On degrees of truth, partial ignorance and contradiction

Didier Dubois
IRIT-CNRS, Université de Toulouse, France
dubois@irit.fr

Abstract

In many works dealing with knowledge representation, there is a temptation to extend the truth-set with values expressing ignorance and contradiction. This is the case with partial logic and Belnap bilattice logic. This is also true in interval-valued extensions of fuzzy sets. This paper shows that ignorance and contradiction cannot be viewed as additional truth-values nor processed in a truth-functional manner.

Keywords: Partial logic, Belnap logic, interval-valued fuzzy sets.

1 Introduction

From the inception of many-valued logics, there have been attempts to attach an epistemic flavor to truth degrees. This has led to a very confusing state of facts, and has probably hampered the development of applications of these logics. Indeed, belief is never truth-functional [9]. Fuzzy logic is often attacked because it is truth-functional. A well-known example is by Elkan [12] criticising the usual fuzzy connectives $\max, \min, 1-$, as leading to an inconsistent approach. Looking at these critiques more closely, it can be seen that the root of the controversy also lies in a confusion between degrees of truth and degrees of belief. Fuzzy logic is not specifically concerned with belief representation, only with gradual (not black or white) concepts [16]. However this misunderstanding seems to come a long way. For instance, a truth-value strictly between true and false was named "possible" [19], a word which refers to uncertainty modelling and modalities. Other logics seem to suffer from this kind of confusion such as partial logic closely related to Kleene 3-valued logic, and Belnap's allegedly useful four-valued logic. We claim (see [8] for details) that we cannot consistently reason under incomplete or conflicting information about propositions by augmenting the set of Boolean truth-values *true* and *false* with epistemic notions like "unknown" or "contradictory", modeling them as additional genuine truth-values of their own. Then we consider the case of truth-functional extensions of fuzzy set algebras like interval-valued fuzzy sets and membership/nonmembership pairs of Atanassov.

2 Truth vs. belief in classical logic

In a previous paper [10] we pointed out that while classical (propositional) logic is always presented as the logic of the true and the false, this description neglects the epistemic aspects of this logic. Namely, if a set B of well-formed Boolean formulae is understood as a set of propositions believed by an intelligent agent (a *belief base*) then the underlying uncertainty theory is ternary and not binary: it is con-

L. Magdalena, M. Ojeda-Aciego, J.L. Verdegay (eds): Proceedings of IPMU'08, pp. 31–38
Torremolinos (Málaga), June 22–27, 2008

ceivable that some proposition is neither believed nor is disbelieved by a particular agent. Namely

1. p is believed (or known), which is the case if B implies p;
2. its negation is believed (or known), which is the case if B implies $\neg p$;
3. neither p nor $\neg p$ is believed, which is the case if B implies neither $\neg p$ nor p.

It is clear that belief refers to the notion of validity of p in the face of B and is a matter of consequencehood, not truth-values. In fact, one can represent belief by means of *subsets of possible truth-values* enabled for p by taking propositions in B for granted. Full belief in p corresponds to the singleton $\{1\}$ (only the truth-value "true" is possible); full disbelief in p corresponds to the singleton $\{0\}$; the situation of total uncertainty relative to p for the agent corresponds to the set $\{0, 1\}$. This set is to be understood disjunctively (both truth-values for p remain possible due to incompleteness, but only one is correct). Under such conventions, the characteristic function of $\{0, 1\}$ is viewed as a possibility distribution π (Zadeh[24]). Namely, $\pi(0) = \pi(1) = 1$ means that both 0 and 1 are possible. It contrasts with other uses of subsets of truth-values, interpreted conjunctively, whereby $\{0, 1\}$ is understood as the *simultaneous* attachment of "true" and "false" to p (expressing a contradiction, see Dunn [11]). This is another convention based on necessity degrees $N(0) = 1-\pi(1); N(1) = 1-\pi(0)$. Then clearly, $N(0) = 1 = N(1)$ indicates a strong contradiction. It must be emphasized that $\{0\}, \{1\}$, and $\{0, 1\}$ are not truth-values of propositions in B. They express what can be called *epistemic values* whereby the agent believes p, believes $\neg p$, or is ignorant about p respectively.

