Aggregation Selection for Hierarchical Fuzzy Signatures: A Comparison of Hierarchical OWA and WRAO

B.S.U. Mendis
Department of Computer Science,
The Collage of Engineering
and Computer Science,
The Australian National University,
Australia
sumudu@anu.edu.au

T.D. Gedeon
Department of Computer Science,
The Collage of Engineering
and Computer Science,
The Australian National University,
Australia
tom@cs.anu.edu.au

Abstract

In general, intelligent decision making systems receive information that is very sparse and which is likely to be hierarchically correlated. Our previous research has show that hierarchical Fuzzy Signatures are effective, efficient, robust and flexible with such inputs. Earlier, we introduced the generalised Weighted Relevance Aggregation Operator (WRAO) for hierarchical Fuzzy Signatures. In this paper, we compare the generalised Ordered Weighted Averaging (GOWA) operator with WRAO to select the best aggregation method for hierarchical Fuzzy Signatures. Additionally, we show a method of learning hierarchical GOWA using the Levenberg-Marquardt optimization Method.

Keywords: Hierarchical Fuzzy Signatures, Vector Valued Fuzzy Sets, Aggregation Functions, OWA, GOWA, WRAO, Generalized Means, Levenberg-Marquardt Method.

1 Introduction

The vector valued fuzzy sets concept has been further generalized in [7] to introduce the fuzzy signature concept. Fuzzy signatures can model sparse and hierarchically correlated data with the help of hierarchically structured vector valued fuzzy sets and a set of not-necessarily homogenous and hierarchically organised aggregation functions. The set of aggregation functions map the different universes of discourse of the hierarchical fuzzy signature structure, from lower branches to the higher branches. We argue that these properties help fuzzy signatures to model problems similar to the nature of human comprehensible hierarchical approaches to problem solving. An important advantage of the fuzzy signature concept is that it can be used to compare degree of similarity or dissimilarity of two slightly different objects, which have the same fuzzy signature skeleton. Additionally, fuzzy signatures are capable of dealing with missing input data. Thus, medical and economic diagnoses, web and document information retrieval, data mining are the obvious applications of fuzzy signatures.

In [12] we enhanced the inference in fuzzy signatures, by introducing the Weighted Relevance Aggregation (WRA) method. The concept behind the Weighted Relevance aggregation method is that the weights in each branch of the fuzzy signature are the observations of the relevance of that branch to its higher level branches in the hierarchical fuzzy signature structure. Thus, this method introduces extra knowledge to the fuzzy signature structure to classify vague data. In addition, it enhances the adaptability of hierarchical fuzzy signatures to different problem domains.

Later, we further generalized these Weighted Relevancies and aggregation functions in hierarchical fuzzy signatures, into one operator

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called Weighted Relevance Aggregation Operator (WRAO) [9]. WRAO allows users to learn both aggregation function and weighted relevance at the same time for one node in the hierarchical fuzzy signature structure. Thus, WRAO simplifies the learning of hierarchical fuzzy signature models from data. In [10] we have shown a successful way to extract WRAO for hierarchical Fuzzy Signatures based on the Levenberg-Marquardt (LM) optimization method [8]. Experiments in [10] showed that the LM method can learn both aggregations and weighted relevancies for hierarchical fuzzy signatures.

The Ordered Weighted Averaging (OWA) operators [14] have gained a considerable attention in the Multi-Criteria Decision Making field. OWA provides a class of mean type aggregation selections based on weight parameters. This paper compares the performance of WRAO with a hierarchically organised Ordered Weighted Averaging (OWA) operator [15].

The rest of the paper is organised as follows. In section 2, we discuss the concept of hierarchical Fuzzy Signatures and Weighted Relevance Aggregation (WRA). Section 3 reviews the Levenberg-Marquardt (LM) optimization method for learning weights in WRAO and OWA. Finally, in section 4, we compare these two aggregation method to select the best method for hierarchical Fuzzy Signatures.

