Cardinalities of Granules of Vague Data

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Abstract

Accumulation of vague data in a vague category is studied. Axiomatic definitions of scalar and fuzzy cardinalities of a granule of vague data are proposed. Some approaches to the construction of scalar and fuzzy cardinalities are suggested and illustrated with a sample of vague data.

Keywords: Vague data, Fuzzy sets, Cardinality, Frequency functions, Histogram.

1 Introduction

Vague data incorporating non-statistical uncertainty are often analyzed by methods of fuzzy sets. We will consider a sample of vague observations described by fuzzy numbers defined on a closed interval of real numbers. A crisp or fuzzy partition of this interval will discretize the sample into a collection of crisp or fuzzy granules. Because a vague observation may belong to different granules with different degrees of membership, cardinality of each granule is, in general, non-precise. Work on accumulation of vague data has already been initiated by several researchers with the goal to construct a "fuzzy histogram". Viertl [12] suggested a method for the construction of a histogram from fuzzy quantities distributed over crisp intervals. His work was generalized in [2] where the notion of a generalized histogram was proposed. In [1], the approximation of fuzzy quantities by their α-cuts was used for evaluation of the fuzziness and the roughness of a generalized histogram. The inclusion of α-cuts of fuzzy numbers in a crisp interval was considered by Viertl and Trutching [13] as the basis for definitions of the lower and the upper relative frequencies at level α. A quasi-continuous histogram was introduced by Comby and Strauss [4]. In this type of histogram, the lower and the upper boundaries for non-precise counts of vague data were evaluated by means of possibility theory.

Cardinality belongs to the most fundamental characterizations of a set. A measure of cardinality of a fuzzy set can be either a real number (scalar cardinality) or a fuzzy set defined over the non-negative integers (fuzzy cardinality). Scalar cardinalities were considered, e.g., by De Luca and Termini [6], Ralescu [11] and Dubois and Prade [8]. Fuzzy cardinalities were explored, e.g., by Zadeh [18], Dubois and Prade [8] and Delgado et al [5]. In recent years, more attention has been given to axiomatization of cardinalities. Wygralak [14, 15] developed an axiomatic theory of scalar cardinalities of fuzzy sets with finite support. Casasnovas and Torrens [3] introduced axiomatization of fuzzy cardinalities of a finite fuzzy set. Deschrijver and Král' [7] proposed an axiomatic theory of cardinalities of interval-valued fuzzy sets. We suggest axiomatic definitions of scalar and fuzzy cardinalities of granules of vague data. Our work is organized as follows: Section 2 provides some basic notions and notations from the theory of fuzzy sets. Axiomatic definitions of scalar and fuzzy cardinalities of finite fuzzy sets as proposed by Wygralak [14, 15] and Casasno-

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vas and Torrens [3] are recalled. In Section 3 we explain the meaning of a granule of vague data and we illustrate it with a small sample of non-precise observations. Definitions of scalar and fuzzy cardinalities of a granule of vague data are presented in Section 4 and Section 5, respectively. Concluding remarks are in Section 6.

2 Preliminaries

In this paper we will adopt, in general, notations from [14] and [3]. We will use the standard notation $|.|$ for the cardinality of a crisp set. Let M be a universal set, finite or not. A fuzzy set A on M [19] defined by membership function $A : M \to [0,1]$ is a finite fuzzy set (ffs) if the support of A, $suppA = \{x \in M : A(x) > 0\}$, is a finite subset of M. We will use notation $FFS(M)$ for the family of all ffs on M, and notation T for the ffs such that $T(x) = 0$ for all $x \in M$. For $A, B \in FFS(M)$, $x \in M$, we use $(A \cup B)(x) = A(x) \vee B(x)$ and $(A \cap B)(x) = A(x) \wedge B(x)$, where $\vee$ stands for the supremum and $\wedge$ for the infimum. The height of A is $h(A) = \bigvee_{x \in M} A(x)$. A fuzzy singleton is a ffs on M denoted by a/x, such that: $a \in [0,1]$, $x \in M$ and $a/x(y) = 0$ if $y \neq x$. Wygralak [14, 15] proposed an axiomatic definition of a scalar cardinality of a finite fuzzy set as follows:

Definition 1 *A function* $\sigma : FFS(M) \to [0, \infty)$ *will be called a scalar cardinality if the following postulates are satisfied for each* $a, b \in [0,1]$, $x, y \in M$ *and* $A, B \in FFS(M)$:
P1. $\sigma(1/x) = 1$,
P2. $a \leq b \Rightarrow \sigma(a/x) \leq \sigma(b/y)$,
P3. $A \cap B = T \Rightarrow \sigma(A \cup B) = \sigma(A) + \sigma(B)$.