3 Partial Logic vs. Supervaluations

Partial logic starts from the claim that truth-values of propositions can be left open, and that such undefinedness may stem from a lack of information. This program is clearly in the scope of theories of uncertainty and partial belief, introduced as well to cope with limited knowledge. Other interpretations of partiality exist, that are not considered here. From a historical perspective, the formalism of partial logic is not so old, but has its root in Kleene [17]'s three-valued logic, where the third truth-value expresses the impossibility to decide if a proposition is true or false. The reader is referred to the dissertation of Thijsse [20] and a survey paper by Blamey [4].

At the semantic level, the main idea of partial logic is to change interpretations into partial interpretations (also called *coherent situations*) obtained by assigning a Boolean truth-value to some (but not all) of the propositional variables forming a set $Prop = \{a, b, c, \ldots\}$. A coherent situation can be represented as any conjunction of literals pertaining to distinct propositional variables. Denote by s a situation, S the set of situations, and $V(a, s)$ the partial function from $Prop \times S$ to $\{0, 1\}$ such that $V(a, s) = 1$ if a is true in s, 0 if a is false in s, and is undefined otherwise. Then, two relations are defined for the semantics of connectives, namely *satisfies* ($\models_T$) and *falsifies* ($\models_F$):

$s \models_T a$ if and only if $V(s, a) = 1$; $s \models_F a$ if and only if $V(s, a) = 0$;
$s \models_T \neg p$ if and only if $s \models_F p$; $s \models_F \neg p$ if and only if $s \models_T p$;
$s \models_x p \wedge q$ if and only if $s \models_x p$ and $s \models_x q$,
$s \models_x p \vee q$ if and only if $s \models_x p$ or $s \models_x q$,
for $x = T, F$.

In partial logic a coherent situation can be encoded as a truth-assignment t_s mapping each propositional variable to the set $\{0, \frac{1}{2}, 1\}$, understood as a partial Boolean truth-assignment in $\{0, 1\}$. Let $t_s(a) = 1$ if atom a appears in s, 0 if $\neg a$ appears in s, and $t_s(a) = \frac{1}{2}$ if a is absent from s. The basic partial logic can thus be described by means of a three-valued logic, where $\frac{1}{2}$ (again) means *unknown.* The connectives can be expressed as follows: $1 - x$ for the negation, max for disjunction, min for the conjunction, and $\max(1 - x, y)$ for the implication. Note that if $t_s(p) = t_s(q) = \frac{1}{2}$, then also $t_s(p \vee q) = t_s(p \wedge q) = t_s(p \to q) = \frac{1}{2}$ in this approach. Besides, this (Kleene-like) three-valued logic is isomorphic to the set of non-empty intervals on $\{0, 1\}$, using the ordering $0 < 1$, equipped with the interval extension of classical connectives, viewing $\frac{1}{2}$ as the interval $\{0, 1\}$, the other ones being the singletons $\{0\}$ and $\{1\}$.

Since these definitions express truth-functionality in a three-valued logic, this logic fails to satisfy all classical tautologies. But this anomaly stems from the same difficulty again, that is, no three-element set can be endowed with Boolean algebra structure! (nor is the set of non-empty intervals on $\{0, 1\}$). A coherent situation s can be interpreted as a special set $A(s)$ of standard Boolean interpretations, and can be viewed as a disjunction thereof. A coherent situation can be encoded as a formula whose set of models $A(s)$ can be built just completing s by all possible assignments of 0 or 1 to variables not assigned yet. It represents an epistemic state reflecting a lack of information. If this view is correct, the equivalence $s \models_T p \vee q$ if and only if $s \models_T p$ or $s \models_T q$ cannot hold under classical model semantics. Indeed $s \models_T p$ supposedly means $A(s) \subseteq [p]$ and $s \models_F p$ supposedly means $A(s) \subseteq [\neg p]$, where $[p]$ is the set of interpretations where p is true. But while $A(s) \subseteq [p \vee q]$ holds whenever $A(s) \subseteq [p]$ or $A(s) \subseteq [q]$ holds, the converse is invalid!

