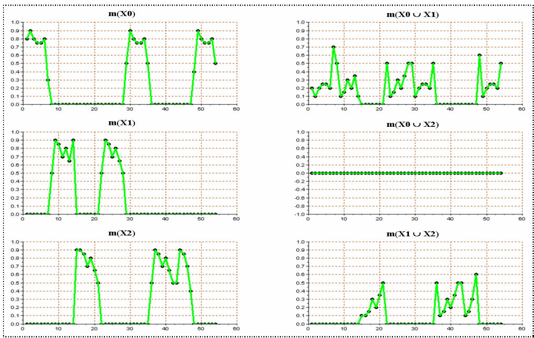

Figure 4 : Temporal evolution of the observed set of masses $m_{observation}$

This temporal evolution is divided into seven phases:

From t=0 to 7, the sensor is in the state X0,

From t=8 to 14, the sensor is in the state X1,

From t=15 to 21, the sensor is in the state X2,

From t=22 to 28, the sensor is in the state X1,

From t=29 to 35, the sensor is in the state X0,

From t=36 to 47, the sensor is in the state X2,

From t=48 to 54, the sensor is in the state X0.

We can observe 2 abnormal phases. At t=36, the sensor goes from A0 to A2, which is an impossible transition. At t=48, the sensor goes from A2 to A0, which is impossible too.

The Figure 5 presents the predictions based on the observed set of masses seen previously.

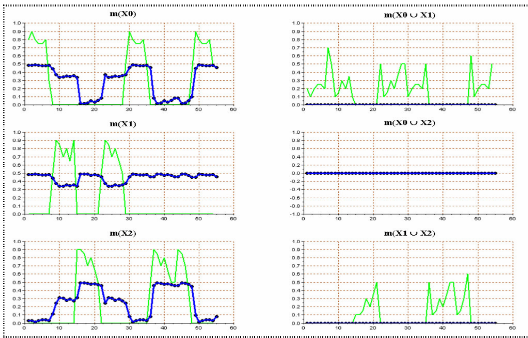

Figure 5 : Temporal evolution of the observed and predicted set of masses

This figure presents the observed set of masses (thin lines) seen on Figure 4 and the predicted set of masses (thick lines) based on the sensor MCM from Figure 2.

On the left, the masses on singleton hypothesis are presented and on the right, the masses on the disjunction of hypothesis. A particular case is at t=0. In this case, we arbitrary choose to begins the prediction process at t=1.However, at t=0, the predicted set of masses is equal to the observed set of masses.

In a normal behaviour, the predicted set of masses follows the observed set of masses tendency. However, in case of abnormal behaviour, at t=36, the predicted and observed set of masses are in conflict:

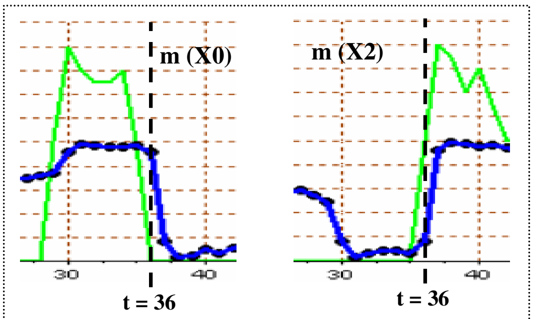

Figure 6 : Set of masses during an abnormal behaviour

At t=35, the sensor is in the state X0. The Markovian prediction provides a mass m(X0) and m(X1) for the next moment but a null mass for m(X2). This is the normal behaviour defined by the sensor MCM.