2 Hierarchical Fuzzy Signatures

Fuzzy signatures can describe, compare and classify objects with complex structure and interdependent features. The hierarchical organisation of fuzzy signatures express the structural complexity of a problem. The local preference relations among the hierarchies and sub-branches of a fuzzy signature can be used to approximate the global preference relation of a decision problem.

2.1 Vector Valued Fuzzy Sets

The fuzzy signature concept is a generalization of the Vector Valued Fuzzy Sets (VVFS) concept. The early work of Kóczy [6] introduced the Vector Valued Fuzzy Sets concept. The VVFS is a special form of an L-fuzzy set, and can be denoted in the following form:

$$A : X \to [0,1]^n . \qquad (1)$$

It is clear that $L = [0,1]^n$ is in (1) and thus VVFS is L-fuzzy. The qualitative meaning of an object is represented by the quantities of the VVFS. The notation of the vector valued fuzzy set A is written as $\overline{A} = \left(x, q_{\overline{A}}\right)$ and the membership function $q_{\overline{A}}$ can be defined as, $q_{\overline{A}} : X \to [0,1]^n$, where $x \in X$.

2.2 Hierarchical Fuzzy Signature Structure

Fuzzy signatures are fuzzy descriptors of real world objects. They represent objects with the help of a sets of quantities that are arranged in a hierarchical structure expressing interconnectedness and set of non-homogeneous qualitative measures, which are the interdependencies among the quantities of each set, to aggregate these hierarchies. Thus, fuzzy signatures are capable of handling problems that are complex and inherently hierarchical.

Additionally, the fuzzy signature concept is a good solution to the rule explosion problem in fuzzy logic, as fuzzy signatures are hierarchically structured and inherently sparse. In this section, we discuss the concept of fuzzy signatures as a practical approach that organised and aggregate data hierarchically. Now, we recall the fuzzy signature concept introduced in [7].

Definition 1 *Fuzzy Signature is a VVFS, where each vector component is another VVFS (branch) or a atomic value (leaf), and denoted by,*

$$A : X \to [a_i]_{i=1}^{k} \left(\equiv \prod_{i=1}^{k} a_i \right) . \qquad (2)$$

$$where\ a_i = \begin{cases} [a_{ij}]_{j=1}^{k_i} & ; if\ branch \\ [0,1] & ; if\ leaf \end{cases}$$

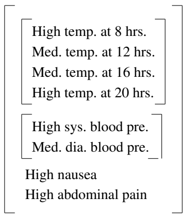

Figure 1: A Fuzzy Signature

and Π *describes the Cartesian product.*

The figure 1 shows an example fuzzy signature [13]. This fuzzy signature describes an individual SARS patient, which is a data point among many SARS data collected in the year 2003.

2.3 Weighted Relevance Aggregation (WRA)

Weighted Relevance Aggregation provides an additional meaning to the fuzzy signature structure by introducing the weighted relevance of each branch to its higher branches of the fuzzy signature structure. That is, the weighted relevance reflects the idea that some branches provide higher values to the next level (or to the parent branch) of the fuzzy signature structure. Some other branches in the same parent branch provide relatively lower values to the next level (or to the parent branch) of the fuzzy signature structure. In this way WRA enhances the accuracy of the final results of the Fuzzy Signature. In [12], we discussed a method of learning weights in WRA automatically. In [11], we have shown the successfulness of the weights extraction method in [12].

We further generalise the weights and the aggregation into one operator called Weighted Relevance Aggregation Operator (WRAO) [9]. This subsection briefly describes the generalised Weighted Relevance Aggregation (WRAO) operator [9] for fuzzy signatures. In [10], we showed that WRAO enhances the accuracy of the results of fuzzy signatures, by allowing better adaptation to the meaning of the decision making process. Further, WRAO helps to reduce the number of individual fuzzy signatures needed for the decision making process, by absorbing more patterns into these recognized by one Fuzzy Signature.