This is the point made by Van Fraassen [21] who first introduced the notion of supervaluation to account for this situation. A supervaluation SV over a coherent situation s is (in our terminology) a function that assigns, to each proposition in the language and each coherent situation s, the *super-truth-value* $SV(p, s) = 1$ (0) to propositions that are true (false) for all Boolean completions of s. It is clear that p is "super-true" ($SV(p, s) = 1$) if and only if $A(s) \subseteq [p]$, so that supervaluation theory recovers missing classical tautologies by again giving up truth-functionality: $p \vee \neg p$ is always super-true, but $SV(p \vee q, s)$ cannot be computed from $SV(p, s)$ and $SV(q, s)$. The term "super-true" in the sense of Van Fraassen stands for "certainly true" in the terminology of possibilistic belief management in classical logic. The belief calculus at work in propositional logic covers the semantics of partial logic as a special case. It exactly coincides with the semantics of the supervaluation approach. Assuming compositionality of epistemic annotations by means of Kleene three-valued logic provides only an imprecise approximation of the actual Boolean truth-values of complex formulas [5].

4 Belnap Four-Valued Logic

Two seminal papers of Belnap [2] [3] propose an approach to reasoning both with incomplete and with inconsistent information. It relies on a set of truth-values forming a bilattice, further studied by scholars like Ginsberg and Fitting. Belnap logic, considered as a system for reasoning under imperfect information, suffers from the same difficulties as partial logic, and for the same reason. Indeed one may consider this logic as using the three epistemic values already considered in the previous sections (*certainly true, certainly false and unknown*), along with an additional one that accounts for epistemic conflicts.

4.1 The contradiction-tolerant setting

Belnap considers an artificial information processor, fed from a variety of sources, and capable of anwering queries on propositions of interest. In this context, inconsistency threatens, all the more so as the information processor is supposed never to subtract information. So the basic assumption is that the computer receives information about atomic propositions in a cumulative way from outside sources, each asserting for each atomic proposition whether it is true, false, or being silent about it. The notion of *epistemic set-up* is defined as an assignment, of one of four values denoted $\mathbf{T}, \mathbf{F}, \mathbf{BOTH}, \mathbf{NONE}$, to each atomic proposition $a, b, \ldots$:

1. Assigning **T** to a means the computer has only been told that a is true.
2. Assigning **F** to a means the computer has only been told that a is false.
3. Assigning **BOTH** to a means the computer has been told at least that a is true by one source and false by another.
4. Assigning **NONE** to a means the computer has been told nothing about a.

In view of the previous discussion, the set $\mathbf{4} = \{\mathbf{T}, \mathbf{F}, \mathbf{BOTH}, \mathbf{NONE}\}$ coincides with the power set of $\{0,1\}$, namely $\mathbf{T} = \{1\}$, $\mathbf{F} = \{0\}$, the encoding of the other values depending on the adopted convention: under **Dunn Convention**, $\mathbf{NONE} = \emptyset$; $\mathbf{BOTH} = \{0,1\}$. It expresses accumulation of information by sources. This convention uses Boolean necessity degrees, i.e. **BOTH** means $N(0) = N(1) = 1$, **NONE** means $N(0) = N(1) = 0$. With **possibility degree convention**, $\mathbf{NONE} = \{0,1\}$; $\mathbf{BOTH} = \emptyset$. These subsets represent constraints, i.e., mutually exclusive truth-values, one of which is the right one. **NONE** means $\pi(0) = \pi(1) = 1$, **BOTH** means $\pi(0) = \pi(1) = 0$. Then $\emptyset$ corresponds to no solution.

The approach relies on two orderings in **4**.
The information ordering, $\sqsubset$, such that $\mathbf{NONE} \sqsubset \mathbf{T} \sqsubset \mathbf{BOTH}; \mathbf{NONE} \sqsubset \mathbf{F} \sqsubset \mathbf{BOTH}$. This ordering reflects the inclusion relation of the sets $\emptyset$, $\{0\}, \{1\}$, and $\{0,1\}$, using Dunn convention. It intends to reflect the amount of (possibly conflicting) data provided by the sources.
The logical ordering, $\prec$, according to which $\mathbf{F} \prec \mathbf{BOTH} \prec \mathbf{T}$ and $\mathbf{F} \prec \mathbf{NONE} \prec \mathbf{T}$ each reflecting the truth-set of Kleene's logic. It corresponds to the idea of "less true than", even if this may sound misleadingly suggesting a confusion with the idea of graded truth. In fact $\mathbf{F} \prec \mathbf{BOTH} \prec \mathbf{T}$ canonically extends the ordering $0 < 1$ to intervals on $\{0,1\}$, under Dunn convention and $\mathbf{F} \prec \mathbf{NONE} \prec \mathbf{T}$ does the same under possibility degree convention.