At t=36, the sensor goes from X0 to X2, which is an impossible transition. When the fusion between the predicted and observed set of masses at t=36 is done, we observe a high conflict:

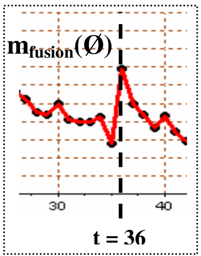

Figure 7 : Conflict due to an abnormal behaviour

The corresponding set of masses at t=36 are :

$m_{Obervation}(X0)=0$	$m_{Prediction}(X0)=0.4575$	$m_{fusion}(X0)=0$
$m_{Obervation}(X1)=0$	$m_{Prediction}(X1)=0.4575$	$m_{fusion}(X1)=0.22875$
$m_{Obervation}(X2)=0.5$	$m_{Prediction}(X2)=0.0825$	$m_{fusion}(X2)=0.12375$
$m_{Obervation}(X0 \cup X1)=0$	$m_{Prediction}(X0 \cup X1)=0$	$m_{fusion}(X0 \cup X1)=0$
$m_{Obervation}(X0 \cup X2)=0$	$m_{Prediction}(X0 \cup X2)=0$	$m_{fusion}(X0 \cup X2)=0$
$m_{Obervation}(X1 \cup X2)=0.5$	$m_{Prediction}(X1 \cup X2)=0$	$m_{fusion}(X1 \cup X2)=0$
$m_{Obervation}(X0 \cup X1 \cup X2)=0$	$m_{Prediction}(X0 \cup X1 \cup X2)=0$	$m_{fusion}(X0 \cup X1 \cup X2)=0$
		$\mathbf{m_{fusion}(Ø)=0.68625}$

The Figure 8 presents the complete fusion signal. On this signal, we observe the appearance of peaks of conflict corresponding to impossible jumps of state.

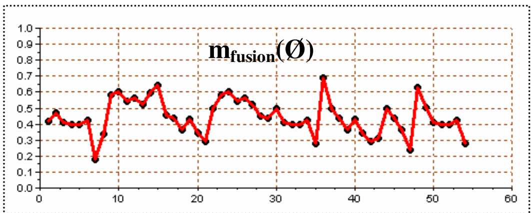

Figure 8 : Conflict signal

To isolate the conflict peaks, indicating a sensor failure, we use three experts that study the conflict resulting from fusion.

The Frame Of Discernment (FOD) of those three expert is the same one and is composed of the two hypothesis 'YES' and 'NO', with 'YES' meaning 'the behaviour of the sensor is normal' and 'NO' meaning 'the behaviour of the sensor is abnormal'. For each of those experts, a Basic Belief Assignment (BBA) is defined.

The first BBA presented Figure 9 provides a set of masses $m_{expert1}$ about the direction between two consecutive points. This BBA is applied on the derivative of the conflict signal and is based on the difference between two consecutive points to determine the direction. This method enables to characterize an increase or a decrease of the conflict.

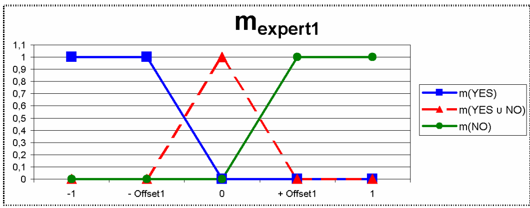

Figure 9 : The first BBA concerning the direction between two consecutive points

In this BBA, when the conflict is increasing, its corresponding derivative is negative, when the conflict is decreasing, its corresponding derivative is positive. The offset named 'Offset1' enables to define the doubt when the direction is close to zero meaning that the conflict is constant. In this case, the conflict is not increasing and not decreasing.

The second BBA presented Figure 10 provides a set of masses $m_{expert2}$ about the amplitude between two consecutive points. This BBA is applied on the conflict signal and is based on the difference between two consecutive points. This method enables to detect high peak of conflict.

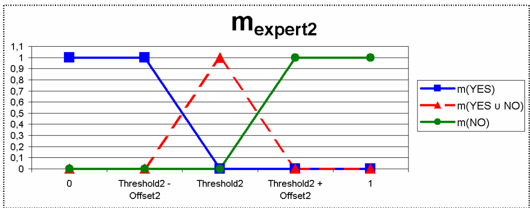

Figure 10 : The second BBA concerning the difference between two consecutive points

In this BBA, when the amplitude of the conflict exceeds the threshold named 'threshold2', it means that it is a high conflict, when it is lower than this threshold, it means that there is a low conflict. 'Offset2' enables to define doubt zone about the amplitude of the conflict.