Now, we recall the definition of WRAO in [9]. All the notation in the definition 2 refer to the arbitrary fuzzy signature in figure 2.

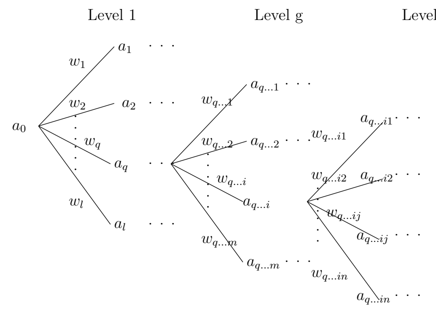

Figure 2: An Arbitrary Fuzzy Signature

Definition 2 *The generalised Weighted Relevance Aggregation Operator (WRAO) of an arbitrary branch* $a_{q\ldots i}$ *with* n *sub branches,* $a_{q\ldots i1}, a_{q\ldots i2}, \ldots, a_{q\ldots in} \in [0,1]$*, and weighted relevancies,* $w_{q\ldots i1}, w_{q\ldots i2}, \ldots, w_{q\ldots in} \in [0,1]$*, for a fuzzy signature is a function* $g : [0,1]^{2n} \to [0,1]$ *such that,*

$$a_{q\ldots i} = \left[\frac{1}{n}\sum_{j=1}^{n}(a_{q\ldots ij} \bullet w_{q\ldots ij})^{p_{q\ldots i}}\right]^{\frac{1}{p_{q\ldots i}}} \quad (3)$$

The WRAO must satisfy the following three properties,

(i) $w_{q\ldots ij} \in [0,1]$

(ii) $\bigvee_{j=1}^{n} w_{q\ldots ij} \leq 1$

(iii) $p_{q\ldots i} \neq 0$

In [9], we prove the following two properties for WRAO.

Theorem 1 *Let* $a_{q\ldots i}$ *be an arbitrary branch with* n *sub branches,* $s_{q\ldots i1}, s_{q\ldots i2}, \ldots, s_{q\ldots in}$ *, and weighted relevancies,* $w_{q\ldots i1}, w_{q\ldots i2}, \ldots, w_{q\ldots in}$*, for an arbitrary fuzzy signature (figure 2). Then WRAO in definition (2) holds the following properties.*

(i) Idempotent w.r.t $s_{q\ldots ij}$, *when all* $w_{q\ldots ij}$ *are fixed and vice versa,*

(ii) Commutative, and

(iii) Monotonic w.r.t $s_{q\ldots ij}$ *when all* $w_{q\ldots ij}$ *are fixed and vice versa.*

Above properties are adequate to satisfy the requirement to be an aggregation function [2] as weights, $w_{q\ldots i1}, w_{q\ldots i2}, \ldots, w_{q\ldots in}$, in WRAO are fixed for any instance of a fuzzy signature in the decision making phase, and both weights and aggregation operators vary simultaneously only in the learning phase.

Theorem 2 *The WRAO in definition (2) holds the following characteristics.*

(a) $p_{q\ldots i} \to 0$ *then WRAO* $\to$ *geometric mean*

(b) $\lim_{p_{q\ldots i} \to +\infty} g(s_{q\ldots i1}, \ldots s_{q\ldots in}; w_{q\ldots i1}, \ldots, w_{q\ldots in})$

$$= max(s_{q\ldots i1}w_{q\ldots i1}, \ldots s_{q\ldots in}w_{q\ldots in})$$

(c) $\lim_{p_{q\ldots i} \to -\infty} g(s_{q\ldots i1}, \ldots s_{q\ldots in}; w_{q\ldots i1}, \ldots, w_{q\ldots in})$

$$= min(s_{q\ldots i1}w_{q\ldots i1}, \ldots s_{q\ldots in}w_{q\ldots in})$$

(d) $p = 1$ *then WRAO*$\to$ *arithmetic mean*

(e) $p = -1$ *then WRAO*$\to$ *harmonic mean*

2.4 Ordered Weighted Average (OWA) Operators

OWA is widely used and popular in multi-criteria decision making.