Then, connectives of negation, conjunction and disjunction are defined truth-functionally on the bilattice. The set **4** is isomorphic to $2^{\{0,1\}}$ equipped with two lattice structures: **The information lattice**, a Scott approximation lattice based on union and intersection of sets of truth-values using Dunn convention. For instance, in this lattice the maximum of **T** and **F** is **BOTH**; **the logical lattice**, based on the interval extension of min, max and $1-$ from $\{0,1\}$ to $2^{\{0,1\}} \setminus \{\emptyset\}$ respectively under Dunn Convention (for **BOTH**) and possibility degree convention (for **NONE**). These logical connectives respect the following constraints: (i)They reduce to classical negation, conjunction and disjunction on $\{\mathbf{T}, \mathbf{F}\}$; (ii) They are monotonic w.r.t. ordering $\sqsubset$; (iii) $p \wedge q = q$ if and only if $p \vee q = p$; (iv)They satisfy commutativity, associativity of $\vee, \wedge$, De Morgan laws.

As a consequence, the restrictions of all connectives to the subsets $\{\mathbf{T}, \mathbf{F}, \mathbf{NONE}\}$ and $\{\mathbf{T}, \mathbf{F}, \mathbf{BOTH}\}$ coincide with Kleene's three-valued truth-tables, encoding **BOTH** and **NONE** as $\frac{1}{2}$. The conjunction and disjunction operations $\vee$ and $\wedge$ exactly correspond to the lattice meet and joint for the logical lattice ordering. In fact, **BOTH** and **NONE** cannot be distinguished by $\prec$ and play symmetric roles in the truth-tables. The major new point is the result of combining conjunctively and disjunctively **BOTH** and **NONE**. The only possibility left for such combinations is that $\mathbf{BOTH} \wedge \mathbf{NONE} = \mathbf{F}$ and $\mathbf{BOTH} \vee \mathbf{NONE} = \mathbf{T}$. This looks intuitively surprising but there is no other choice and this is in agreement with the information lattice.

4.2 This is not really how a computer should think

Belnap's calculus is an extension of partial logic to the truth-functional handling of inconsistency. In his paper, Belnap does warn the reader on the fact that the four values are not ontological truth-values but epistemic ones. They are qualifications referring to the state of knowledge of the agent (here the computer). The set-representation of Belnap truth-values after Dunn [11] rather comforts the idea that these are not truth-values. For instance $\{1\}$ is a subset of $\{0, 1\}$ while 1 is an element thereof. Interpreting Belnap's epistemic truth-values as genuine truth-values comes down to confusing elements of a set and singletons included in it.

Belnaps explicitly claims that the systematic use of the truth-tables of **4** "tells us how the computer should answer questions about complex formulas, based on a set-up representing its epistemic state"([2], p. 41). However, since the truth-tables of conjunction and disjunction extend the ones of partial logic so as to include the value **BOTH**, Belnap's logic inherits all difficulties of partial logic regarding the truth-value **NONE**. Moreover, equalities $\mathbf{BOTH} \wedge \mathbf{BOTH} = \mathbf{BOTH}$, $\mathbf{BOTH} \vee \mathbf{BOTH} = \mathbf{BOTH}$ are hardly acceptable when applied to propositions of the form p and $\neg p$.

Another issue is how to interpret the results $\mathbf{BOTH} \wedge \mathbf{NONE} = \mathbf{F}$ and $\mathbf{BOTH} \vee \mathbf{NONE} = \mathbf{T}$. One may rely on bipolar reasoning and argumentation to defend that when p is **BOTH** and q is **NONE**, $p \wedge q$ should be $\mathbf{BOTH} \wedge \mathbf{NONE} = \mathbf{F}$. Suppose there are two sources providing information, say S_1 and S_2. Assume S_1 says p is true and S_2 says it is false. This is why p is **BOTH**. Both sources say nothing about q, so q is **NONE**. So one may consider that S_1 would have nothing to say about $p \wedge q$, but one may legitimately assert that S_2 would say $p \wedge q$ is false. In other words, $p \wedge q$ is **F**: one may say that there is one reason to have $p \wedge q$ false, and no reason to have it true. However, suppose two atomic propositions a and b with $E(a) = \mathbf{BOTH}$ and $E(b) = \mathbf{NONE}$. Then $E(a \wedge b) = \mathbf{F}$. But since Belnap negation is such that $E(\neg a) = \mathbf{BOTH}$ and $E(\neg b) = \mathbf{NONE}$, we also get $E(\neg a \wedge b) = E(a \wedge \neg b) = E(\neg a \wedge \neg b) = \mathbf{F}$. Hence $E((a \wedge b) \vee (\neg a \wedge b) \vee (a \wedge \neg b) \vee (\neg a \wedge \neg b)) = \mathbf{F}$ that is, $E(\top) = \mathbf{F}$ which is hardly acceptable again. See Fox [13] for a related critique.

