Aggregation operators for conditional information without probability

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Abstract

In an axiomatic way we give some forms of aggregation operators for group of $n, n \in [0, +\infty]$ conditional informations without probability on fuzzy setting.

Keywords: Aggregation operators, conditional information, fuzzy sets.

1 Introduction

An aggregation operator [3, 6] is a procedure by which a unique value can be associated to the results obtained through different tests or different values of a data base.

The unique value is a kind of mean or average.

Our framework is in the theory of information without probability [4, 5, 1].

In previous papers [7, 8, 9, 10] we have introduced the conditional information and the conditional entropy on fuzzy setting.

The aim of this paper is the characterization of classes of aggregation operators for the conditional information of some given fuzzy sets, all conditionated by another fuzzy set.

As regard to the aggregation operators, we propose an approach based on the axioms of the paper [2]: only the natural properties of idempotence, monotonicity and continuity from below are requested. These properties will been transformed into a system of functional equations. Some classes of solutions will been obtained.

2 Preliminaires

On crisp and fuzzy setting [7, 8, 9, 10] we have studied the conditional information without probability and conditional entropy without fuzzy measure for sets and partitions, respectively.

In [1] we have defined the information measure (without a fuzzy measure) for fuzzy sets: it is a map $J(\cdot) : \mathcal{F} \to [0, +\infty]$ such that:

(a) $J(X) = 0 \quad , \quad J(\emptyset) = +\infty \quad ,$

(b) $F_1 \subset F_2 \Longrightarrow J(F_1) \geq J(F_2) \quad .$

Given an information measure J as below, let X be an abstact space, $\mathcal{F}$ a family of fuzzy sets $F \in \mathcal{F}$ [11] and H another fuzzy set in $\mathcal{F}$ with $J(H) \neq +\infty$, which makes conditional the event F.

From axiomatic point of view, the conditional information is a function $J_H(\cdot) : \mathcal{F} \to [0, +\infty]$ such that:

(i) $F \subseteq F' \Longrightarrow J_H(F) \geq J_H(F')$,

(ii) $J_H(F) = J(F)$,

if F is not conditioned by H.

It follows that $J_H(\emptyset) = +\infty$ and $J_H(X) = 0$.

In [7] we have chosen the previous axioms because they agree with the typical axioms of non-conditioned information.

3 Statment of the problem

Let $\mathcal{J}$ the family of the conditional information $J_H(\cdot)$. We characterize the aggrega-

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tion operator $L : \mathcal{J} \to [0,\mathrm{k}]$, $0 \le \mathrm{k} \le [0,+\infty]$ of $n \in [0,+\infty]$ conditional informations $J_H(F_1), J_H(F_2), ..., J_H(F_n)$, with $F_i \in \mathcal{F}$, $\forall\ i = 1,...,n$, and $J(H) \neq +\infty$:

$$L\Big(J_H(F_1), J_H(F_2), ..., J_H(F_n)\Big) = r \quad ,$$

$0 \le r \le k$.

L has the following properties:

(I) *idempotence*

$$J_H(F_i) = c, \forall i = 1,...,n \implies L(\underbrace{c, c, ..., c}_{n}) = c \ ,$$

(II) *monotonicity*

$$J_H(F_1) \le J_H(F_1') \implies L\Big(J_H(F_1), ..., J_H(F_n)\Big) \le L\Big(J_H(F_1'), ..., J_H(F_n)\Big) \ ,$$

(III) *continuity from below*

$$J_H(F_{1m}) \nearrow J_H(F_1) \implies L\Big(J_H(F_{1m}), ..., J_H(F_n)\Big) \nearrow L\Big(J_H(F_1), ..., J_H(F_n)\Big) \ .$$

From [(I)-(III)], putting $J_H(F_i) = x_i \in [0,+\infty), i = 1,...,n$, we have the following system of functional equations:

$$\begin{cases} (I')\ L(c, c, ..., c) = c \ , \\ (II')\ x_1 \le x_1' \implies \\ L(x_1, ..., x_n) \le L(x_1', ..., x_n) \ , \\ (III')\ x_{1m} \nearrow x_1 \implies \\ L(x_{1m}, ..., x_n) \nearrow L(x_1, ..., x_n) \ . \end{cases}$$

4 Solution of the problem

In this paragraph, some classes of solutions of the system [(I')-(III')] are obtained.

Proposition 1 *Two solutions of the system [(I')-(III')] are:*

$$L(x_1, x_2, ..., x_n) = \bigwedge_{i=1}^{n} x_i \ ,$$

and

$$L(x_1, x_2, ..., x_n) = \bigvee_{i=1}^{n} x_i \ .$$

Proof. It is immediate the chech.

Proposition 2 *A class of solution of the system* [(I')-(III')] *is:*

$$L(x_1, x_2, ..., x_n) = h^{-1}\Big(\frac{h(x_1) + h(x_2) + ... + h(x_n)}{n}\Big)$$

where $h : [0,+\infty] \to [0,+\infty]$ *is a continuous, strictly function, with* $h(0) = 0$ *and* $h(+\infty) = +\infty$.

Proof. The proof is immediate.

Remark If the function h is linear, then the aggregation operator L is the aritmetic average.

5 Conclusion

In this paper we have given some classes of solutions of the system of functional equations [(I')-(III')]. We have found that the aggregation operator L of n conditional informations can be:

$$L\Big(J_H(F_1), ..., J_H(F_n)\Big) = \bigwedge_{i=1}^{n} J_H(F_i),$$

$$L\Big(J_H(F_1), ..., J_H(F_n)\Big) = \bigvee_{i=1}^{n} J_H(F_i),$$

$$L\Big(J_H(F_1), ..., J_H(F_n)\Big) = h^{-1}\Big(\frac{h(J_H(F_1)) + h(J_H(F_2)) + ... + h(J_H(F_n))}{n}\Big),$$

where $h : [0,+\infty] \to [0,+\infty]$ is a continuous, strictly function, with $h(0) = 0$ and $h(+\infty) = +\infty$.

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