Definition 3 *The OWA operator of* n *input arguments,* $a_1, a_2, \ldots, a_n$ *, is a mapping,* $f : [0,1]^n \to [0,1]$, *that has an associated weighting vector* $W = [w_1, w_2, \ldots, w_n]^T$ *such that,*

(i) $w_i \in [0,1]$

(ii) $\sum_{i=1}^{n} w_i = 1$

and where

$$f(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \qquad (4)$$

where b_j *is the* j*th largest element of the* n *input argument vector.*

OWA operators include *min*, *max*, and *mean*. Further, Yager [14] has shown that OWA satisfies the basic properties of monotonicity, idempotency, and generalized commutativity.

Definition 4 *A mapping* $M : I^n \to I$ *is called a generalized ordered weighted aggregation (GOWA) operator of* n *input arguments if,*

$$M(a_1, a_2, \ldots, a_n) = \left(\sum_{j=1}^{n} w_j b_j^p \right)^{\frac{1}{p}} \qquad (5)$$

where $p \in [-\infty, \infty]$, b_j *is the* j*th largest input, and* w_j *are collection of weights satisfying*

(I) $w_j \in [0,1]$

(II) $\sum_{j=1}^{n} w_j = 1$

In [15], Yager shown his interest to use hierarchically organised OWA and weighted average aggregation methods to solve decision making problems. In this paper, we use hierarchically organised OWA as an aggregation method for the Fuzzy Signatures.

3 Levenberg-Marquardt Learning of WRAO and OWA from Real World Data

Methods of learning the weights in OWA and GOWA can be found in [3, 1] respectively. In this sub section we explain the Levenberg-Marquardt learning of both aggregation and weights factors for WRAO and GOWA in details.

3.1 Levenberg-Marquardt Optimization Method

The LM algorithm is a widely used advanced optimization algorithm that outperforms simple gradient descent and other gradient methods when applied in a wide variety of problems. The major drawback with the steepest gradient descent method is that if there is no line search method combined with it, there is no guarantee of convergence. LM is a pseudo-second order method in which the Hessian matrix is estimated using the gradients [4]. The LM algorithm is a Sum of Squared Error (SSE) based minimization method that is the function to be minimized is of the following special form [8]:

$$f(s) = \frac{1}{2}\sum_{i=1}^{n}(t_i - s_i)^2 = \frac{1}{2}\|\bar{t} - \bar{s}\| \qquad (6)$$

Where $\bar{t}$ stands for the target vector, $\bar{s}$ for the predicted output vector of the fuzzy signature, and $\|\|$ denotes the 2-norm. Also, it will be assumed that there are m parameters to be learned and there are n records in the training data set, such that $n > m$.

Now, the next update of the LM can be written as:

$$\underline{u}[k] = \underline{par}[k] - \underline{par}[k-1] \qquad (7)$$

Where the vector $\underline{par}[k]$ contains all learning parameters, ie. all aggregation factors and weights factor, of WRAO in equation (3) and GOWA in equation (5) for the kth iteration. The LM defines next update $\underline{u}[k]$ in following manner:

$$(J^T[k]J[k] + \alpha I)\underline{u}[k] = -J^T[k]\underline{e}[k] \qquad (8)$$

Where J is the jacobian matrix of the equation (6), I is the identity matrix of J, and α is a regularisation parameter, which control both search direction and the magnitude of the next update $\underline{u}[k]$.