A fuzzy cardinality of a ffs is expressed by a generalized natural number (gnn), which is a fuzzy set on the set of all natural numbers N. We will use notation $CGNN$ for the set of convex finite generalized natural numbers. For $\alpha, \beta \in CGNN$ extended addition [16] is defined by

$$(\alpha \oplus \beta)(k) = \bigvee \{\alpha(i) \wedge \beta(j) : i + j = k\}. \quad (1)$$

Casasnova and Torrens [3] introduced an axiomatic definition of a fuzzy cardinality of a finite fuzzy set as follows:

Definition 2 *A function* $\gamma : FFS(M) \to CGNN$ *is a fuzzy cardinality if and only if it satisfies the following conditions for each* $a, b \in [0,1]$, $x, y \in M$ *and* $A, B \in FFS(M)$:
P1. If $A \cap B = T$ *then* $\gamma(A \cup B) = \gamma(A) \oplus \gamma(B)$.
P2. If $i > |suppA|$ *and* $j > |suppB|$ *then* $\gamma(A)(i) = \gamma(B)(j)$.
P3. If A *is a crisp subset of* M, *then* $\gamma(A)(i) \in \{0,1\}$ *for all* $i \in N$, *and if* $n = |suppA|$, *then* $\gamma(A)(n) = 1$.
P4. If $a \leq b$ *then* $\gamma(a/x)(0) \geq \gamma(b/y)(0)$, *and* $\gamma(a/x)(1) \leq \gamma(b/y)(1)$.

Special cases of generalized natural numbers are FGCounts, FLCounts and FECounts developed by Zadeh [20]. They are based on the following idea: For $A \in FFS(M)$, $t \in [0,1]$ and $k \in N$,

$$A_t = \{x \in M : A(x) \geq t\}, \quad (2)$$

and

$$[A]_k = \bigvee \{t \in [0,1] : |A_t| \geq k\}. \quad (3)$$

Then $FGC(A)(k) = [A]_k$, $FLC(A)(k) = 1 - [A]_{k+1}$ and $FEC(A) = FGC(A) \cap FLC(A)$. Here "F" stands for "fuzzy", "L" stands for "less than or equal to" and "G" and "E" stand for "greater than or equal to" and "equal to", respectively. We will show how scalar and fuzzy cardinalities of a ffs can help to evaluate non-precise accumulation of fuzzy numbers in a fuzzy interval.

3 Granules of vague data

It is generally agreed that vague data can be modeled by fuzzy intervals [12]. Fuzzy intervals are special fuzzy sets defined on the real line $\mathbf{R}$. We will denote the family of all fuzzy intervals by $\mathcal{F}(\mathbf{R})$. A membership function of a fuzzy interval D is a function $D : \mathbf{R} \to [0,1]$ such that for all $t \in (0,1]$ the set D_t given by (2) is a non-empty crisp interval. A fuzzy interval with the membership grade equal to 1 for exactly one real number will be called in this paper a fuzzy number. Let a, b, c, d be real numbers such that $a \leq b \leq c \leq d$. Then a trapezoidal fuzzy interval D denoted by quadruple $< a, b, c, d >$ has the membership function

$$D(x) = \begin{cases} \frac{x-a}{b-a} & \text{if } a < x < b, \\ 1 & \text{if } b \leq x \leq c, \\ \frac{d-x}{d-c} & \text{if } c < x < d, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

If $b = c$ then D is a triangular fuzzy number. Fuzzy set A is a subset of fuzzy set B if and only if $A(x) \leq B(x)$ for all $x \in \mathbf{R}$. Cardinality of an infinite fuzzy set A on $\mathbf{R}$ will be evaluated by $|A| = \int_{\mathbf{R}} A(x)\,dx$, if the integral exists. If the support of A is a bounded interval, we will use the notation $||suppA||$ for its length.

