A Restriction Level Approach to the Representation of Imprecise Properties

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Abstract

In this paper we introduce an alternative representation of imprecise properties that verify all the logical equivalencies of the crisp case. An imprecise property (or proposition) on the objects of an universe X is represented by a collection of crisp realizations, each one corresponding to a Restriction Level (RL). A fuzzy subset of X can be seen as a summary of a RL-representation, both representations being equivalent only for the combination of atomic properties via maximum and minimum. Our approach allows us to extend crisp operations to the imprecise case, keeping all the properties of the crisp case, notably those involving negation. **Keywords:** Representation of imprecision, fuzzy sets, restriction level, negation

1 Introduction

Fuzzy sets are one of the best models for representing and reasoning with imprecise properties. They have been successfully employed to deal with imprecision in many different areas. This has been possible because the classical set operations and concepts defined on sets have been extended to the fuzzy case. In general, there are many different ways to extend operations and definitions. A well known example are set intersection, union, and complement, that can be extended to the fuzzy case in a virtually infinite number of ways via t-norms, t-conorms, and fuzzy negations.

In this process of extension of operations and definitions to the fuzzy case, many properties of their classical counterparts can be lost or must be redefined in different ways. As an example, the law of excluded middle does not hold for most of the combinations of t-norm, t-conorm, and negation. One important problem of fuzzy logic is how to determine the most suitable fuzzy extensions of a certain crisp operation, what properties should be kept and what are its semantics.

This problem arises frequently when the operations and definitions we want to extend involve negations (eq. complement). This is the case with the definition of set difference. Given crisp sets R and S, $R\backslash S = R \cap \bar{S} = T$ so that $T \cup (R \cap S) = R$. However, for many of the most employed t-norms, t-conorms and negations, the previous properties do not hold. In particular, if F is a fuzzy set defined on a crisp universe X, $X\backslash F = X \cap \bar{F} = \bar{F}$ but $F \cup \bar{F} = X$ is not always true since the law of excluded middle does not hold in general. Another problem involving negation is the extension of implication as $\neg F \vee G$ and the corresponding set interpretation as a degree of inclusion (in particular, the original definitions of fuzzy set inclusion and equality are crisp).

These problems, and others, are very important in certain applications. The extension of set difference plays an important role in fuzzy

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database queries and in fuzzy arithmetic when we want to operate with cardinalities of fuzzy sets (without the law of excluded middle, one cannot expect $card(\bar{F}) = card(X) - card(F)$ in general for a cardinality measure $card$ yielding fuzzy integers). Implication is crucial for fuzzy reasoning and in the extension of properties that include implication in its definition, like transitivity of fuzzy relations for instance, i.e., $(R(a,b) \wedge R(b,c)) \rightarrow R(a,c)$. There are many other examples.

In this paper we introduce an alternative extensive representation of imprecision that verify all the classical properties of crisp sets, including those involving negation. The main contribution of our approach is that the extension of crisp concepts like operations, definitions and measures is direct and simple, and maintain all the properties of the crisp case, making the approach useful for applications that require those properties. The representation is based on the idea of restriction level (RL). An imprecise property is represented by a collection of crisp realizations or representatives, each one corresponding to a RL. Fuzzy sets, as represented by a collection of α-cuts, are a particular case of a RL representation where each restriction level corresponds to a value α. However, the new representation is more general, since the representatives are not necessarily nested with respect to inclusion. Fuzzy sets are very important for our representation since they are a way to obtain the RL representation of atomic properties, and can also be employed to obtain a summary of the representation for non-atomic properties.

2 Restriction-level representation

2.1 Atomic Properties and Restriction levels

Given a knowledge representation problem, an atomic property is a property that cannot be defined in terms of other properties in our problem. If the atomic property is imprecise, fulfilment is a matter of degree, i.e., some objects verify the property, some others don't, and the rest verify it to a certain extent. In this paper we shall assume the usual representation of fuzzy sets for atomic properties, i.e., degrees are in $[0,1]$ with the usual meaning.