The LM algorithm uses the restricted step size method to find best quasi-optimal solution. The following algorithm has been given in [4] for calculating the next update $\underline{\alpha}[k]$ and using a trust region $r[k]$, which is given in equation (9), to achieve a good convergence:

I. Given $\underline{par}[k]$ and $\underline{\alpha}[k]$, use (8) to find $\underline{u}[k]$

II. Calculate $\underline{r}[k]$ use (9)
If $\underline{r}[k] < 0.25$ set $\underline{\alpha}[k+1] = 4\underline{\alpha}[k]$
If $\underline{r}[k] > 0.75$ set $\underline{\alpha}[k+1] = \frac{\underline{\alpha}[k]}{2}$
Otherwise $\underline{\alpha}[k+1] = \underline{\alpha}[k]$

III. If $\underline{r}[k] \leq 0$ set $\underline{par}[k+1] = \underline{par}[k]$
Else $\underline{par}[k+1] = \underline{par}[k] + \underline{u}[k]$

IV. Find new error $\underline{e}[k+1]$, use equation (6)
If $\underline{e}[k+1] > \textit{threshold}$ then go to I.
Else stop learning.

Where k is the current iteration number. The trust region $\underline{r}[k]$ calculation for above algorithm is given by:

$$\underline{r}[k] = \frac{f(\underline{par}[k-1]) - f(\underline{par}[k])}{f(\underline{par}[k-1]) - q(\underline{par}[k])} \qquad (9)$$

where approximation of the error, $q(\underline{par}[k]) = \{J^T[k]\underline{u}[k] + \underline{e}[k]\}$. Initially, the algorithm starts by choosing an arbitrary $\underline{\alpha}[k] > 0$ and arbitrary values for $\underline{par}[k]$.

3.2 Learning of WRAO for Fuzzy Signatures

In this subsection we explain the method of learning WRAO from real world data briefly, with more detailed explanations to be found in [9]. First, to avoid the first 2 constraints on the weighted relevance factor $w_{q\ldots ij}$ in definition 2, we replaced it by the following sigmoid function,

$$w_{q\ldots ij} = \frac{1}{1 + e^{-\lambda_{q\ldots ij}}} \qquad (10)$$

where $\lambda_{q\ldots ij} \in \mathbb{R}$. Now, the equation (3)can be modified as follows,

$$a_{q\ldots i} = \left[\frac{1}{n}\sum_{j=1}^{n}\left(s_{q\ldots ij} \bullet \left[\frac{1}{1 + e^{-\lambda_{q\ldots ij}}}\right]\right)^{p_{q\ldots i}}\right]^{\frac{1}{p_{q\ldots i}}} \qquad (11)$$

The $p_{q\ldots i}$ and $\lambda_{q\ldots ij}$ are called the aggregation factors of branch $q\ldots i$ and weighted relevance factor of sub branch $q\ldots ij$ of the fuzzy signature in figure 2, respectively. This form of WRAO (11) can be readily used for gradient based learning.

The parameters we need to learn are the aggregation factor $p_{q\ldots i}$ and weighted relevance factors $\lambda_{q\ldots ij}$ for each WRAO at each node of the fuzzy signature structure in figure 2. First we can obtain the partial derivatives of the equation (11) w.r.t. $p_{q\ldots i}$,

$$\frac{\partial a_{q\ldots i}}{\partial p_{q\ldots i}} = \left[\frac{a_{q\ldots i}^{1-p_{q\ldots i}}}{np_{q\ldots i}^{2}}\right]\left\{\sum_{j=1}^{n} t\ln(t) - nt^{'}\ln\left(t^{'}\right)\right\} \tag{12}$$

where $t = (a_{q\ldots ij}w_{q\ldots ij})^{p_{q\ldots i}}$ and $t^{'} = a_{q\ldots i}^{p_{q\ldots i}}$. Similarly, we obtain the patrial derivatives of the equation (11) w.r.t. $\lambda_{q\ldots ik}$

$$\frac{\partial a_{q\ldots i}}{\partial \lambda_{q\ldots ik}} = \left[\frac{1}{n}\sum_{j=1}^{n}(s_{q\ldots ij}\bullet w_{q\ldots ij})^{p_{q\ldots i}}\right]^{\frac{1}{p_{q\ldots i}}-1} \bullet T \tag{13}$$

where $w_{q\ldots ij} = \frac{1}{1+e^{-\lambda_{q\ldots ij}}}$ and $T = \left\{\frac{d\left(\left[s_{q\ldots ik}\bullet w_{q\ldots ik}\right]^{p_{q\ldots i}}\right)}{d\lambda_{q\ldots ik}}\right\}$.