Assume a sample S of n non-precise observations described by fuzzy numbers $X_1, \ldots, X_n$. For each $i \in N_n = \{1, \ldots, n\}$ the support of X_i is the interval of real numbers (a_i, b_i). Let $a_S = \min_{i \in N_n} a_i$ and $b_S = \max_{i \in N_n} b_i$. Then interval $R_S = [a_S, b_S]$ will be called the range of sample S. Let $C = \{C_1, \ldots, C_m\}$ be a partition of R_S into m crisp or fuzzy intervals such that $\max_{X_i \in S} ||suppX_i|| < \min_{C_j \in C} ||suppC_j||$ and $\sum_{j=1}^{m} C_j(x) = 1$ for all $x \in \mathbf{R_S}$. The partition of S due to C can be described by the family of granules $S/D = \{S/C_1, \ldots, S/C_m\}$. For each $C_j \in C$, the cardinality of S/C_j can be interpreted as the "height" of the " fuzzy bar" of a fuzzy histogram displaying accumulation of elements from S in class C_j. We will illustrate granulation of vague data on a small data set from [1].

Example 1 The variable of interest is water level X repeatedly measured (observed) in cm on 12 different locations in a river. From the repeated measurements on the i-th location, a triangular fuzzy number $X_i =< a_i, b_i, c_i >$ was constructed as follows: a_i = first quartile, b_i = median and c_i = third quartile of the measurements. Non-precise observations characterized by triangular fuzzy numbers are in Table 1. The range of the sample $R_S = [65, 175]$ is divided into vague categories *low water level* described by fuzzy interval $C_1 =< 65, 65, 75, 105 >$, *medium water level* described by $C_2 =< 75, 105, 135, 165 >$ and *high water level* described by $C_3 =< 135, 165, 175, 175 >$. For all $x \in R_S$ we have that $\sum_{j=1}^{3} C_j(x) = 1$ and therefore $C = \{C_1, C_2, C_3\}$ is a fuzzy partition of R_S. Sample S is depicted in Figure 1. Granulation of S due to C is shown in Figure 2.

Table 1: Numerical representation of sample S from Example 1

X_i	a_i	b_i	c_i
X_1	65	70	75
X_2	75	80	85
X_3	80	85	95
X_4	85	95	105
X_5	95	100	105
X_6	105	115	125
X_7	115	120	130
X_8	120	130	135
X_9	125	140	145
X_{10}	150	155	160
X_{11}	155	160	170
X_{12}	165	170	175

In the next section we will study how to evaluate accumulation of fuzzy numbers from S in fuzzy intervals from C.

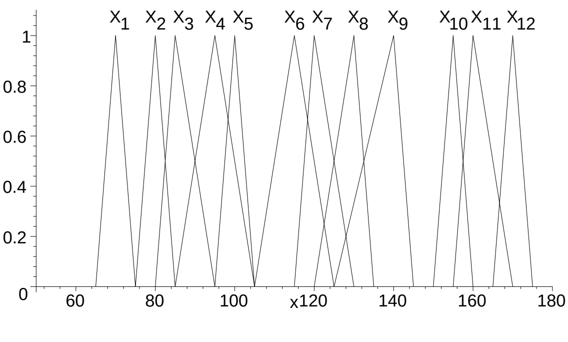

Figure 1: Graphical representation of sample S from Example 1

4 Scalar cardinality

Let D be a fuzzy interval and let S be a family of fuzzy numbers as described in Section 3. Evaluation of accumulation of fuzzy numbers

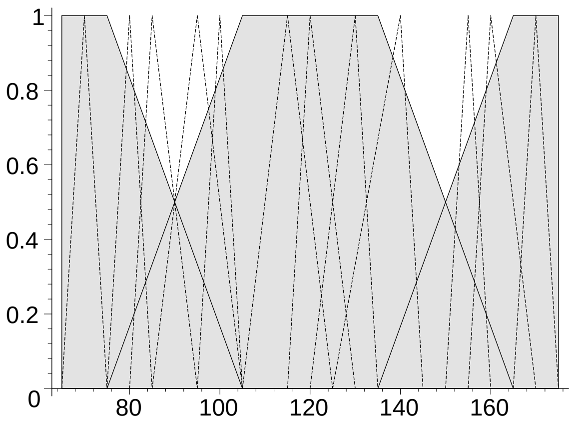

Figure 2: Granulation of sample S

from S in D is described as the cardinality of granule S/D. A fuzzy number $X_i \in S$ can be included in D fully or partially or not at all. Therefore the total count of fuzzy numbers in D is, in general, non-precise. A non-precise scalar count (scalar cardinality of a granule of vague data) will be evaluated by a non-negative real number.