In our approach, a *restriction* is a rule employed to obtain a crisp realization of an imprecise property. This rule specifies how to make a crisp decision about whether a certain object verifies or not the imprecise property if we use a certain level of restriction, i.e., if we are strict to a certain extent or level.

For the sake of coherence, in the case of atomic properties with imprecision degrees in $[0,1]$, this kind of rules are of the form $degree \geq \alpha$ with $\alpha \in (0,1]$, and hence restriction levels are associated to values $\alpha \in (0,1]$. In the same case, the crisp realization of an atomic imprecise property represented by a fuzzy set F in the restriction level α corresponds to the α-cut F_α. In other words, F_α is a crisp realization (representative, version) of F at the level α if we relax our restriction on the fulfilment of F so that $\forall x \in X$ we accept $x \in F$ iff $F(x) \geq \alpha$.

In this paper we consider that an imprecise property A is represented by a finite set of crisp realizations corresponding to a set of RLs $\Lambda_A \subseteq (0,1]$ containing at least the level $\alpha = 1$. This is not a practical limitation since humans are able to distinguish a limited number of restriction or precision levels and, in practice, the limit in precision and storage of computers allows us to work with a finite number of degrees (and consequently, of levels) only. We introduce the following definition:

Definition 2.1 *A* RL-set Λ *is a finite set of restriction levels* $\Lambda = \{\alpha_1, \ldots, \alpha_m\}$ *verifying* $1 = \alpha_1 > \alpha_2 > \cdots > \alpha_m > \alpha_{m+1} = 0$, $m \geq 1$.

In a practical situation, the RL-set for an atomic property represented by a fuzzy set A is defined as follows:

Definition 2.2 *Let A be a fuzzy set on X. Then*

$$\Lambda_A = \{A(x) \mid x \in support(A)\} \cup \{1\} \quad (1)$$

The RL-set employed to represent an imprecise property is obtained as the union of the

RL-sets of the atomic properties in terms of which the property is defined. We explain this idea in section 3.

2.2 Representation

We define a restriction-level representation (RL-representation for short) of an imprecise property on X as follows:

Definition 2.3 *A RL-representation is a pair* (Λ, ρ) *where* Λ *is a RL-set and* ρ *is a function*

$$\rho : \Lambda \to \mathcal{P}(X) \tag{2}$$

The function ρ indicates the crisp realization that represents the imprecise property for each restriction level. As an example, the RL-representation for an atomic imprecise property defined by a fuzzy set A is the pair (Λ_A, ρ_A), where Λ_A is obtained using equation (1), and $\rho_A(\alpha) = A_\alpha \ \forall \alpha \in \Lambda_A$. We introduce the following definition:

Definition 2.4 *Given an imprecise property* P *represented by* (Λ_P, ρ_P)*, we define the set of crisp representatives of* P*,* Ω_P*, as*

$$\Omega_P = \{\rho_P(\alpha) \mid \alpha \in \Lambda_P\} \tag{3}$$

For an atomic property A, the set of crisp representatives Ω_A is the set of significant α-cuts of A, as we have seen. However, notice that in definition 2.3 there is no restriction about the possible crisp representatives for non-atomic properties. In particular, as a consequence of operations, they don't need to be nested. We shall see examples coming from operations in section 3.

In order to define properties by operations, it is convenient to extend the function ρ to any RL $\alpha \in (0, 1]$ as follows:

Definition 2.5 *Let* (Λ, ρ) *be a RL-representation with* $\Lambda = \{\alpha_1, \ldots, \alpha_m\}$ *verifying* $1 = \alpha_1 > \alpha_2 > \cdots > \alpha_m > \alpha_{m+1} = 0$*. Let* $\alpha \in (0, 1]$ *and* $\alpha_i, \alpha_{i+1} \in \Lambda$ *such that* $\alpha_i \geq \alpha > \alpha_{i+1}$*. Then*

$$\rho(\alpha) = \rho(\alpha_i) \tag{4}$$

This idea is natural if we think of α-cuts since in the conditions of definition 2.5, for any fuzzy set A, $A_\alpha = A_{\alpha_i}$. Using this definition, we define the equivalence of RL-representations as follows:

Definition 2.6 *Let* (Λ, ρ) *and* (Λ', ρ') *be two RL-representations on* X*. We say that both representations (and the corresponding properties) are* equivalent*, noted* $(\Lambda, \rho) \equiv (\Lambda', \rho')$*, iff* $\forall \alpha \in (0, 1]$

$$\rho(\alpha) = \rho'(\alpha).$$

In particular, the following proposition holds:

Proposition 2.1 *Let* (Λ, ρ) *be a RL-representation and let* $\Lambda \subseteq \Lambda'$*. Then* $(\Lambda, \rho) \equiv (\Lambda', \rho)$*.*

Proof: Trivial. □

2.3 Fuzzy summary of a RL-representation

Given a RL-representation (Λ_A, ρ_A) for an atomic property A, the values of Λ_A can be interpreted as values of possibility of a possibility measure defined $\forall \rho_A(\alpha_i) \in \Omega_A$ as

$$Pos(\rho_A(\alpha_i)) = \alpha_i. \tag{5}$$

Following this interpretation we define a basic probability assignment in the usual way:

Definition 2.7 *Let* (Λ, ρ) *be a RL-representation with crisp representatives* Ω*. We define the associated probability distribution* $m : \Omega \to [0, 1]$ *as*

$$m(Y) = \sum_{\alpha_i \mid Y = \rho(\alpha_i)} \alpha_i - \alpha_{i+1}. \tag{6}$$

It is easy to show that

$$\sum_{Y \in \Omega} m(Y) = \sum_{\alpha_i \in \Lambda} m(\rho(\alpha_i)) = 1 \tag{7}$$

Using this basic probability assignment it is possible to obtain an object-centered summary of the information available about the imprecise property as a fuzzy set. We introduce the following definition:

Definition 2.8 *Let (Λ, ρ) be a RL-representation. We define the associated fuzzy summary $\mu : X \to [0,1]$ as*

$$\mu(x) = \sum_{Y \in \Omega \mid x \in Y} m(Y) \tag{8}$$

The following proposition holds for atomic properties, but not in general:

Proposition 2.2 *Let (Λ_A, ρ_A) be the RL-representation for an atomic property represented by a fuzzy set A. Then $\forall x \in X$*

$$\mu_A(x) = A(x). \tag{9}$$

Proof: Trivial. □

As a consequence, fuzzy sets and RL-representations are equivalent for atomic properties, but not for derived properties. In the latter case, fuzzy sets are a way to summarize the information given by the RL-representation in an easy-to-understand way. However, both representations are not equivalent in general as a result of the way we define operations. We shall see our definitions of operations and examples of the non-equivalence between RL-representations and fuzzy sets in the next section.

3 Logical operations

Atomic properties can be combined to form other properties via conjunction, disjunction, and negation in the usual way. Our approach to operations relies on the following ideas:

1. There is a single membership scale, i.e., let F and G be fuzzy subsets of X, then $F(x) = G(x')$ means that x is F just like x' is G. Hence, it makes sense to consider the representation of a knowledge base in a certain restriction level as a knowledge base consisting of the representation in that RL of all the properties and concepts that form the KB.

2. Crisp operations are extended to imprecise properties by operating in each restriction level. In general, let $f : \mathcal{P}(X)^n \to \mathcal{P}(X)$ be a crisp operation. Then, f is extended to RL-representations as follows: let $(P_1, \ldots, P_n)$ be imprecise properties defined on X with P_i represented by a RL-representation $(\Lambda_{P_i}, \rho_{P_i})$. Then, $f(P_1, \ldots, P_n)$ is represented by $(\Lambda_{f(P_1,\ldots,P_n)}, \rho_{f(P_1,\ldots,P_n)})$ where

$$\Lambda_{f(P_1,\ldots,P_n)} = \bigcup_{1 \le i \le n} \Lambda_{P_i} \tag{10}$$

and, $\forall \alpha \in \Lambda_{f(P_1,\ldots,P_n)}$,

$$\rho_{f(P_1,\ldots,P_n)}(\alpha) = f(\rho_{P_1}(\alpha), \ldots, \rho_{P_n}(\alpha)) \tag{11}$$

The representation in a certain RL of the extension of a certain crisp operation is the result of that operation on the representatives in the same RL of the arguments involved.