3.3 Learning of GOWA for Fuzzy Signatures

Beliakov [1] has shown a way of learning weights in GOWA. We use our LM based method for learning [10, 9], of both weights and parameter p in GOWA operators. First, to avoid the 2 constraints on the weights $w_{q\ldots ij}$ in definition 4, we replaced it by the following function [3],

$$w_{q\ldots ik} = \frac{e^{\lambda_{q\ldots ik}}}{\sum_{j=1}^{n} e^{\lambda_{q\ldots ij}}} \tag{14}$$

where $\lambda_{q\ldots ij} \in [0,1]$ and $k \in [1,n]$. Now, the equation (5) can be modified as follows,

$$a_{q\ldots i} = \left(\sum_{j=1}^{n}\left[\frac{e^{\lambda_{q\ldots ij}}}{\sum_{k=1}^{n} e^{\lambda_{q\ldots ik}}}\right] b_{q\ldots ij}^{p_{q\ldots i}}\right)^{\frac{1}{p_{q\ldots i}}} \tag{15}$$

Now, the parameters need to be learnt are the $p_{q\ldots i}$ and $\lambda_{q\ldots ij}$ for each GOWA at each node of the fuzzy signature structure in figure 4. First we obtain the partial derivatives of the equation (5) w.r.t. $p_{q\ldots i}$,

$$\frac{\partial a_{q\ldots i}}{\partial p_{q\ldots i}} = \left[\frac{a_{q\ldots i}^{1-p_{q\ldots i}}}{p_{q\ldots i}^{2}}\right]\left\{\sum_{j=1}^{n} t - \left[t^{'}\ln(t^{'})\right]\right\} \tag{16}$$

where $t = [w_{q\ldots ij}b_{q\ldots ij}\ln(b_{q\ldots ij})]$ and $t^{'} = \sum_{j=1}^{n} w_{q\ldots ij}b_{q\ldots ij}$. Similarly, we obtain the partial derivatives of the equation (15) w.r.t. $\lambda_{q\ldots ik}$

$$\frac{\partial a_{q\ldots i}}{\partial \lambda_{q\ldots ik}} = \frac{t^{\left(\frac{1}{p_{q\ldots i}}-1\right)}}{p_{q\ldots i}}\left[w_{q\ldots ik}(b_{q\ldots ij}^{p_{q\ldots i}} - t)\right] \tag{17}$$

where $t = \sum_{j=1}^{n} w_{q\ldots ij}b_{q\ldots ij}^{p_{q\ldots i}}$.

4 Experiments: GOWA vs WRAO for Fuzzy Signatures

Now, we explain the results of 2 experiments to extract the weights in hieratically organised WRAO and GOWA from real world data.

4.1 High Salary Selection Fuzzy Signature

The High Salary Selection problem has been discussed in [5]. This problem is to find the degree of relevance of having a high salary based on *contacts*, *age*, and *work experience* of a employee. The figure 3 shows the polymorphic fuzzy signature, which is obtained using domain experts knowledge, for the high salary selection problem.

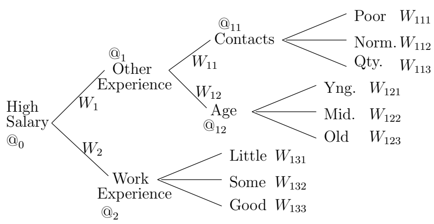

Figure 3: High Salary Selection Fuzzy Signature

Note that in figure 3, $@_i$ and w_i represent the aggregation function and weighted relevance of node i, respectively.

We learnt the weights and aggregations for WRAO and GOWA for each node in the High

Salary Selection fuzzy signature structure, in figure 3 using the method explain in section 3. Table 1 shows training and testing results (Mean Squared Error (MSE)) of the experiment for learning weights of WRAO and GOWA respectively.