Definition 3 *Let* $S = \{X_1, \ldots, X_n\}$ *be a family of fuzzy numbers and let* $\mathcal{F}_S(\mathbf{R}) = \{D \in \mathcal{F}(\mathbf{R}) : \max_{X_i \in S} ||suppX_i|| < ||suppD||\}$. *A scalar cardinality of granule* S/D, $D \in \mathcal{F}_S(\mathbf{R})$, *is the value* $\rho(D)$ *of the function* $\rho : \mathcal{F}_S(\mathbf{R}) \to [0, n]$, *which satisfies the following properties:*

P1. if for all $X_i \in S$, $X_i \cap D = T$ *then* $\rho(D) = 0$,
P2. if for all $X_i \in S$, $X_i \subset D$ *then* $\rho(D) = n$,
P3. if for $B, D \in \mathcal{F}_S(\mathbf{R})$, $B \subset D$ *then* $\rho(B) \leq \rho(D)$,
P4. if for $B, D \in \mathcal{F}_S(\mathbf{R})$, $D \cap B = T$ *and* $D \cup B \in \mathcal{F}_S(\mathbf{R})$ *then* $\rho(D \cup B) = \rho(D) + \rho(B)$.

One of the possible constructions of function ρ from Definition 3 is given in the following procedure.

Procedure 1
Step 1: Find fuzzy relation (degree of inclusion) $\varepsilon : S \times \mathcal{F}_S(\mathbf{R}) \to [0, 1]$ such that $\rho_\varepsilon(D) = \varepsilon(X_i, D)$ satisfies properties $P1 - P4$ from Definition 3 for $n = 1$.
Step 2: Create fuzzy set $\varphi_D : S \to [0, 1]$ such that for all $X_i \in S$

$$\varphi_D(X_i) = \varepsilon(X_i, D). \quad (5)$$

Step 3: Define $\rho(D)$ as a scalar cardinality of fuzzy set φ_D. Any scalar cardinality satisfying axioms from Definition 1 can be used.

Example 1 - continued
Different inclusion grades ε and different scalar cardinalities of fuzzy set φ can be used in Procedure 1. For more information about different degrees of inclusion see [9] and for more information about different scalar cardinalities of a finite fuzzy set see [15]. We will choose

$$\varepsilon(X_i, C_j) = \frac{|X_i \cap C_j|}{|X_i|} \quad (6)$$

and

$$\sigma(\varphi_{C_j}) = \sum_{i=1}^{n} \varphi_{C_j}(X_i). \quad (7)$$

Functions φ_{C_j} representing inclusion of X_i in C_j, $i = 1, \ldots, 12$, $j = 1, 2, 3$, are in Table 2.

Table 2: Inclusion of X_i in C_j

X_i	$\varphi_{C_1}(X_i)$	$\varphi_{C_2}(X_i)$	$\varphi_{C_3}(X_i)$
X_1	1.000	0.000	0.000
X_2	0.970	0.288	0.000
X_3	0.856	0.600	0.000
X_4	0.500	0.875	0.000
X_5	0.286	0.970	0.000
X_6	0.000	1.000	0.000
X_7	0.000	1.000	0.000
X_8	0.000	1.000	0.000
X_9	0.000	0.980	0.144
X_{10}	0.000	0.546	0.880
X_{11}	0.000	0.191	0.979
X_{12}	0.000	0.000	1.000

From Table 2 we obtain that $\rho(C_1) = 3.162$, $\rho(C_2) = 7.45$, and $\rho(C_3) = 3.003$. Because categories C_1, C_2 and C_3 are not mutually disjunctive, $\rho(C_1) + \rho(C_2) + \rho(C_3) \neq 12$.

Scalar cardinalities listed above provide quick information about the "size" of each granule. They can be used, for example, for ordering of granules. We can see that the largest granule is the category of observations labeled as the "medium water level" followed by the categories "low water level" and "high water level". However, interpretation of scalar cardinalities in terms of counts is not clear. If, for example, $\rho(C_1) = 1$, it does not necessarily mean that exactly one observation is included in C_1. There could be several observations partially included in C_1. Better interpretation of accumulation of vague data in terms of "counts" can be provided by fuzzy cardinality. It will be discussed in the next section.

5 Fuzzy cardinality

It is obvious that a precise count (a natural number) may represent the accumulation of vague data in a vague category only to a certain degree (possibility). A fuzzy set defined on the set of all natural numbers will be used for characterization of the fuzzy cardinality of a granule of vague data.