Using these ideas, the logical operations between imprecise properties represented by RL-representations can be defined by extending the corresponding set operators in their extensive representations. Conjunction and disjunction are extended as follows:

Definition 3.1 *Let P, Q be imprecise properties represented by (Λ_P, ρ_P), (Λ_Q, ρ_Q). Then, $P \wedge Q$ and $P \vee Q$ are imprecise properties represented by $(\Lambda_{P\wedge Q}, \rho_{P\wedge Q})$ and $(\Lambda_{P\vee Q}, \rho_{P\vee Q})$, respectively, where*

$$\Lambda_{P\wedge Q} = \Lambda_{P\vee Q} = \Lambda_P \cup \Lambda_Q \tag{12}$$

with, $\forall \alpha \in (0,1]$,

$$\rho_{P\wedge Q}(\alpha) = \rho_P(\alpha) \cap \rho_Q(\alpha) \tag{13}$$

and

$$\rho_{P\vee Q}(\alpha) = \rho_P(\alpha) \cup \rho_Q(\alpha) \tag{14}$$

The idea of operation by crisp representatives of the same RL is not new. In the case of fuzzy sets, fuzzy intersection and union can be performed by α-cuts. As it is well known, the results (also in the case of RL-representation of atomic properties) are equivalent to combine degrees via minimum and maximum, respectively. This means in particular that imprecise properties derived from atomic properties by using $\wedge$ and $\vee$ only are compatible with fuzzy sets when these operations are performed via minimum and maximum, in the sense that the corresponding RL-representation yields the usual nested α-cut representation of fuzzy sets.

However, this is not true in general when negation is employed. We define the negation operator as follows:

Definition 3.2 *Let P be an imprecise property represented by (Λ_P, ρ_P). Then, $\neg P$ is an imprecise property represented by $(\Lambda_{\neg P}, \rho_{\neg P})$, where*

$$\Lambda_{\neg P} = \Lambda_P \tag{15}$$

and, $\forall \alpha \in (0, 1]$,

$$\rho_{\neg P}(\alpha) = \overline{\rho_P(\alpha)} \tag{16}$$

where $\overline{Y}$ is the usual set complement of a crisp set Y.

As an example of negation, consider an universe $X = \{x_1, \ldots, x_5\}$ and the atomic property A on X defined by the fuzzy set

$$A = 1/x_1 + 0.8/x_2 + 0.5/x_3 + 0.4/x_5$$

Table 1 shows the RL-representations for A and $\neg A$ on the RL-set

$$\Lambda_A = \Lambda_{\neg A} = \{1, 0.8, 0.5, 0.4\}.$$

α	$\rho_A(\alpha)$	$\rho_{\neg A}(\alpha)$
1	$\{x_1\}$	$\{x_2, x_3, x_4, x_5\}$
0.8	$\{x_1, x_2\}$	$\{x_3, x_4, x_5\}$
0.5	$\{x_1, x_2, x_3\}$	$\{x_4, x_5\}$
0.4	$\{x_1, x_2, x_3, x_5\}$	$\{x_4\}$

Table 1: Negation of A.