5 Interval-valued fuzzy sets

IVFs were introduced by Zadeh [26], along with some other scholars, in the seventies, as a natural truth-functional extension of fuzzy sets. Variants of these mathematical objects exist, under various names (vague sets for instance). The IVF calculus has become popular in the fuzzy engineering community of the USA because of many recent publications by Jerry Mendel and his colleagues. This section points out the fact that if intervals of membership grades are interpreted as partial ignorance about precise degrees, the calculus of IVFs suffers from the same flaw as partial logic, of which it is a many-valued extension.

5.1 Definitions

An interval-valued fuzzy set is defined by an interval-valued membership function. Independently, Atanassov [1] introduced the idea of defining a fuzzy set by ascribing a membership function and a non-membership function separately, in such a way that an element cannot have degrees of membership and non-membership that sum up to more than 1. Such a pair was given the misleading name of "Intuitionistic Fuzzy Sets" but corresponds to an intuition that differs from IVFs, although both turned out to be mathematically equivalent notions (e.g. G. Deschrijver, E. Kerre [6]).

An IVF is defined by a mapping F from the universe U to the set of closed intervals in $[0, 1]$.

Let $F(u) = [F_*(u), F^*(u)]$. The union, intersection and complementation of IVF's is obtained by canonically extending fuzzy set-theoretic operations to interval-valued operands in the sense of interval arithmetic. As such operations are monotonic, this step is mathematically obvious. For instance, the most elementary fuzzy set operations are extended as follows, for conjunction $F \cap G$, disjunction $F \cup G$ and negation F^c, respectively:

$$[\min(F_*(u), G_*(u)), \min(F^*(u), G^*(u))];$$

$$[\max(F_*(u), G_*(u)), \max(F^*(u), G^*(u))];$$

$$F^c(u) = [1 - F^*(u), 1 - F_*(u)].$$

IVFs are a special case of L-fuzzy sets in the sense of Goguen [14], so as a mathematical object, it is not of special interest. An IVF is also a special case of type 2 fuzzy set (also introduced by Zadeh [25]). Of course all connectives of fuzzy set theory were extended to interval-valued fuzzy sets and their clones. IFVs are being studied as specific abstract algebraic structures [7], and a multiple-valued logic was recently proposed for them, called the triangle logic [22]. See [23] for a careful study of connectives for type 2 fuzzy sets; their results apply to the special case of IVFs.

5.2 Paradoxes and their solution

Paradoxes of IVFs are less blatant than those of Kleene and Lukasiewicz three-valued logics (when the third truth-value refers to ideas of incomplete knowledge) because in the latter case, the lack of excluded-middle law on Boolean propositions is a striking anomalous feature. In the case of fuzzy logic, some laws of classical logic are violated anyway. However, the fact that interval-valued fuzzy sets have a weaker structure than the fuzzy set algebra they extend should act as a warning. Indeed, since fuzzy sets equipped with fixed connectives have a given well-defined structure, this structure should be valid whether the membership grades are known or not.

For instance, the fact that $\min(F(u), F^c(u)) \leq 0.5$ should hold whether $F(u)$ is known or not. This is a weak form of the contradiction law. However, applying the truth-tables of interval-valued fuzzy sets to the case when $F(u) = [0, 1]$ (total ignorance) leads to $\min(F(u), 1 - F(u)) = [0, 1]$, which means a considerable loss of information. The same feature appears with the weak excluded middle law, where again $\max(F(u), F^c(u)) = [0, 1]$ is found, while $\max(F(u), F^c(u)) \geq 0.5$ should hold in any case. More generally, if the truth-value $t(p) = F(u)$ is only known to belong to some subinterval $[a, b]$ of the unit interval, the truth-functional calculus yields $t(p \wedge \neg p) = \min(F(u), 1 - F(u)) \in [\min(a, 1 - b), \min(b, 1 - a)]$, sometimes not included in $[0, \frac{1}{2}]$.