Definition 4 *Let S be a family of n fuzzy numbers. Then for $D \in \mathcal{F}_S(\mathbf{R})$, fuzzy set $w_D : N \to [0,1]$ is called a fuzzy cardinality of granule S/D if and only if the following properties are satisfied:*

P1. if for all $X_i \in S$, $X_i \cap D = T$ then $w_D(t) = 0$ for $t > 0$ and $w_D(0) = 1$,
P2. if for all $X_i \in S$, $X_i \subset D$ then $w_D(t) = 0$, for all $t \neq n$ and $w_D(n) = 1$,
P3. if for $B, D \in \mathcal{F}_S(\mathbf{R})$, $D \subset B$ we have that $w_D(t_1) = w_B(t_2) = 1$ then $t_1 \leq t_2$,
P4. if for $B, D \in \mathcal{F}_S(\mathbf{R})$, $D \cap B = T$ and $D \cup B \in \mathcal{F}_S(\mathbf{R})$ we have that $w_D(t_1) = w_B(t_2) = w_{D\cup B}(t_3) = 1$ then $t_3 = t_1 + t_2$,
P5. if $t_1 \leq t_2 \leq t_3$ then $w_D(t_2) \geq \min\{w_D(t_1), w_D(t_3)\}$.

One of the possible constructions of function w_D from Definition 4 is given in the following procedure.

Procedure 2
Assume the degree of inclusion ε and fuzzy set φ_D as in Procedure 1.
Step 1: Create fuzzy set $\underline{w}_D : N \to [0,1]$ such that $\underline{w}_D(t) = FGC(\varphi_D)(t)$ for all $t \in N$. Note that $\underline{w}_D(t)$ can be interpreted as the degree of possibility that at least t elements from S are included in D. Function $\underline{w}_D$ will be called the lower fuzzy frequency function associated with D.
Step 2: Create fuzzy set $\overline{w}_D : N \to [0,1]$ as follows: $\overline{w}_D(t) = FLC(\varphi_D)(t)$ for all $t \in N$. Note that $\overline{w}_D(t)$ can be interpreted as the degree of possibility that at most t elements from S are included in D. Function $\overline{w}_D$ will be called the upper fuzzy frequency function associated with D.
Step 3: Create fuzzy set $w_D : N \to [0,1]$ as follows: $w_D = FEC(\varphi_D)$. The value of $w_D(t)$ evaluates the degree of possibility that the total number of elements from S included in D is exactly t. Function w_D (fuzzy cardinality of granule S/D) can also be called the fuzzy frequency function associated with D.

Example 1 - continued
We will use the degree of inclusion ε given by (6). Then the lower fuzzy frequency function associated with C_1 is

$$\underline{w}_{C_1}(t) = \begin{cases} 1 & \text{if } t \in \{0,1\}, \\ 0.970 & \text{if } t = 2, \\ 0.856 & \text{if } t = 3, \\ 0.500 & \text{if } t = 4, \\ 0.286 & \text{if } t = 5, \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

The lower fuzzy frequency function associated with C_2 is

$$\underline{w}_{C_2}(t) = \begin{cases} 1 & \text{if } t \in \{0,1,2,3,\}, \\ 0.980 & \text{if } t = 4, \\ 0.970 & \text{if } t = 5, \\ 0.875 & \text{if } t = 6, \\ 0.600 & \text{if } t = 7, \\ 0.546 & \text{if } t = 8, \\ 0.288 & \text{if } t = 9, \\ 0.191 & \text{if } t = 10, \\ 0 & \text{otherwise,} \end{cases} \tag{9}$$

and the lower fuzzy frequency function associated with C_3 is

$$\underline{w}_{C_3}(t) = \begin{cases} 1 & \text{if } t \in \{0,1\}, \\ 0.979 & \text{if } t = 2, \\ 0.884 & \text{if } t = 3, \\ 0.144 & \text{if } t = 4, \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Lower fuzzy frequency functions can be used for graphical representation of "fuzzy bars" in a fuzzy histogram. The fuzzy bar associated with fuzzy interval C_2 is depicted in Figure