Table 2 shows another example of negation for the atomic property $B = 0.9/x_1 + 0.6/x_3 + 0.5/x_4$. In this case,

$$\Lambda_B = \Lambda_{\neg B} = \{1, 0.9, 0.6, 0.5\}.$$

As we can see, the representation of the negation of an atomic property (e.g. $\neg A$) is not the set of α-cuts of the complement of the corresponding fuzzy set (e.g. the fuzzy set $\overline{A}(x) = 1 - A(x)$). This is also true for B in table 2. However, the following proposition holds:

α	$\rho_B(\alpha)$	$\rho_{\neg B}(\alpha)$
1	$\emptyset$	X
0.9	$\{x_1\}$	$\{x_2, x_3, x_4, x_5\}$
0.6	$\{x_1, x_3\}$	$\{x_2, x_4, x_5\}$
0.5	$\{x_1, x_3, x_4\}$	$\{x_2, x_5\}$

Table 2: Negation of B.

Proposition 3.1 *Let A be a fuzzy set representing an atomic property. Then*

$$\mu_{\neg A}(x) = \overline{A}(x) = 1 - A(x) \tag{17}$$

Proof: Let $x \in X$ with degree $A(x)$. Then,

$$\sum_{Y \in \Omega_A \mid x \in Y} m_A(Y) = A(x)$$

Since $\sum_{Z \in \Omega_{\neg A}} m_{\neg A}(Z) = 1$ and $x \in Y \in \Omega_A$ implies $x \notin \overline{Y} \in \Omega_A$, and $m_A(Y) = m_{\neg A}(\overline{Y})$, we have

$$\sum_{\overline{Y} \in \Omega_{\neg A} \mid x \in \overline{Y}} m_{\neg A}(\overline{Y}) = 1 - A(x)$$

□

Proposition 3.1 tell us that the summary of the RL-representation of $\neg A$ is the fuzzy complement of the fuzzy set A by means of the standard negation $n(x) = 1 - x$. This means that, from the point of view of operations on degrees, both are equivalent. However, the structure of the RL-representation of $\neg A$ and the RL-representation obtained from the fuzzy set $\overline{A}$ are not the same.

As an example of combination of operations, tables 3 and 4 show the representation of several properties derived from A and B. In particular, in table 3 we have an example of RL-representation, that of the property $A \wedge \neg B$, in which the different crisp realizations corresponding to RL's are not nested.

The following is one important property of RL-representations with the operations we have defined in this section:

α	$\rho_A(\alpha)$	$\rho_{\neg A}(\alpha)$	$\rho_B(\alpha)$	$\rho_{\neg B}(\alpha)$	$\rho_{A\wedge\neg B}(\alpha)$	$\rho_{B\wedge\neg A}(\alpha)$
1	$\{x_1\}$	$\{x_2,x_3,x_4,x_5\}$	$\emptyset$	X	$\{x_1\}$	$\emptyset$
0.9	$\{x_1\}$	$\{x_2,x_3,x_4,x_5\}$	$\{x_1\}$	$\{x_2,x_3,x_4,x_5\}$	$\emptyset$	$\emptyset$
0.8	$\{x_1,x_2\}$	$\{x_3,x_4,x_5\}$	$\{x_1\}$	$\{x_2,x_3,x_4,x_5\}$	$\{x_2\}$	$\emptyset$
0.6	$\{x_1,x_2,x_3\}$	$\{x_4,x_5\}$	$\{x_1,x_3\}$	$\{x_2,x_4,x_5\}$	$\{x_2\}$	$\emptyset$
0.5	$\{x_1,x_2,x_3\}$	$\{x_4,x_5\}$	$\{x_1,x_3,x_4\}$	$\{x_2,x_5\}$	$\{x_2\}$	$\{x_4\}$
0.4	$\{x_1,x_2,x_3,x_5\}$	$\{x_4\}$	$\{x_1,x_3,x_4\}$	$\{x_2,x_5\}$	$\{x_2,x_5\}$	$\{x_4\}$

Table 3: Several properties derived from the atomic properties A and B (I).