The third BBA presented Figure 11 provides a set of masses $m_{expert3}$ about the variation value between N consecutive points. This BBA is applied on the conflict signal and is based on the difference between N consecutive points. It enables to detect short conflict.

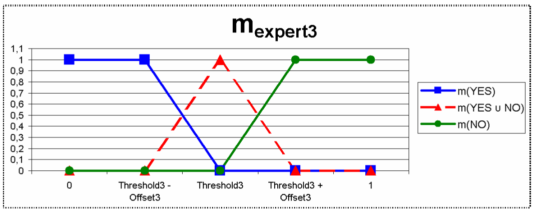

Figure 11 : The third BBA concerning the duration of a peak

In this BBA, when the variation between N points exceeds the threshold named ‘threshold3’, it means that it is a peak of conflict, when it is lower than this threshold, it means that it is not a peak of conflict, just a little increase of it.

The fusion of those three experts allows to isolate the impossible transition phases like from *X0* to *X2* or from *X2* to *X0*, which are theoretically impossible with the chosen transition matrix *T*.

The fusion of those three experts is done using the Smets operator and provides a set of masses $m_{failure} = m_{expert1} \cap m_{expert2} \cap m_{expert3}$ with $m_{expert12} = m_{expert1} \cap m_{expert2}$ and $m_{failure} = m_{expert12} \cap m_{expert23}$.

The conflict coming from the fusion of the three experts is analysing in that way:

When conflict appears during fusion of expert 1 and 2, it comes from: a negative high peak of conflict or a positive low peak. In those two cases, this conflict represents the hypothesis ‘YES’. So we report the conflict value on $m_{expert12}$(YES). In the same way, when conflict appears during fusion of expert 12 and 3, it comes from: a small peak of conflict, a decrease or an increase of the conflict for a long duration. In those two cases, this conflict represents the hypothesis ‘YES’. So we report the conflict value on $m_{expert123}$(YES).

The Figure 12 presents the temporal view of the three opinions coming from the experts, the fusion of those three experts and the final decision based on the pignistic decision process.

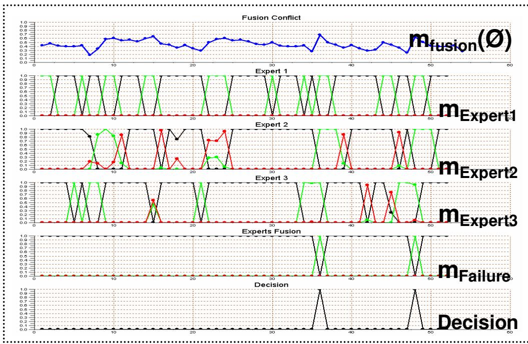

Figure 12 : Fusion conflict, fusion of three experts and pignistic decision signals

To obtain those results, we define Offset1 = 0.001, threshold2 = 0.10 Offset2= 0.01, threshold3 = 0.20 Offset3= 0.02 and N=2. This setting allows to efficiently detect the two sensor failure at t=36 and t=48.

We tested our method with a second example. We used the same process with another MCM shown Figure 13.

T=

	X0	X1	X2
X0	1/2	1/2	0
X1	0	1/2	1/2
X2	0	1/2	1/2

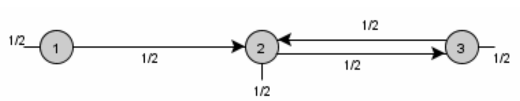

Figure 13 : Transition matrix T and its corresponding directed graph

This MCM describes the following behaviour : from X0 at t, the process can remain in X0 or goes to X1 at t+1. From X1 at t, the process can remain in X1 or goes to X2 at t+1. When the process is in X2 at t, the process can remain in X2, or goes in X1 at the next moment. With this MCM, the impossible transitions are from X0 to X2, from X1 to X0 and from X2 to X0. Like the first example, we use the experimental data presented Figure 4. Following the MCM, we can note failures at t = 30, t = 36 and t=48.

The obtained results are shown on Figure 14.