3. The fuzzy histogram describing distribution of vague observations from S in vague categories from C is in Figure 4.

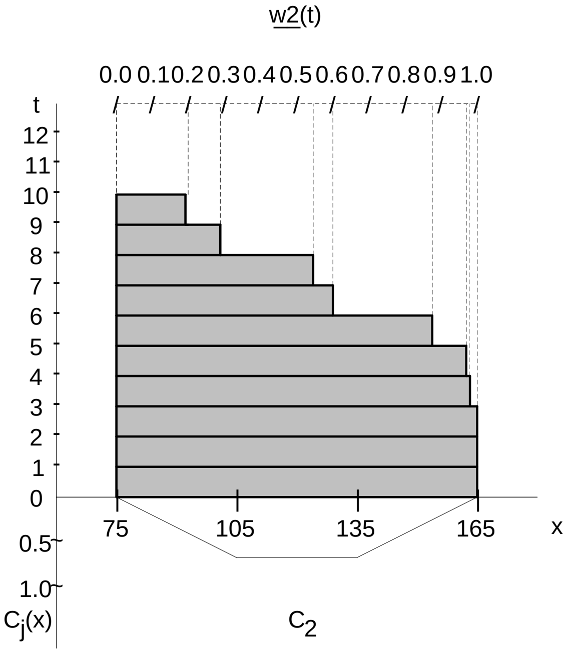

Figure 3: Fuzzy bar associated with fuzzy interval C_2

Using Steps 2 and 3 from Procedure 2, we obtain the following results: the fuzzy cardinality of granule S/C_1 is

$$w_{C_1}(t) = \begin{cases} 0.030 & \text{if } t = 1, \\ 0.114 & \text{if } t = 2, \\ 0.500 & \text{if } t = 3, \\ 0.500 & \text{if } t = 4, \\ 0.286 & \text{if } t = 5, \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

the fuzzy cardinality of granule S/C_2 is

$$w_{C_2}(t) = \begin{cases} 0.020 & \text{if } t = 3, \\ 0.030 & \text{if } t = 4, \\ 0.125 & \text{if } t = 5, \\ 0.400 & \text{if } t = 6, \\ 0.454 & \text{if } t = 7, \\ 0.546 & \text{if } t = 8, \\ 0.288 & \text{if } t = 9, \\ 0.191 & \text{if } t = 10, \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

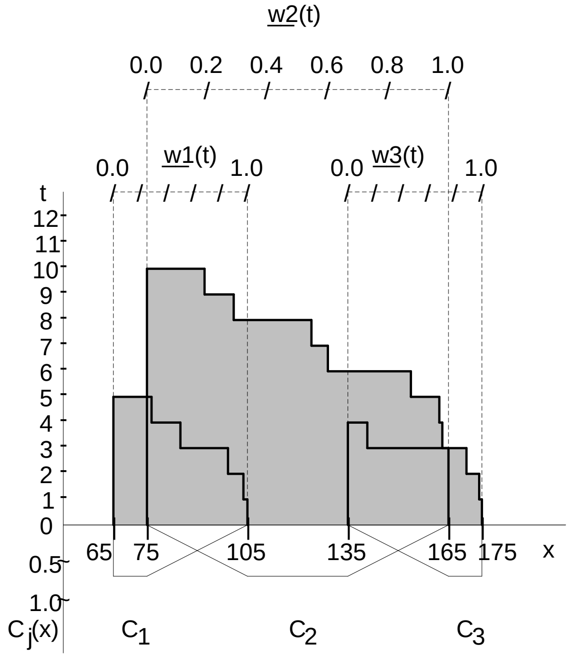

Figure 4: Fuzzy histogram associated with fuzzy partition C

and the fuzzy cardinality of granule S/C_3 is

$$w_{C_3}(t) = \begin{cases} 0.021 & \text{if } t = 1, \\ 0.116 & \text{if } t = 2, \\ 0.856 & \text{if } t = 3, \\ 0.144 & \text{if } t = 4, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

We can see that there is an equal possibility (0.5) that the total number of "low water level" observations from sample S is 3 or 4. The total number of "medium water level" observations from sample S is 8 with possibility 0.546 and the total number of "high water level" observations from sample S is 3 with possibility 0.856. Fuzzy cardinalities of granules S/C_1, S/C_2 and S/C_3 are depicted in Figures 5, 6 and 7, respectively.