α	$\rho_{A\wedge B}(\alpha)$	$\rho_{A\vee B}(\alpha)$	$\rho_{\neg(A\wedge B)}(\alpha)$	$\rho_{\neg(A\vee B)}(\alpha)$	$\rho_{\neg A\vee\neg B}(\alpha)$
1	$\emptyset$	$\{x_1\}$	X	$\{x_2,x_3,x_4,x_5\}$	X
0.9	$\{x_1\}$	$\{x_1\}$	$\{x_2,x_3,x_4,x_5\}$	$\{x_2,x_3,x_4,x_5\}$	$\{x_2,x_3,x_4,x_5\}$
0.8	$\{x_1\}$	$\{x_1,x_2\}$	$\{x_2,x_3,x_4,x_5\}$	$\{x_3,x_4,x_5\}$	$\{x_2,x_3,x_4,x_5\}$
0.6	$\{x_1,x_3\}$	$\{x_1,x_2,x_3\}$	$\{x_2,x_4,x_5\}$	$\{x_4,x_5\}$	$\{x_2,x_4,x_5\}$
0.5	$\{x_1,x_3\}$	$\{x_1,x_2,x_3,x_4\}$	$\{x_2,x_4,x_5\}$	$\{x_5\}$	$\{x_2,x_4,x_5\}$
0.4	$\{x_1,x_3\}$	X	$\{x_2,x_4,x_5\}$	$\emptyset$	$\{x_2,x_4,x_5\}$

Table 4: Several properties derived from the atomic properties A and B (II).

Proposition 3.2 *Operations on RL-representations verify all the ordinary properties of logical equivalence.*

Proof: Immediate since the logical operations between RL-representations are performed via the corresponding set operations on crisp sets in each restriction level, that verify the corresponding set-equality properties, and the equivalence of RL-representations is defined in terms of equality of the representatives in all levels. □

By proposition 3.2, properties like $\neg\neg A \equiv A$, De Morgan's laws ($\neg(A\wedge B) \equiv (\neg A\vee\neg B)$, one of them, is illustrated in table 4) and the law of excluded middle in any of its versions hold. The latter can be expressed as $A \wedge \neg A \equiv \bot$ or $A \vee \neg A \equiv \top$, where $\top$ and $\bot$ are atomic properties representing respectively a tautology (RL-representation obtained from X) and a contradiction (whose representation is obtained from $\emptyset$).

4 Discussion

The representation of imperfect properties by means of a collection of crisp sets has been employed in many ways, in particular in the representation theorems of fuzzy sets, rough sets [5], and evidence theory [8], among others. The idea of representing imprecise properties starting from atomic properties has been employed before with the same objective of keeping the equivalence properties of the crisp case [6, 7], and is something usual in the area of fuzzy description logics. In this paper we obtain the RL-representation of an atomic property from a fuzzy set; however, there are other possibilities that will be considered in forthcoming papers.

With respect to fuzzy sets, RL-representations need some more storage space and time for operations (though a finite, suitable for the problem number of RL's will maintain a linear complexity), and the definitions of conjunction, disjunction, and negation are unique; however, t-norms and t-conorms can be employed as aggregation operators in order to obtain the degrees of atomic properties. The interpretability of RL-representations is easier by using the idea of fuzzy summary; we shall introduce linguistic summaries in forthcoming papers.

The new representation is very useful for those applications where we need to keep the properties of logical equivalence of the crisp case. Some approaches based on α-cuts to problems like cardinality of fuzzy sets [2], evaluation of quantified sentences [4], fuzzy description logics [1], and representation and arithmetic of fuzzy integers [3] are based on the representation of atomic properties by means of α-cuts and operations by means of maximum and minimum. These approaches can be extended and improved by using RL-representations, in particular with respect to the fulfilment of properties involving negation. We are currently working in this line. In particular, by means of the extension of measures, we have developed a representation and arithmetic of imprecise numbers (from naturals to complex numbers) that verify all the arithmetic properties. The arithmetic of these RL-numbers has the remarkable feature that they don't increase the imprecision of the results of arithmetic operations, contrary to the representation of imprecise numbers by means of collections of intervals. This work is described in a currently submitted paper.

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