In fact, treating fuzzy sets with ill-known membership functions as a truth-functional calculus of IFVs is similar to the paradoxical treatment of partial logic by means of Kleene's three-valued logics, where the third truth value is interpreted as total ignorance. Kleene's three valued logic is more naturally truth-functional when viewed as a simplified variant of *fuzzy logic*, where the third truth-value means half-true. The loss of classical tautologies then looks more acceptable. In fact, partial logic is debatably construed as an interval-valued truth-functional extension of classical logic (isomorphic to Kleene logic), and is to classical sets what IVFs are to fuzzy sets.

The basic point is that IVFs lead to a multiple-valued logic where the truth set [0, 1] is turned into the set of intervals on [0, 1], i.e. *intervals are seen as genuine truth-values*. This approach does not address the issue of ill-known membership grades, where the latter are nevertheless supposed to be precise, even if out of reach. Choosing intervals for truth-values is a matter of adopting a new convention, while reasoning about ill-known membership grades does not mean changing the truth set. When reasoning about ill-known membership grades, the truth set remains [0, 1] and truth-values obey the laws of some multiple-valued calculus, while intervals model epistemic states about truth-values, just like elements in Belnap **4**. A logic that reasons about ill-known membership grades cannot be truth-functional. It should handle weighted formulas where the weight is an interval representing our knowledge about the truth-value of the formula, similar to Pavelka's logic [16], Lehmke's weighted fuzzy logic [18], and exploit the algebraic properties of the underlying logic as constraints. Interval-weighted formulas are also signed formulas in many-valued logic. Reasoning about ill-known membership grades is then a matter of constraint propagation, especially interval analysis, and not only simple interval arithmetics on connectives. Automated reasoning methods based on signed formulae in multiple-valued logics follow this line and turn inference into optimization problems[15].

The generic reasoning problem in interval-valued fuzzy logic is of the following form: Given a set of weighted many-valued propositional formulas $\{p_i, [a_i, b_i]), i = 1, \ldots, n\}$, find the most narrow interval $[a, b]$ such that $(p, [a, b])$ can be deduced. It corresponds to the following optimization problem: maximize (resp. minimize) $t(p)$ under the constraints $t(p_i) \in [a_i, b_i], i = 1, \ldots, n$.

This problem cannot be solved by a truth-functional interval-valued fuzzy logic. A simpler instance of this problem is the one of finding the membership function of a complex combination of IVFs. It comes down to finding the interval containing the truth-value of a many-valued formula, given intervals containing the truth-values of its atoms. For instance, finding the membership function of $F \cap F^c$ when F is an IVF comes down to solving for each element of the universe of discourse the following problem: maximize (resp. minimize) $f(x) = \min(x, 1 - x)$ under the constraint $x \in [a, b]$. Since the function f is not monotonic, the solution is obviously not (always) the interval $[\min(a, 1 - b), \min(b, 1 - a)]$ suggested by IVF connectives, it is as follows: $f(x) \in [a, b]$ if $b \leq 0.5$; it is $[\min(a, 1 - b), 0.5]$ if $a \leq 0.5 \leq b$; it is $[1 - a, 1 - b]$ if $a \geq 0.5$. Only the first and the third case match the IVF connectives solution.

In Lukasiewicz logic, using the bounded sum and linear product connectives, inferring in the interval-valued setting comes down to solving linear programming problems [15]. Especially the condition $F \cap F^c = \emptyset$ is always trivially valid using linear product, even if F is an IFV, since $\max(0, x + (1 - x) - 1) = 0$.

In conclusion, there is a pervasive confusion between truth-values and the epistemic values an agent may use to describe a state of knowledge: the former are compositional by assumption, the latter cannot be consistently so. This paper suggests that such difficulties appear in partial logic, Belnap logic, and interval-valued fuzzy logic. In logical approaches to incompleteness and contradiction, the goal of preserving tautologies of the underlying logic (classical or multivalued) should supersede the goal of maintaining a truth-functional setting. Considering subsets or fuzzy subsets of a truth-set as genuine truth-values leads to new many-valued logics that do not address the issue of uncertain reasoning on the underlying original logic. Such "powerset logics" are special cases of lattice-valued logic that need another motivation than reasoning under uncertainty. Our critique encompasses the truth-functional calculus of type 2 fuzzy sets[23] as well, since it again considers fuzzy sets of truth-values as truth-values. In that respect, the meaning of "fuzzy truth-values" proposed in [25] is sometimes misunderstood, as they cannot be at the same time genuine truth-values and ill-known ones.