Table 1: Results of High Salary Fuzzy Signature

	MSE Train	MSE Test
GOWA	0.1643	0.2014
WRAO	0.0130	0.0130

4.2 SARS Patient Classification Fuzzy Signature

Next experiments is SARS patient classification Fuzzy Signature. Medical practitioners know that for a certain disease, such as SARS, they need to check the patient for possible fever, hypertension, conditions of nausea, and abdominal pain. In addition, it is fairly important to monitor fever regularly during the day as well as blood pressure. Figure 4 shows a SARS polymorphic fuzzy signature, which is constructed based on domain expert knowledge. Each symptom check has been divided into a number of doctors diagnosis levels, such as *slight*, *moderate*, and *high* for body temperature (fever), *low*, *normal*, and *high* for both measurements of blood pressure, *slight*, *medium*, and *high* for nausea, and *slight*, and *high* for abdominal pain.

We learnt the weights for WRAO and GOWA for each node in the SARS patients classification Fuzzy Signature structure, in figure 4 using the method explain in section 3. Table 2 shows training and testing results (MSE) of the experiment for learning weights of WRAO and GOWA respectively.

Table 2: Results of SARS Patients Classification Fuzzy Signature

	MSE Train	MSE Test
GOWA	0.1608	0.1611
WRAO	0.0125	0.0079

According to the results of two experiments,

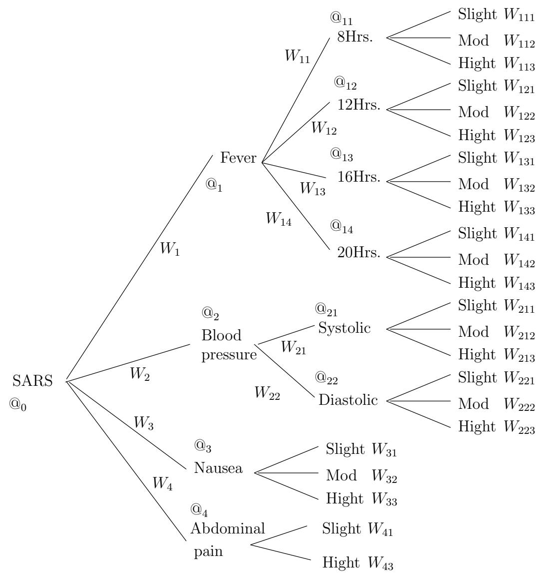

Figure 4: SARS Fuzzy Signature

it is clearly significant that WRAO outperform the GOWA for the aggregation of hierarchical Fuzzy Signatures. Our observation for the results of these two experiments is that Fuzzy Signatures inherently use fuzzyfied data to describe objects. The WRAO has been design to handle the redundancies exists with fuzzy data such that their weights express the relevance of fuzzy data to its parent node in a hierarchical structure and uses aggregation factor to find the optimal mean value based on the situation. Further, importantly, weights in WRAO are non-additive compared to that of GOWA. When we use GOWA in the hierarchical Fuzzy Signatures, GOWA needs to handle some redundancy in fuzzy data. But unlike WRAO, the nature of GOWA is to use both their weights and aggregation factor to find a mean type aggregation and thus GOWA leaning may fail to converge to a good solution. These experiments lead to new future research directions to compare GOWA and WRAO with hierarchically organised non-fuzzy data, and non-hierarchical organised (flat) non-fuzzy data.

5 Conclusion

Hierarchical organisation of GOWA and WRAO have been discussed. The Levenberg-

Marquardt Learning method has been used for the learning of both aggregation factor and weights of GOWA and WRAO for two real world problems. The two experiment, concluded that WRAO is better than GOWA for hierarchical Fuzzy Signature. Our assumption is that OWA is not suitable for the aggregation of fuzzyfied data in hierarchical Fuzzy Signatures.

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