An evaluation of the scalar cardinality of granule S/D, $D \in \mathcal{F}_S(\mathbf{R})$ can be derived from an evaluation of its fuzzy cardinality by choosing an appropriate numerical characterization of the fuzzy cardinality membership function w_D. For example, let us denote by N^* the set of all $t^* \in N$ such that $w_D(t^*) = h(w_D)$. Then the arithmetic average from all integers in N^* fulfills properties P1-P4 from Definition 3. Approximation of fuzzy cardinality w_D by

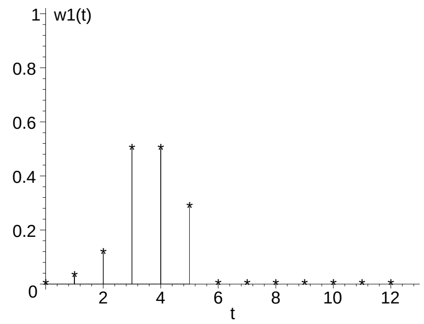

Figure 5: Fuzzy cardinality of granule S/C_1

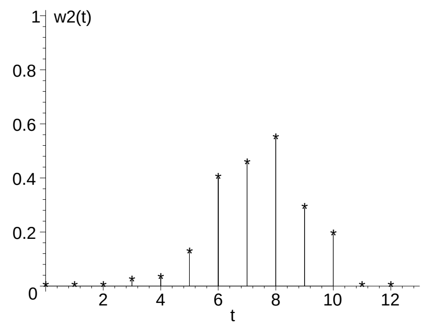

Figure 6: Fuzzy cardinality of granule S/C_2

a single count $t^* \in N^*$ is justified only if the following conditions are satisfied:

1. $|N^*| = 1$,
2. $w_D(t^*)$ is reasonably large,
3. nonspecificity of fuzzy set w_D is reasonably low.

Let us recall that nonspecificity is uncertainty associated with a set of possible alternatives. The lower is the number of alternatives, the lower is nonspecificity. For any $A \in FFS(M)$

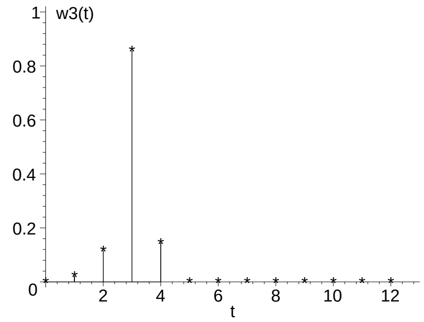

Figure 7: Fuzzy cardinality of granule S/C_3

we can evaluate nonspecificity by, e.g., U-uncertainty [10] defined as

$$U(A) = \frac{1}{h(A)} \int_0^{h(A)} log_2|A_t|\, dt. \qquad (14)$$

Observe that $0 \leq U(A) \leq log_2|M|$. More information about measures of nonspecificity of fuzzy sets can be found in [10, 17]. The vague notions "reasonably large" and "reasonably low" can be specified by appropriate numerical thresholds for $w_D(t^*)$ and $U(w_D)$, respectively.

Example 1 - continued
We will say that the height of a fuzzy cardinality w_{C_j} is reasonably large if $h(w_{C_j}) \geq 0.6$. We will consider the nonspecificity of a fuzzy cardinality w_{C_j} reasonably low if $U(w_{C_j}) \leq 0.4log_2|S| = 0.4log_2 12 = 1.434$. Fuzzy cardinality w_{C_1} attains its maximum 0.5 at $t^* = 3$ and $t^* = 4$ and $U(w_{C_1}) = 2.702$. Fuzzy cardinality w_{C_2} has height 0.546 at $t^* = 8$ and $U(w_{C_2}) = 1.671$. In both cases the height is below the specified threshold and U-uncertainty is above the specified level. Fuzzy cardinality w_{C_3} has height 0.856 at $t^* = 3$ and $U(w_{C_3}) = 0.258$. We conclude that only fuzzy cardinality of the vague category C_3 (high water level) can be reasonably approximated by count 3.

6 Conclusion

We introduced axiomatic definitions of scalar and fuzzy cardinalities of a granule of vague data. We showed how these cardinalities can be evaluated by using degrees of inclusion of fuzzy sets and scalar and fuzzy cardinalities of finite fuzzy sets. Other approaches to the construction of scalar and fuzzy counts of vague data will be explored in our future work. We discussed the lower and the upper fuzzy frequency functions associated with a fuzzy interval defined on the range of a sample of vague data. It was shown that the lower fuzzy frequency function can be graphically represented as a "fuzzy bar" of a fuzzy histogram. Computational steps illustrated on a small example can provide guidelines for application of suggested procedures in practice.

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