References

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets & Syst., 20 (1986), 87-96.

[2] N.D. Belnap. How a computer should think. In G. Ryle, ed., Contemporary As-

pects of Philosophy, Oriel Press, 30–56 (1977)

[3] N. D. Belnap. A useful four-valued logic. In J.M. Dunn and G. Epstein, editors, Modern Uses of Multiple-Valued Logic. D.Reidel, Dordrecht, 1977.

[4] S. Blamey Partial Logic, Handbook of Philosophical Logic, Vol. 3, D. Reidel Publishing Company, 1–70 (1985).

[5] G. De Cooman From possibilistic information to Kleene's strong multi-valued logics. In Fuzzy Sets, Logics and Reasoning about Knowledge (D. Dubois et al., Eds.) Kluwer Academic Publishers (1999).

[6] G. Deschrijver, E. Kerre, On the relationship between some extensions of fuzzy set theory, Fuzzy Sets and Systems, 133 (2004), 227-235

[7] G. Deschrijver, E. Kerre, Advances and challenges in interval-valued fuzzy logic, Fuzzy Sets & Syst., 157, (622-627) 2006.

[8] D. Dubois On ignorance and contradiction considered as truth-values, Logic Journal of IGPL, 2008, on line.

[9] D. Dubois, H. Prade: Can we enforce full compositionality in uncertainty calculi? Proc. AAAI'94, Seattle, WA, 149-154 (1994).

[10] D. Dubois, H. Prade. Possibility theory, probability theory and multiple-valued logics: A clarification. Ann. Math. and AI. 32, 35-66(2001).

[11] J.M. Dunn. Intuitive semantics for first-degree entailment and coupled trees. Philosophical Studies, 29:149–168 (1976).

[12] Ch. Elkan (1993) The paradoxical success of fuzzy logic. In Proc. AAAI'93, Washington, DC, 698-703. Extended (with discussions), IEEE Expert, 9(4), 2-49 (1994).

[13] J. Fox: Motivation and Demotivation of a Four-Valued Logic. Notre Dame Journal of Formal Logic 31(1): 76-80 (1990).

[14] Goguen J.A. L-fuzzy sets, J. Math. Anal. Appl., 8(1967), 145-174.

[15] R. Haehnle, Proof Theory of Many-Valued Logic - Linear Optimization - Logic Design: Connections and Interactions, Soft Computing, 1, 107-119.

[16] P. Hájek The Metamathematics of Fuzzy Logics. Kluwer Academic (1998).

[17] S.C. Kleene Introduction to Metamathematics. North Holland (1952).

[18] S. Lehmke Logics which Allow Degrees of Truth and Degrees of Validity. PhD, Universität Dortmund, Germany, 2001.

[19] J. Łukasiewicz Philosophical remarks on many-valued systems of propositional logic (1930), In Selected Works (Borkowski, ed.), North-Holland, 1970, 153-179.

[20] E.G.C. Thijsse. Partial Logic and Knowledge Representation. PhD thesis, University of Tilburg (1992).

[21] B. C. van Fraassen, "Singular Terms, Truth -value Gaps, and Free Logic," Journal of Philosophy, 63, 481 -495 (1966).

[22] B. Van Gasse, C. Cornelis, G. Deschrijver and E.E. Kerre Triangle algebras: A formal logic approach to interval-valued residuated lattices, Fuzzy Sets & Syst., In Press,

[23] C. L. Walker and E. A. Walker ,The algebra of fuzzy truth values, Fuzzy Sets & Syst., 149 (2005) 309-347.

[24] L. A. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets & Syst., 1, 3-28 (1978).

[25] L. A. Zadeh, Fuzzy Logic and approximate reasoning (In memory of Grigore Moisil), Synthese, 30, 407-428 (1975).

[26] L. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, Inform. Sci. 8(1975), 199–249.