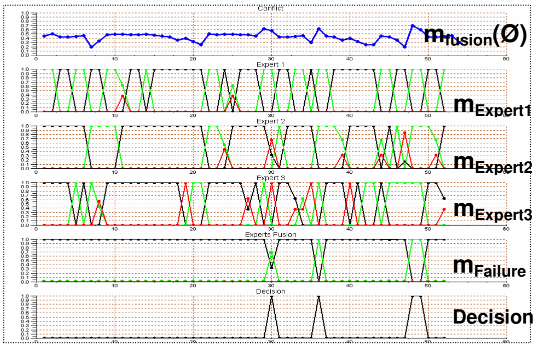

Figure 14 : Fusion conflict, fusion of three experts and pignistic decision signals

To obtain those results, we modify the previous settings and define Offset1 = 0.001, threshold2 = 0.085, Offset2= 0.01, threshold3 = 0.15 Offset3= 0.02 and N=2. This setting allows to efficiently detect the two sensor failures at t=30, t=36 and t=48.

5 Conclusion

In this paper, we present a method for sensor failure detection, modelled by a Markovian stochastic process, within the TBM framework by studying the conflict resulting from fusion between an observed set of masses and a predicted set of masses. With the conflict resulting from this fusion and by using three experts about the shape of the fusion conflict signal, we manage to determine the failure phases of the sensor. However, the expert settings are not easy to do. Actually, we are working on a new version of the predictor to obtain a more accurate state prediction to make easier the conflict analysis.

References

[1] Faust, N.J., « Reliability Analysis by Use of Markov Modeling » - RAC Technical Brief, March 1989.

[2] Infantes G., Ingrand F., Ghallab M. – « Learning stochastic models of activities for planning and execution control » - RFIA 2006, January 25-27 2006, Tours, France.

[3] Metz J.A.J., Dienske H., De Jonge G., Putters F.A.– « Continuous time Markov chains as models for animal behaviour ». Bull. Math. Biol. 45, 643-658, 1983.

[4] Ricquebourg V., Durand D. ,Menga D., Marhic B., Delahoche L., Logé C., Jolly-Desodt A.M. - "Context inferring in the Smart Home : An SWRL approach" – IEEE UCI07 - May 21-23 2007 - Niagara Falls, Canada.

[5] Smets Ph. – « The Transferable Belief Model for uncertainty representation » - Technical Report, 1995

[6] Ross S.M. – « Introduction to probability model » - Academic Press, 9 edition, November 21, 2006.

[7] Taylor H.M., Karlin S. - « An Introduction to Stochastic Modeling » - Academic Press; 3 edition - February 1998.

[8] Ye N. – « A markov chain model of temporal behavior for anomaly detection » - In Proceedings of the 2000 IEEE Systems, Man, and Cybernetics Information Assurance and Security Workshop, 2000.

[9] Zhang, Y., Augenbroe, G., Vidakovic B. « Uncertainty analysis in using markov chain model to predict roof life cycle performance » - ICDBMC 05, April 17-20 2005, Lyon, France.

[10] Li Z., Koh B.H., Nagarajaiah S ., Phan M. Q. – « Sensor failure detection using interaction matrix formulation » - Smart Structures and Materials 2006. SPIE, Volume 6174, pp. 430-440 (2006)

[11] Caron F et al. - « Particle Filtering for Multisensor Data Fusion with Switching Observation Models. Application to Land Vehicle Positioning» - IEEE Transactions on Signal Processing. To appear.

[12] Jaikaeo C., Srisathapornphat C. and Shen C.-C.. Communications, 2001. ICC 2001. IEEE International Conference on , Volume: 5 , 2001, pp. 1627 -1632.

[13] Zhu, Jianhan and Hong, Jun and Hughes, John G. – « Using Markov Chains for link prediction in adaptive web sites » - Soft-Ware 2002 Belfast, Northern Ireland, April 8-10, 2002. Lecture Notes in Computer Science, 2311. Springer, pp. 60-73.

[14] Kohler, M.A. Yarlagadda, R.K. – « Markov chain prediction for missing speech frame compensation » - IEEE Workshop on Speech Coding – pp 75-77, Delavan, WI, USA, 2000